Behavioral and Personal Finance
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Lecture # 36
Numerical Example when Excitation is Impulse

Welcome to lecture number 36. And we have been discussing how to solve differential equation in time domain when the excitation function is an impulse function. Today I will highlight remember that to show this.

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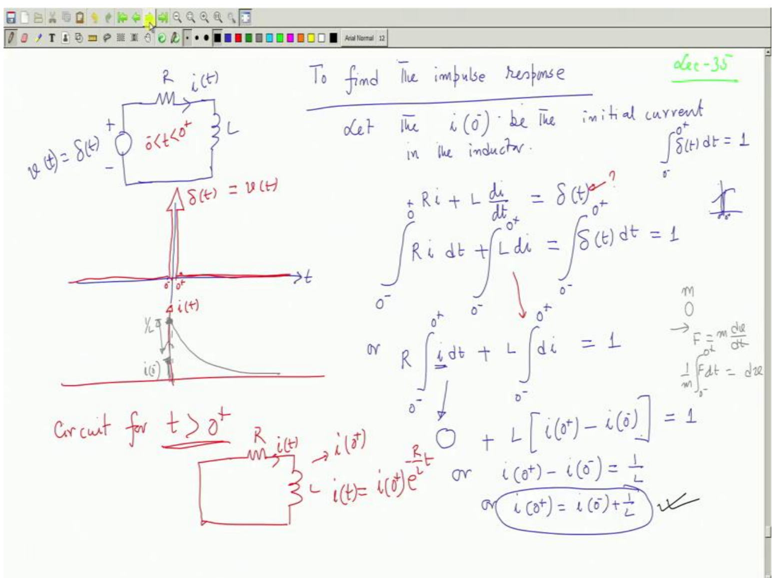

For example and RL circuit is there is an impulse applied, what happens and we want to know the current for time t greater than 0 we want to solve the current. Now for t greater than 0 + there is no excitation impulse came at equal to around equal to 0 and then also vanish. So, after that the circuit will be like this here. Now, the question is certain circuit can have time current provided there was some initial current, if there was initial current i 0 - which will be known to me. So, what impulse does it change is that initial current at I 0 + following this relationship got the point. So, this is the point now, let me solve.

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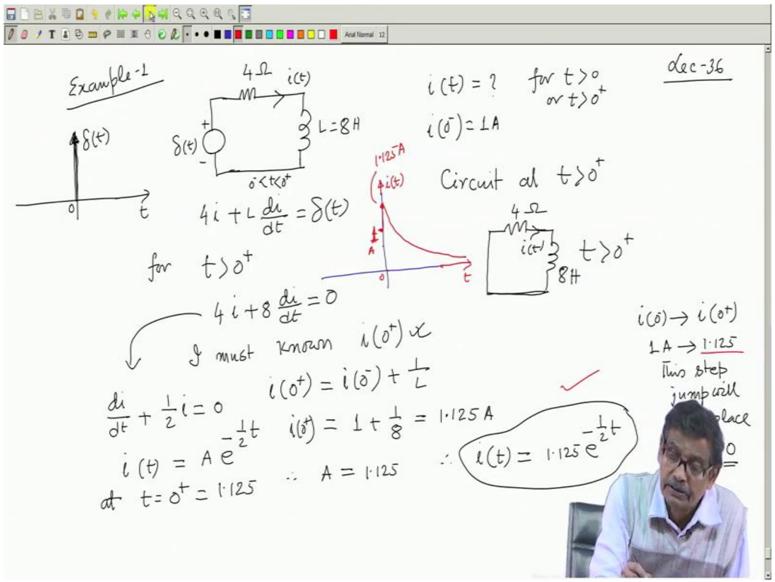

Supposed example 1 suppose we have a circuit where r = 4 ohm and L = suppose 8 henry and I will excite this with an impulse I will excited with an impulse function Delta t that is what I will be doing unit impulse function. See problem at hand is this 1, what will be current it and impulses sitting at the origin. So i t will be equal to how much for the greater than 0 or t greater than 0 + for me there is no difference between t = 0 and t = 0 +.

So, this is the circuit i 1 to solve. So, the equation of the circuit will be r i + L di dt = delta t. And suppose, it is given that I 0 - = suppose 1 it is given inductor inherited this current because of some previous operations from some other circuit then suddenly it is connected here. So, I 0 - is 1 MPI. So, this delta t exist at between 0 - 2 t 2 0 + this fellow only exist for t less than you do it was not there I 0 - was 1 ampere and for t greater than 0 + now, the circuit at t greater than 0 + will look like this for an inductor L.

And there is no input now, it is shorted delta t because delta t looks like this is delta t and this time and this is t 0 t = 0 something happens, what happens is this 1 now to so, for t greater than 0 + what will be the equation I will state right this 4 + L di dt and that I am allowed to write it to be 0 because now this is for t greater than the + this is a 10 and this is 4 ampere this situation and this current I want to find out it.

Now, to solve this differential equation, I should know the initial condition, what is the initial condition I must know i 0 + x to solve this differential equation, how to know i 0 + we have seen that I 0 + will be equal to i 0 - + 1 by L is not that is what we have seen. Now i 0 - is 1 ampere. So, 1 + 1 by 8, 1 by 8 is point 125 so, 1.125 ampere therefore, here is another circuit which has got an initial current 1.125 ampere.

So, how it will decay this let me write it as a temporary and then we know how to solve this equation is nothing but di dt divide by 8 + half by = 0. So, I can straight away write i t will be = A into e to the power – half t and then I will apply this boundary condition. So, at t = 0 + the current is 1.125 therefore, the value of a will be 1.125. So, the solution will be 1.125 into the power - that is the thing.

That is the impulse at = 0 will change the boundary condition i 0 - which was existing just prior to applying the impulse that initial current will suddenly jump from i 0 - to i 0 + that is from 1 ampere to 1.125 ampere and this sudden change will take place take jump off I will take place at = 0 this step jump will take place at t = 0 from t = 0 - 2 0 +, that is all. Therefore, the solution of this problem will be, if you escape this current waveform, I should draw it like this t = 0 - the current was 1 ampere.

Say this is 1 ampere here It was there this is equal to 0, this is t at t = 0 + this current shoots up to 1.125. So, in no time e was there, this is 1 ampere it now goes to 1.125 ampere and then of course, it will decay like this decay of current, this axis is your it for me it does not matter whether it is 0 + 0 what is the point so, this will be the complete solution of this 1. Let us do it for the second problem.

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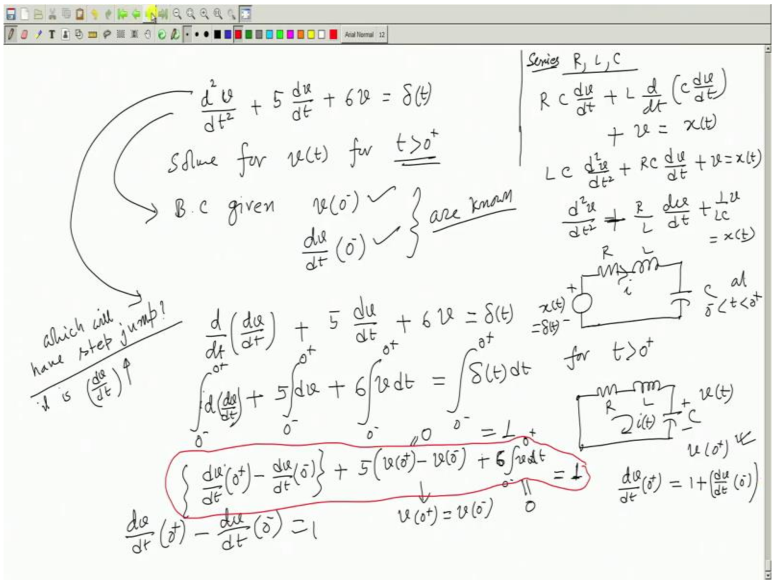

Second problem is like this suppose you have differential equation say d2 I d2 v say capacitor voltage something like this need to be d 2 v + 5 dv dt + 6 v. And suppose this circuit is energized with an impulse function it is looking at the equation it looks like some models is arcade voltage across the capacitor. Recall that in an RLC circuit differential equation will be R i and R i is dv dt at just am writing this R i + L d dt of i and that is c dv dt c dv dt.

And + of v and that is the left hand side and any excitation x t. Now, if this 1 you write it will be LC is d 2 v dt 2 + Rc is dv dt and + v = x t theory called this and then you write it like this d 2 dv t 2 = R v + R by L dv dt t + 1 by LC into v = your ex team for any excitation to the series RLC circuit series. So, suppose this equation you have said the value of RL and C such that for this circuit by RL is 5 and 1 by LC 6 like that.

So, anyway this is the voltage across the capacitor. So, this was the circuit See, I want to know the voltage across the capacitor here is your x t we have done so many times this is R this is a dv dt. So, the values of RLC are such that R by L is 5 and 1 by LC 6 and on the right hand side you have the excitation whatever it is this is now. Now in such a situation I must know what a what the initial conditions I am asking. So what is the problem at hand problem and hand will be solve for V t for t greater than 0 +, that way I will be solving.

So, and what are the boundary conditions given there must be 2 boundary condition 1 is v 0 - is given and another is dv dt at 0 - is given. These 2 boundary conditions unknown are given to you Now for t greater than So, x t if it is equal to delta t for t greater than 0 + the circuit looks like impulse does not exist at that time and the circuits will look like this. These the x t is 0 impulse only exists this is the circuit at t = 0 - between t = 0 - 2 0 + that is what because delta t only makes its presence felt during 0 - 2 0 +.

So, for t = 0 + it is 1 second 0. Now, there were some initial conditions given to me and I have to I will do it because this is secondary question. I will do it from once again fundamental because I have forward now these are really Anyway, see, it will also tell you what to do, because delta t is there, so multiply both sides with delta t. So, this was this. So, this was actually d dt of dv dt is not + it was 5 dv dt and it was 6 V and that is = delta t. It was there. Now what I will do, I will multiply both sides by dt. So that it will become dv dt it will become 5 dv and it will become 6 vdt and that will be = delta t dt.

Then will integrate both sides from 0 - to 0 + this 1 0 - 2 0 + and this 1 from 0 - 2 0 + and this 1 from 0 - 2 kilo + and right hand side will become equal to 1 root O + Oh there you are correct dt I have multiplied, so, this dt will go, this will be equal to d of dv dt is not it. So, I will clean it right cleanly. So, this term will become d of dv dt this is the thing. Now if you integrate this first term it will become dv dt only. And then upper limit is 0 + So, this must be at 0 + this value – dv dt at 0 -, this will be the first term. Second term will be 5 V 0 + - V 0 - this will be the second term.

And is not it and + this integral and V dt and this = 1. This will be the situation. Now I told you last time in a second order situation like this right hand side delta T which thing which of these that is dv dt V are dv dt will have a step jump we cannot have step jump if we have a step jump, then this will be delta t dv dt if we have a step jump, then dv dt must have a delta and d2v dt 2 will be delta .t but there is no Delta dot t on the right hand side to balance it off.

Therefore, only quantity in this expression will have which will have stepped jump by step jump will only we will have in dv dt will have a step jump because if dv dt has a step jump then this fellow will produce an impulse. So, that left hand side right hand side has 1 impulse only, no Delta .t nothing like that delta t. Therefore, this was the idea to understand what is going in. So, which will have step jump it is dv dt will have a step jump we will not have step jump then this integral v dt over 0 - 2 0 + this 0 + 0 - as small dt becomes so small.

So did this will be eventually 0, I mean no point V cannot have escaped I am therefore we must be continuous. So, this tells me that we 0 + must be equal to be 0 - since it has it is not a step jump case. However, dv dt 0 + will have a step jump from 1 value to the other got the point Therefore, for t = 0 + it is a second order system RLC I want to solve for these vt and if I know vt I can differentiate it to get the current dv dt.

So, this current i t i an interested to solve, I will then say it is a second order differential equation and no excitation maybe capacitor will have some initial voltage do I know the value yes I know v 0 + how much it is, it is same as V 0 - this condition is known, then another condition is necessary, what is that condition that is dv dt at 0 +. So, other condition necessary is dv dt at 0 + Do I know this value dv dt at 0 - is known. So, dv dt at 0 + will be how much this side is 1.

So this must have got the so dv dt at 0 + will be equal to 1 + dv dt x 0 - let me be consistent so dv dt at 0 -how do I get this because this side V 0 + is V 0 - this is 0, this is also 0. So this term will be also 0. So, this 1 so, dv dt at 0 + - dv dt 0 - must be = 1 from this equation from this equation. So, from this equation dv dt at 0 + is known. Therefore, now I have a problem where there is no excitation and the initial conditions are known. And I can solve so, t = 0 +.

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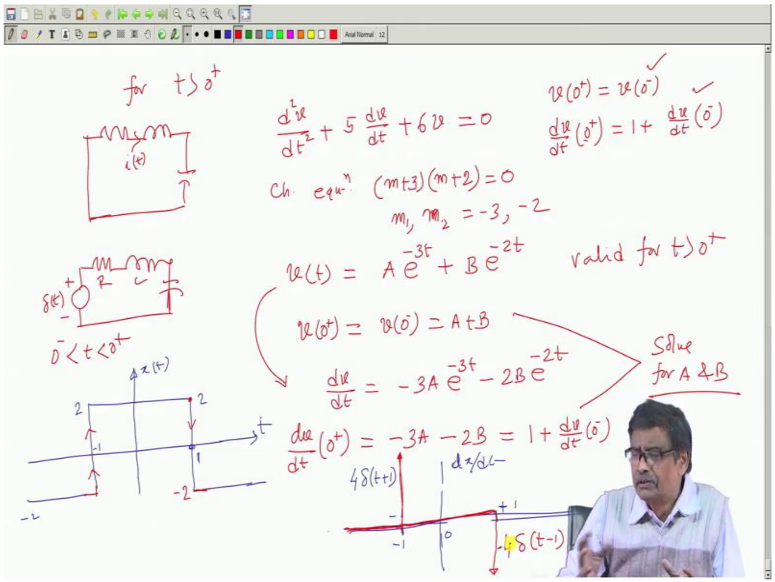

For t greater than 0 +, they circuit was evaluated shorted what was the equation d 2 V dt 2 + 5 dv dt + 6 V and that = 0 this is the solution. So, and what are the boundary condition is v 0 + is known, which = V 0 - and the boundary condition because 2 boundary conditions are needed at 0 + will be = 1 + dv dt at 0 - these were given to me So, I will be able to calculate this to then the characteristic equation. In this case it will be m + 3 stepping quickly writing this has to will be the characteristic equation roots are not equal for this particular problem m 1 m 2 will be = - 3 and - 2 this will be the roots.

And Vt you can write it as A into e to the power - 3 t + B into e to the power - 2 t valid this equation is valid for t greater than 0 + no excitation and then apply these boundary conditions that is v 0 + is v 0 - so, put that so, you right V 0 + whatever is known now, will be equal to happens to be V 0 - = A + B this will be 1 equation and the other equation will be differentiate this that is dv dt at you differentiate and this will be - 3 Ae to the power - 3 t - 2 B e to the power - 2 t this will be dv dt and then you say dv dt at 0 + it will be equal to – 3A e = 0 0 + and t e to the power - 3 0 + is also 1.

So, - 3 v – 2 B and this value is known to be because I am giving dv to this 1 it = 1 + this value which is known to me. So, there are now these 2 equations solve them Solve for A and B we cannot doing you can just try solve them and then come back here then say. So, they the important point is if a differential equation has impulse as its input signal x equal to delta t, then do not apply that rule that is the final solution will be equal to some natural response + linear combination of the forcing function known.

That way it will not, you will not proceed in fact, whatever was derived, we assume that it excludes that impulse function but anyway event impulse function is there, I have now told you how to carry on the calculations to get the response current i t if the it is excited by an impulse and what happens is this impulse only exists around t = 0 and in the rest of the time it is not there. So, for me to get the solution of this arcade for t greater than 0 + is good enough because I do not find any difference for practical purposes between 0 and 0 + what is there.

So, solve for i t, t greater than 0 + circuit is 1 second like this. So, and the circuit was like this source was a really active only during this brief period for t 0 - 2 0 + 8 well that had 0 - as usual we knew the boundary conditions, then I mean capacitance is also there. So, I have to recalculate the initial conditions with physical arguments that is which quantity will have a step jump, because, you know, in any situations for example, that that is what I told for example. So, the conclusion will be that if you have any waveform For example, you have a voltage waveform like this.

This is the suppose - 1, this level is - 1, this is a suppose 2 + 2, this is also 2 this is also + 1 signal goes like this is - 2 what I am telling this is some signal x t it was – 2 for t less than - 1, this point is - 1 for any negative value of time this function had a constant value - 2. Similarly between - 1 and + 1 between these 2 points it has got a + 2 and for time greater than 1 this axis is time we should not forget to mention that and then it will have - 2 values for t greater than + 1.

Now, such a function if I differentiate t if you see the differentiation of the Vst of this signal if you differentiate up to - 1, this is + 1 before - 1 it was constant - 2 so, differentiation will be 0. Now at equal to - 1 I find the signal jumps from - 1 to + 1 in no time this change discontinuity - 1 to + 1. Therefore it has jumped by a step of 3 from - 2 - 1 3. Therefore, we must expect there will be an impulse sitting here dx dt must have an impulse here.

Similarly, between - 1 to + 1 valve is constant this dt will be 0 and then at t = 1 another jump takes place of xt from where + ohm I did a mistake Please forgive me. This was - 1. So, this is also - 1 suppose this level it could be anything but anyway let me write correctly. So that it is. So, this was also this level was - 1 this level. So, - 1 to ah - 2 I have mentioned, on there was - 1 that is very confusing. So, this is also this - 1 I should remove that is the, so, this level was - 2 this is also at - 2. So, it is - 2 this level.

So, here I saw it was actually - 2 to + 2. So, the jump has taken place 4 time So, here will be an impulse whose strength will be 4 and this impulse will have 4 Delta 4 this impulse will have a strength 4 of delta and it is shifted delta function + 1, it is not sitting at origin sitting at = - 1 similarly here there will be another impulse but this time it comes down from + 2 to - 2 in no time. So, it is derivative must give you another impulse whose strength is 4 - 4 is not, final - 2 - 2 - 4 and delta of t - 1 so, shifted delta functions we will get. Anyway so, this you keep in your mind therefore, we have solved now, e are well equipped to solve differential equations.

Whether it is a reasonable function x t excitation R it is any impulse present on the right hand side, I will be able to solve for the current Plassey self. In previous lectures, I told you I have only digressed a bit at that time telling that you see most of the times functions will be reasonable type like sinusoidal, then to find out the solution due to sinusoidal functions, we adopted further approach to get the solution at least solution due to forcing function, anyway, next class I will start the self and material inductances which are very common in electrical circuit. Thank you.