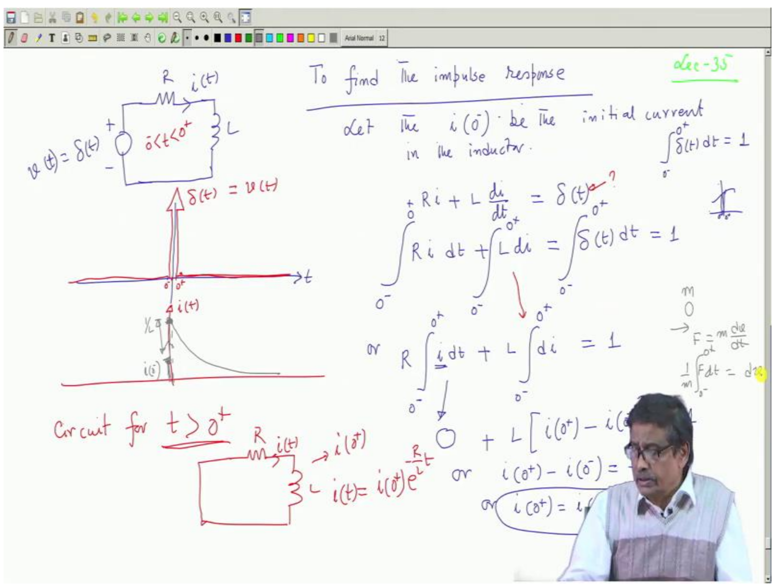
Network Analysis
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology-Kharagpur

Lecture # 35
Solution of Differential Equation
With Impulse Excitation

(Refer Slide Time: 00:26)



So, our lecture number is 35 and in this lecture I will tell you now we have made enough ground to find out the response say for example in RL circuit when it is excited with an impulse voltage x t or V t = delta t, this voltage source vt = delta t. We even need him an unit impulse function in the laboratory you cannot get that sinusoidal square wave does that you get but in it impulsive cannot get, but the important is will show later that if the circuit is linear and time invariant.

Then if you know the impulse response which you will do on pen and paper, then you can predict the performance of the circuit for any other arbitrary inputs signal that is why people studied this. So, we want to find out to find the impulse response so, to find the impulse response that is I want to find out what will be the current height and suppose late the initial current in the inductor was some value i 0. So, far we considered excitations to be reasonable functions and I told you i 0 + to i 0 -, but let us see what happens here. So, let the initial current be i 0 - be the initial current and why i 0 I am writing.

Let it be the initial current lead i 0 - be the initial current in the inductor in this circuit. The way p = 0 do you have impressed and impulse response that is what this is your time t less than 0 - current was 0 in the circuit or not 0 is 0 - something was flowing, because it might have inherited from some other circuits some value of the current at t = 0 - then suddenly connected these inductance and our across an impulse voltage source like this, so much. So, I have applied and impulse voltage here, like this.

So this is delta this is 0 -, this is 0 + this is equal to your vt. Now, I want to know what are the current waveform will look like that is what it will be. Now, the first thing is I will write down the differential equation KBL is to be satisfied. So, Ri + L di / dt I will write it as delta t is the voltage applied but as I told you delta t on the right hand side its amplitude is infinitely large. So, you cannot proceed further with this equation what to do it the right hand side of an equation is infinitely large.

But I know this much that 0 – to 0 + delta t dt = 0 = 1 that is what any unit impulse function is, this is finite. So beta i will transform this equation in this form. So to do this I multiply both sides by dt and this will be equal to delta t dt. This I can do and then I will integrate both sides from 0 - 20 + No why are integrated like that, because this quantity on the right side right hand side becomes finite, this is what has to be. Now, let us see what will be the integration of each of the start.

So, this 1 will be then our integration 0 – to 0 + i dt and this will be like this 0 – to 0 + di and the right hand side is equal to 1. Now, so far as the second term is concerned, this will be equal to this integral will be it only, and this will be i 0 + 2 i 0 - and this side it is equal to 1. Now the question is what will be the fate of this integral there will be some current flowing and integral of a function even it has got straight jump area under discard when dt is vanishingly small will be 0.

Once again I am repeating there will be some response of the current, it is expected we have applied something and I am telling this integral whatever will be that current waveform, you are trying to calculate idt as dt, becoming vanishingly small, the upper and lower integration limit is the idea and that this responsible the response of the current. Something I am not sure what it will be. But if any function you integrate between, 0 - to 0+ that area will become equal to 0 got the point.

So this this integral will become good and then I will say that i 0 + to i 0 - - this has to be equal to 1 / L or I will tell i 0 + will be equal to i 0 – + 1 / L. Mind you here is a case where i 0 + is not equal to i 0 - because, of the presence of an impulse as an excitation. i 0 + will be equal to i 0 - provided right hand side is not an impulse that we have learned earlier. But here in this case 0 – to 0 + is as small and interval as you can think of. So, current earlier it was it 0 - is 1 in no time that is that equal to 0 + it will become some value which will be equal to that initial current i 0 - + 1 / L.

Now, impulse function, some people say it is very simple function, some people say it is simple, but it is disturbing a t = 0, let it disturb at t = 1. But do you see for t greater than 0 + delta t does not exist So, this arcade impulse will exist this circuitry will be valid for t less than 0 - to less than 0 + whatever current you want to find out you can find out because voltage exists only during this time, but for t greater than 0 + for circuit for t greater than 0 + will look like what are RL circuit and these 0 terminals are shorted no voltage source existing this you must first understand this it.

Now will be an RL circuit with no excitation because the excitation ceases to exist for t greater than 0 + for t less than 0 - there was no voltage but these inducted in edited current from some others circuit etc. You have put it that is fine no problem this side but for t at t = 0 do you have applied and impulse means between 0 – to 0 + their existing infinitely long straight for pulse whose it is infinitely large that is what delta t is. That is why integral 0 – to 0 + delta t dt is equal to 1.

So, we started with open mind KVL is satisfied Ri + L di / dt is equal to delta t that is good but then I the question is this differential equation I cannot proceed for that simply because right hand side is in finite. But now I explored the property of impulse function and integrate both the sides in the limit 0 – to 0 + then the right hand side becomes finite a bit comfortable situation now, and the left hand side has got 2 terms, 1 term is this 1 ldi, di if you integrate, it will be either 0 + - i 0 - into L and this other term is the integral of the current response over this small time interval.

Even the current as a step change. The area under this card will be 0 because dt vanishingly small and you are integrating the value of this function will not matter delta t make as small as you think of. So, this integral will give you 0 and from this we will get i 0 + to i 0 -. In other words what I am telling thinking this way, the inductor had some initial current is i 0 - 1 from some other circuit suddenly this inductor is connected at t = 0 to this circuit.

This inductor has been brought in here after the narrating some current i 0 - 1 and suddenly you have connected that is you have impressed and in impulse in this article and then what does the impulse function to impulse function changes the current in no time? It was it 0 - suddenly the current has a step jump it is a constant number by a factor of 1 / L. So, it is it is like a magic 1 impulse touches these whatever was the initial current that changes to a i 0 - + 1 / L.

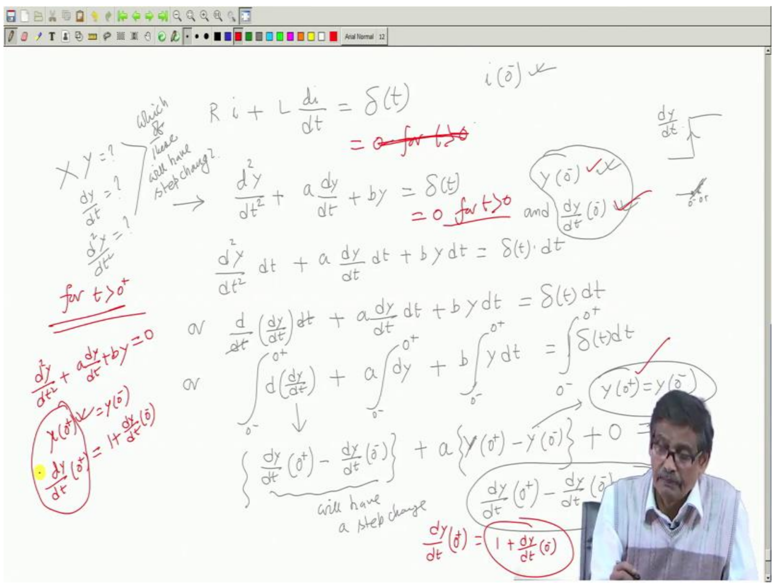
And for t greater than 0 + this is the circuit and this circuit has an initial current i 0 +. So, the to obtain the current for it for all time t greater than 0, I say that initial current if it was - 1 then in that interval 0 – to 0 + the currently have a step jump, 0 – to 0 + what will be the value of the current impulse does this 1 change? Is it 0 – to 0 + that what impulse does and after that impulse is not there in the circuit.

And therefore, this problem we have solid an inductor having some initial current short end. So, response of the current it which I am not solving now, will be that initial current into e to the power - R / L into t got the idea that his impulse will change the initial current in no time 0 - it was it will suddenly make it 0 + and for t greater than 0 + circuit is shorted impulse appeared and it vanished in no time.

This is the circuit for t equal to the 0 + for me t = 0 and t = 0 + and 1 and the same thing. So, what I will say while skating this circuit, it was some it 0 - 1 suppose, I will just show it like this try to understand, this, this is very interesting. This was i 0 - 1 inductor current. At this point this was the current is i 0 - 1 this was the current, this is as close to the origin as you can think of is 0 - impulse appears 0 – to 0 + what happens this current has escaped jump.

This current jumps to this value at t = 0 + what is the value of the current this i 0 - 1 + 1 / L, this will be 1 / L and this is i 0 - 1 this up to this i 0 - 1. So, at t = 0 + current will come here, the moment current comes here impulse then disappeared and it is a circuit like this, there will current will take an exponentially for t greater than 0 +. So, at t = 0 something happening, it is changing the initial current by a certain amount and then it decays down to 0 understood.

(Refer Slide Time: 18:01)



Then there is another way of interpreting this result that is our initial differential equation was like this L di / dt was equal to delta t is not this was the initial current. Now, in this equation the left hand side has to give me an impulse then only it can balance that impulse. Now the question is how that impulse can be created? Can the response I be delta t, suppose I am just arguing from common sense. It is a differential equation right hand side is delta t left hand side, what it will be? Maybe current as responsible of the current is also in the impulse who knows response of the current is impulse.

Then d dt of delta t delta dot t will also appear on the left hand side. So, how it can balance the right hand side therefore, step change of current has to take place, so, that di / dt becomes an impulse that is what it has happened the response of the current here response of the current has taken place in the previous here is this step jump from this to this my whatever amount that will be causing caused by this impulse that is what this equation tells me got the point.

So when a circuit is excited by an impulse it will be like this and the result is not surprising at all, we are told inductor current cannot be changed instantaneously that is fine with reasonable function that is what the natural things are but when an impulse voltage with an impulse voltage if you excite the circuit, which is having an inductance. We now come to know the total current can change also instantaneously almost between, 0 - to 0 + in no time in our school days, while studying mechanics we have learned the term impulse force.

For example, if there is a mass we say that force is equal to m dv / dt if the force is some non-functions reasonable function we say that velocity cannot change instantaneously is not velocity cannot change instantaneously that is the rule you apply be v 0 - is equal to v 0 + whatever velocity the mass was moving with that velocity cannot change instantaneously apply force velocity will start changing which time like that.

But if you apply an impulse force velocity will change instantaneously. Example I suppose some bodies with a bat and I am throwing a ball to him with some non-velocity, so, t = 0 - the velocity was v 0 - then he says 6 with all its might he hits the ball what will be the contact time between the bat and ball very little no time but velocity has changed from this 0 – to 0 + not only its magnitude to change but also introduction if you write in vector form that is the impulse is not.

So, in mechanics also velocity can change instantaneously in no time practically if the mass is subjected to an impulse force, what is the nature of the impulse force the force is very large contact time is very less, but F into delta t is finite that is a finite F into delta t if you bring it here by m is equal to dv. So, between F into delta t this quantity will be finite, so that a sudden change in velocity can take place anyway.

So, this is the thing, only thing I will tell you about this thing is that what happens if it is an RLC circuit second order system. So, suppose I will explain it like this suppose, you have a second order system d2y / dt +. So, in a faster system how many boundary condition you required i 0 -

must be known, then only you can solve the circuit. Now, let us consider a second order differential equation a dy / dt + by suppose it is excited by delta t it could be mechanical electrical whatever be the system now it is a second order differential equation natural response natural response I know how to solve it.

Now, if this is the forcing function solution do you do forcing function it will be how it will look like it will be you see this circuit it is a second order system, how many boundary condition I need I need to 2 boundary condition y 0 - i and I need dy / dt 0 - this value must be known then only I can completely solve the circuit. This is an impulse function of know well. So, what I will do I will fast multiply this with dt to make it a finite thing delta t.

So, multiply both sides by this thing dt 2 + a dy / dt + b y dt and they say will be delta t dt or this can be also written as ddt of dy / dt into dt + these things are. Let me write it clearly so that you understand. Repeat this, this will be y dt and this is equal to delta tdt this will be the thing. Now, this dt has not yet been multiplied. Everything I kept now, I have not dt shown some correct. Now, this thing or I will write it as d of this dt goes here d dy / dt + a dy + b y dt is equal to delta t any mistake point out, this is the thing.

Now, as I told you to make the right hand side some known finite things, I integrate both sides from 0 – to 0 + integrated B is constant so 0 – to 0 + and this is 0 – to 0 + integrate, what will be the integration of this term it will be d / dt at integration will be this thing only dx x, and it has to be evaluated at 0 + - d y / dt at 0 -, this is the faster this 1. This 1 will be a into 0 + - y 0 – this to this thing and this integration why dt 0 – to 0 + this will be 0, because it will be some even if y changes by step it does not matter dt is very small.

So functional value of y really does not change, only dt becoming vanishingly small, so insignificant 0 and they should be equal to 1. So, what are the things I know these 2 things y 0 - and y 0 + and dy / dt 0 - is not this will be the thing. 1 thing you see that, coming back to this equation, right inside delta t some impulse left hand side. Y dy / dt and d2y / dt 2 which of this quantity is it? y is it dy / dt is it d2y / dt2 to reach up this will have stepped in, we have stepped in we jumped this can why you have escaped change, 0 – to 0 +.

If it is step changes then dy / dt will be an impulse and d2y / dt2 to will be derivative of that impulse, but there is no delta dot t terms. So why it cannot be can do idt have a step changes in answer is yes, if dy / dt divided at value changes by a straight vertical line, then d2y / dt2 will give you that delta which is which will be perhaps balancing the right hand side delta. So, the question is this dy / dt to 0 + and dy / dt 0 - this term will have a step change. This term will not have a step change means it will be continuous?

Got the idea any variable as opposed it was doing like this, this is called step change. dy / dt will be something like that yt cannot have escaped change. So 0 - and this is 0 + it will be continuous. So, why cannot change so, y 0 + will be equal to y 0 - it cannot have a step change simply because if why step changes, then you have delta t as well as delta dot t, right hand side only delta t, how this can happen?

So, y 0 + has to be equal to y 0 - and therefore, you must type dy / dt very interesting - dy / dt 0 - this must be equal to 1 So, you have been provided without any boundary condition these 2 now, an impulse come to the second order system there was some y 0 – 1 y 0 + will not have a step change it will be a continuous thing. So, I will now calculate y 0 + but this dy / dt will have a step jump like this. So, dy / dt at 0 - i know so, I will calculate dy / dt 0 + I will be able to calculate will be equal to 1 + dy / dt 0 - these I will be able to calculate.

And for t greater this is what will happen between t = 0 – to 0 + these things will happen practically in no time and for t greater than 0 impulses not their right hand side is 0 this is t = 0 for t greater than 0. Therefore, the same as this 1 is equal to 0 for t greater than 0. So, you will have for t greater than 0 + say this will be the equation will be d2y / dt2 + a dy / dt + by is equal to 0 and the boundary conditioner i 0 + is known. I will start with this and not i dy 0 + is known y 0 + known is equal to y 0 - because this is given to me and I will calculate dy / dt at 0 + as 1 + dy / dt 0-.

So, system will be having no excitation now for t greater than 0 +, but these are the new boundary conditions, apply them and solve. Right hand side you will be 0 hope you are following me will solve some numerical example later that is a pulse whenever it comes, 1 of the quantities of the differential equation will have an abrupt change step change because step change of a signal if differentiated gives rise to an impulse that is what we have established earlier anyway, Thank you.