

Behavioral and Personal Finance
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Lecture # 37
Self and Mutual Inductance

Welcome to my next lecture on network analysis, lecture number 37. So, far we have seen that there are three basic elements resistance inductance and capacitance is, but there will be another, not a new element but there may be a coupled coils present in a network and a coupled coils or they are mutually coupled coils. Then apart from self-inductances of the coils, another term appears which is called mutual inductance.

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Self & Mutual Inductances
: Coupled coils

$\Phi_{t1} = \text{total flux created by coil 1 when it carries } I_1.$

$\Phi_{t1} = \Phi_{m1} + \Phi_{k1}$ → leakage flux

↑
mutual flux in the core

$\Phi_{t1} = \bar{K}_1 \Phi_{m1}$
 \bar{K}_1 is small

Self inductance of coil 1
 $L_1 = \frac{N_1 \Phi_{t1}}{I_1}$

$= \frac{N_1 \Phi_{m1} + N_1 \Phi_{k1}}{I_1}$

$L_1 = \frac{N_1 \Phi_{m1}}{I_1} + \frac{N_1 \Phi_{k1}}{I_1} = L_{m1} + L_{k1}$ → magnitude of inductance → leakage inductance (small)

$L_{m1} \gg L_{k1}$

$L_1 = L_m + L_k$

Today I will just define very basic things about that and also try to calculate the energy stored in a couple of coils. Recall that if you have a coil, if you have a magnetic circuit like this suppose it is magnetic material and if you have a coil with N_1 times and if you pass some current through the coil, say constant value of current, then what happens is this, the current flows like this. So, in this way if you draw it means we are showing correctly the sense of the winding.

Now, when current flows through a coil in this fashion, suppose, current flows in a loop like this, then there will be flux produced. Then direction of which will be indicated by the thumb.

Therefore, here it will be, if current flows at any instant of time in this direction some flux will be created in the core and this flux is created by this current I_1 this is coil number 1 and it will create a flux ϕ_1 .

Now, this flux will be this flux is called the total flux created by the coil. So, ϕ_{t1} is the total flux created by coil 1, when it carries I_1 , and number of turns of this coil is N_1 and we know how to calculate ϕ_{t1} this ϕ_{t1} will have 2 parts, 1 part ϕ_{t1} we can be written as ϕ_{t1} can be written as $\phi_{m1} + \phi_{l1}$ so ϕ_{t1} is the total flux created it will have 2 parts 1 is called ϕ_{m1} and there will be another part which will not go to the which will be linking the coil alone it will not flow through the as a mutual flux.

So, this 1 is called mutual flux in the core, so, this total flux is 1 has 1 component, we have written ϕ_{m1} , 1 corresponding to this coil 1 and this is called Mutual flux it will be in the core and this part is called leakage flux we will be completing their paths through a part of the core through the first coil and through the air. Then it is not in the core, but nonetheless total flux created by the coil number 1 when it carries current I_1 is this 1 and what is self-inductance.

By definition self-inductance have coil 1 will be is defined as flux linkage per ampere, so, flux is total flux that is $N_1 \phi_{t1}$ total flux linkage when you pass 1 ampere current that is you divide by 1, this is what is self-inductance all about clear. And this can be also written as N_1 if you put this $\phi_{m1} + N_1 \phi_{l1}$ divided by I_1 and it can be written as 2 components that is $N_1 \phi_{m1} / I_1 + N_1 \phi_{l1} / I_1$ mind you this leakage flux will be a little fraction of the total flux created.

For example, ϕ_{l1} can be expressed as some say k_1 bar into ϕ_{t1} a little portion, if the coupling if all the flux can be 95, 98% of the total flux created will be confined to the core maybe 2% 3% is leaking. So, k_1 takes care of that anyway So, this is the thing so, self-inductance of a coil of course, to calculate self-inductance total flux linkage is to be taken because that is the definition of inductance of a coil sometimes what is called it this inductance is called the magnetizing inductance L_{m1} on and this is called the leakage inductance L_{l1} .

So, this term is called magnetizing inductance and this is called leakage inductance. So, total inductance self-inductance of a coil has got 2 components in general a component which is contributed by mutual flux, this ϕ_{m1} is mutual flux and the another component is leakage flux and these inductance will be pretty small K_1 is small number is small. Small means few percentage of the total flux created, but anyway this is the thing. So, self-inductance of the coil is L_{m1} and L_{l1} , L_{m1} is much larger than L_{l1} that also we can like L_{m1} is much larger than these. So, this is this thing.

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Mutual inductance
 $M_{21} = \frac{\text{flux linkage with the 2nd coil}}{I_1}$

$$M_{21} = \frac{N_2 \phi_{m1}}{I_1} = \frac{N_2}{I_1} \frac{N_1 I_1}{R}$$

$$M_{21} = \frac{N_1 N_2}{R} = M_{12}$$

Note
 $H_1 = \frac{N_1 I_1}{l}$
 $B_{m1} = \mu_0 \mu_r \frac{N_1 I_1}{l}$
 $\phi_{m1} = B_{m1} A = A \mu_0 \mu_r \frac{N_1 I_1}{l}$
 $\phi_{m1} = \frac{N_1 I_1}{R}$

2nd coil carrying no current

Now, the, inductance of a coil so, henceforth I will not draw the magnetic circuit, I will simply draw this 1 and I will say that the inductance of the coil is L_1 and it is suppose getting a current and I which may be time ready . If it is a time varying current then we know that there will be induced voltage in the current $L_1 di_1 / dt$ we have done so many currents and since inductances a can store energy sometimes we will represent these also as a source with this polarity of the 1 and mind you L_1 consists of those 2 component of fluxes while calculating L_1 .

So, this is also because inductor inductance of capacitors can store energy they may be also represented like this without any loss of t and depending upon the value of di_1 / dt , sometimes this inductance is going to store energy sometimes it will come back energy to the supply here there is some sources connected and so on. So, anyway that is the case. So, this is how we

presenting. Now, I will draw the same diagram, this part is very important you know trying to understand these your magnetic circuit.

This is your coil number 1 within 1 turns and in carries a current of I_1 and suppose there is another coil like this having into turns and suppose this second coil. This is the second coil, I am not passing any current, no current getting no current only first coil get the current. And if first coil carries current we have seen that it will produce a liquid flux here. Φ_{11} this indicates who has produced this flux coil 1 current I_1 has produced that is where you want and there will be flux mutual flux like this, which is fine.

And if it is DC current there will be no induced voltage here because this Φ_{11} and current I_1 will have constant value by applying this rule right and rule it will have constant value and nothing is going to happen. Now, meeting mutual inductance between these 2 coils is defined as mutual inductance M_{21} that is how I will write the second 1 is that coil 1 is getting current, what is the flux linkage with the second coil when 1 ampere flows in the first coil.

So for either 1 whatever is the flux linkage, so for 1 ampere what is the flux linkage per ampere or the second coil. Now, obviously you can see it is the mutual flux which is going to link the Φ_{11} will only link the first coil, so I should write it then flux linkage with the second coil will be $N_2 \Phi_{m1}$ divided by I_1 that will be the thing is not M_{21} . No, this mutual flux can be calculated note that this coil is carrying a current of value what is the mmf which has produced this 1 it is $N_1 I_1$ want is not that is what has produced this mutual flux I am interested in mutual flux Crossley to know how much will be the flux linkage per unit can be here.

And if I_1 ampere is here what will be the flux linkage there. So, note that Φ_{m1} how to calculate Φ_{m1} anyway I will doing like this. Note that H_1 produced by first coil $N_1 I_1$ divided by L . What is L , L is the length of this flux spot, average length and length of the flux but we know this H_1 . How much will be B_{m1} it will be $\mu_0 \mu_r$ into $N_1 I_1$ by l this L is the flux of the length of the mutual flux part.

So, this will be this what will be ϕ_{m1} will be by B_{m1} multiplied by the cross sectional area suppose it is uniformly cross sectional area. So, it will be B_{m1} into A and let us assume linear magnetic circuit are constant at least you have constant inductance I must say that it is operating in the linear so, B_{m1} into A is cross sectional area of the core. So, it will then be equal to $\mu_0 \mu_r N_1 I_1$ into A substituting the value of B_{m1} . So, in $N_1 I_1$ divided by l , this will be the area value strength of the mutual flux.

Then by definition M_{21} will be equal to N_2 and mind you this quantity $N_1 I_1 / l$ is called the mmf and bring everything down. It will be $1 / \mu_0 \mu_r$ by l / A I can bring it there this quantity is called the reluctance $\phi_{m1} = N_1 I_1$ divided by called this quantity is called reluctance constant it depends on the core only core physical dimension and cold properties means you did not mean μ_1 meanwhile, so as the cross sectional area.

So this $1 / \mu_0 \mu_r$ is nothing but into NR / l and ϕ_{m1} I can write it as $M_{11} I_1$ by R reluctance of the magnetics is it. So, M_{21} will simply become equal to anyone into productive number of turns divided by the law that is all into 1 just noted now, the way I have defined mutual inductance it is in this fashion in the same way I could energize only coil to and define the self-inductance of coil to and the mutual inductance caused by coil to let us do that quickly but effectively

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Now excite the 2nd coil alone with I_2

Diagram: A rectangular magnetic core with two windings, N_1 and N_2 . Current I_2 flows through winding N_2 . Flux ϕ_{m2} is shown circulating in the core.

Equations:

$$\phi_{I_2} = \phi_{m2} + \phi_{l2}$$

$$L_2 = \frac{N_2 \phi_{I_2}}{I_2} = \frac{N_2 \phi_{m2}}{I_2} + \frac{N_2 \phi_{l2}}{I_2}$$

$$L_2 = L_{m2} + L_{l2} \quad L_{m2} \gg L_{l2}$$

$\phi_{m2} = B_{m2} A$
 $= \mu_0 \mu_r H A$
 $= \mu_0 \mu_r \frac{N_2 I_2}{l} A$

$M_{12} = \frac{N_1 \phi_{m2}}{I_2}$
 $= \frac{N_1}{I_2} \times \mu_0 \mu_r \frac{N_2 I_2}{l} A$
 $= \frac{N_1 N_2}{l} \frac{\mu_0 \mu_r A}{l} = \frac{N_1 N_2}{\left(\frac{l}{\mu_0 \mu_r A}\right)}$ ← Reluctance

$M_{12} = \frac{N_1 N_2}{\mathcal{R}} = M_{21} = M$

M = the mutual inductance

So, in the second case now excite the 2nd coil alone with some current say I_2 . So let us draw it quickly. So this was the thing. This is the core and there were 2 coils 1 coil was there with in 1 turns but this time I am not going to excite these, this coil and the second coil was there we tend to turn and what I am telling is I am exciting it with some current tied to some source I have connected and current will be flowing like this it is not.

Now if current flows like this, then flux will be produced once again this will produce a flux which will have 2 components 1 is ϕ_{m2} and another flux component will be there which will be leakage flux by ϕ_{l2} . So, when second coil get this current, if ϕ_{t1} ϕ_{t2} I will say total flux created by the second coil will be in the same way which will flux which is going to link the first coil + leakage flux. What these 2 indicates second coil is only getting and this will be the distinct then self-inductance of the second coil will be flux linkage with the second coil.

When the second coil carries 1 ampere current that is per linear current and that per linear current is right I_2 I write it like that and these 2 will have them to component into $N_2 \phi_{m2} + \phi_{l2}$ and this total self-inductance of the second coil can be written as some magnetizing inductance of the second coil caused by mutual flux + the leakage flux of the second coil and L_{m2} will be much higher than L_{l2} to for a coupled coil.

So, this will be L_2 when you have energized the second coil flux is produced here and it is only the mutual flux I am to which will link the first coil then I will say that the mutual inductance of the first coil when second coil carries current I will write him on to and this will be equal to M_{12} mutual flux value is ϕ_{m2} and it is linking in N_1 number of times. So, in $N_1 \phi_{m2}$ by I_2 ampere current in the second coil that is it that that will be the mutual inductance of the second coil way first coil when the second coil get this current.

Here I have not assumed any current carried by the first coil so, this will be the situation. Now, in the same way what will be here I will right straight away in N_1 this let me do also. So, so, how to calculate ϕ_{m1} to you see ϕ_{m2} to will be equal to B_{m2} into cross sectional area, what is A is this cross sectional area of the core. So, B_{m2} is how much it will be called to $\mu_0 \mu_r$ into H

into A. What is H is I/l is the length of the flux remain same. So, $\mu_0 \mu_r$ what is the mmf into $H_2 I_2 / l$ is the unit ampere that is A this will be the thing.

So, ϕ_{m2} to is this put this value of N_1 / I_2 is here into $\mu_0 \mu_r$ into $N_2 I_2 / l$ into A this I_2 goes and you will be getting $N_1 N_2$ into $\mu_0 \mu_r$ into A a divided by l which is nothing but $N_1 N_2$ divided by one over $\mu_0 \mu_r$ by l / A and this quantity we have seen it depends on the physical dimension of the core and the properties of the core same material. So, this is called this is the reluctance this term is reluctance same for the mutual flux either you excite this side or that side. Therefore, M_{12} will be equal to $N_1 N_2$ by R reluctance and in the previous case when you excited on the first coil then also mutual inductance M_{21} was $N_1 N_2$ by R .

So, here also I will say M_{21} is $N_1 N_2 / R$ but these we have seen is same as M_{12} . Therefore, the mutual inductance either you calculate by exciting the primary 1 coil first coil, calculate mutual inductance whatever you will get, if you excite the second coil then also you will get the same mutual inductance got the point. So, M_{21} is equal to M_{12} was there any inconsistency in the suffix of a M_{12} to what I told here, this was there, then this is M_{21} indicating flux linkage there then second coil is current.

So, a 1 2 second coil is the current came on to and we find that this is equal to M_{21} and therefore, instead of writing an M_{12} I now define him by a single variable. So, I say that m is equal to be mutual inductance which is sim whether you calculate by exciting first coil see the flux linkage in the second coil or the excite the second coil calculate the flux linkage in the first coil. Therefore, from this we will further carry on in our discussion of mutually coupled coils, thank you.