

Behavioral and Personal Finance
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Lecture # 39
Mutually Coupled Coils in Series and Parallel

So, welcome to lecture number 39 and we have been considering mutually coupled coils there may be 2 or more coupled coils having their respective self-inductances and mutual inductances between them and it is essential that you must show the polarities instantaneous you must convey to that diagram, the instantaneous polarities of the induced voltages in the respective coils with the help of some dot convention, and I told you that.

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The slide contains the following content:

- Top Diagram:** A circuit with a voltage source $V(t)$ connected in series with two coils. The first coil has terminals A and B, self-inductance L_1 , and a dot at terminal A. The second coil has terminals C and D, self-inductance L_2 , and a dot at terminal C. Mutual inductance M is indicated between the two coils.
- Right Diagram:** A single coil with inductance L_1 and current i_1 entering terminal A. It is shown to be equivalent to a voltage source $L_1 \frac{di_1}{dt}$ with the positive terminal at A.
- Equation 1:**

$$V(t) = \left(L_1 \frac{di}{dt} + M \frac{di}{dt} \right) + \left(L_2 \frac{di}{dt} + M \frac{di}{dt} \right)$$
- Equation 2:**

$$= (L_1 + L_2 + 2M) \frac{di}{dt}$$
- Bottom Diagram:** An equivalent circuit with a voltage source $V(t)$ and current i flowing through four voltage drops: $L_1 \frac{di}{dt}$, $M \frac{di}{dt}$, $L_2 \frac{di}{dt}$, and $M \frac{di}{dt}$.
- Equation 3 (circled):**

$$V(t) = (L_1 + L_2 + 2M) \frac{di}{dt}$$
- Equation 4 (circled):**

$$\therefore L_{eq} = L_1 + L_2 + 2M$$

If suppose and henceforth I will draw coils already mentally does not matter in my mind I know that these 2 coils are ruled on some magnetic circuit like that and if you show this is taught it is sufficient so far as to find out right down the KVL where mutually coupled coils will be present to this much information I must expect so, that I will be able to write down the equations correctly.

Now, in general what will happen both the L_1 and L_2 will carry current. They will be connected to respective supply voltage and recall that if you have a single inductance I told you that this is i

1 mutual coupling with other coils this can be thought of a voltage source, if i_1 is increasing and $\frac{d i_1}{dt}$ is positive, this will be the polarity of the induced voltage $i_1 \frac{d i_1}{dt}$ like that.

Now, suppose let us say let us take a concrete example, suppose these 2 coils I have connected and the terminals of these 2 coils is this is A B and this is a CD and these 2 coils suppose they are connected in cities like this so that it will stem then your concept of convention and here you have connected some Sources alternating sources like this + - and that is causing some current to flow in the circuit say $i(t)$ and these 2 coils are connected in cities, then I want to find out the current response of the circuit.

So, for that what I have to do, I have to write down the KVL there is no escape from that rule, if you want to find out the current if you know the voltage you have to write down the KVL and that is why the polarity of the voltage induced must be known. Now, let us see this circuit is fine. These are having self-inductance L_1 L_2 and them. So, between points A and B let us see what are the things A Current is entering here i_1 there will be a voltage source president whose polarity will be like this because i_1 is I is like that there is their cities connected.

So, i_1 to i_2 same as you can easily see and here I can like $L \frac{d i}{dt}$ that is what I can right then the story is not any story does not end here, because now these 2 coils has got a mutual coupling with mutual inductance same therefore, because of current in this coil, there will be induced voltage in the coil A B with you will be $M \frac{d i}{dt}$ For a single inductance here everything is over but for mutually coupled coils it is not because of self-inductance there will be a voltage source and because of mutual inductance because second coil is carrying current it same current.

So, $M \frac{d i}{dt}$ but I must write down what is the polarity of this induced voltage, that instantaneous polarity. Second coil through the dot current is entering. Therefore, the induced voltage in this side must be this side + these what suppose we add is positive and it is entering here. So, second coil through the If current enters the other dog becomes class terminal So, true to the left hand side it will be + towards a that is what between A and B we have now 2 as a 2 sources of m.

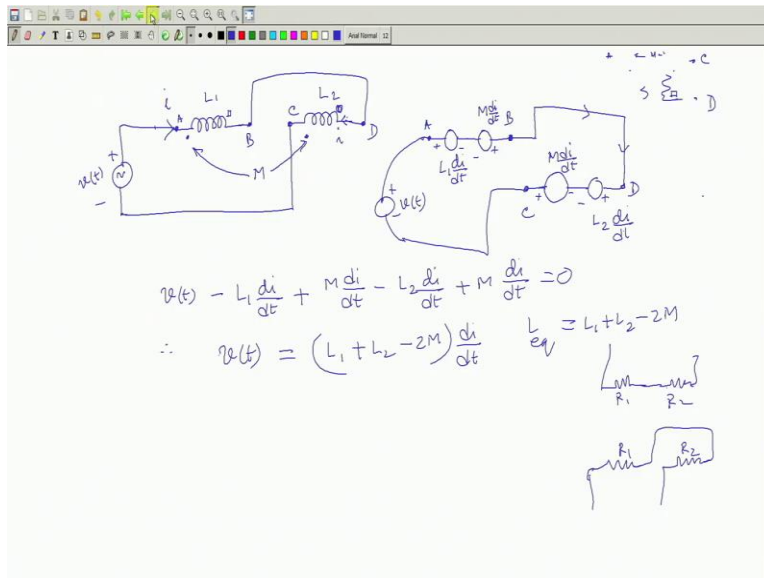
Similarly, then you come to the point C, B and C are sample point known here is i end come to the second end across CD I will say that i look here it has got a self-inductance So, it will have induced voltage $L_2 \frac{di}{dt}$ that is the rule of self-inductance and current is shown entering here. Therefore, this must be + this + b – but then the first coil is carrying a time varying current So, it will Cause and induced voltage here and that will be $M \frac{di}{dt}$ that must be because that question is what should be the polarity of this voltage.

So, polarity of the voltage will be through the dot current is entering in the first coil, it is causing an induced voltage in the second coil whose dot is known so this side will become + and D will become - and then you are done and here is your supply voltage because mind you to find out currents in circuit, you have to write down the KVL then if it is algebraic equation algebraic it is differential equation differential and we know how to handle them.

So, in this circuit now, this is supposed v_t then I will say the KVL equation will be from, this point you start here to hear v_t is a call to then $-L_1 \frac{di}{dt}$ all those terms you take it on the right hand side it will be $L_1 \frac{di}{dt}$ and $M \frac{di}{dt} + L_2 \frac{di}{dt}$. These 2 terms is the voltage across A B. And these 2 terms, $M \frac{di}{dt}$ is the voltage across CD. This is across AB. This term is a CD. This is this story and these of course can be written as $L_1 + L_2 + 2M \frac{di}{dt}$.

So, applied voltage is this $L_1 + L_2 + 2M$ and this is $\frac{di}{dt}$ I can save this much see, I have applied some voltage and it is some constant into $\frac{di}{dt}$. So, I can say L equivalent of this circuit is this is the thing I will tell, but the bond so, L equivalent of the circuit can be calculated, what will be the energy stored in the system up the L equivalent into i^2 so, and similarly, I will not do that.

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But I will tell you what is the difference between these suppose these are the 2 coils but you practice it so that it will give you more confidence in handling mutually coupled acquires. Suppose, I showed that this is dot this is the series connection, I could also do like this the other way now, I will quickly write down so that and this is the applied voltage v_t across the combination.

So, I will assume the current i once again same current clothes and let me call this as A this as B this as C which I called it earlier and this is D. So, I have connected like that. Now, first AB between A and B and mind you these 2 coils has got a mutual inductance which is shown like that. So, across AB, there will be 2 sorts of here m . Now listen carefully here. So far as self-induced emf is concerned $L_1 di/dt$ it was tried to oppose the change in the size.

so, this is + this is - nothing but there will be another volt as source because of the fact that the second coil CD also carries current and what will be the magnitude of this voltage $M di/dt$, di/dt this will be the thing, but this time you should be careful. In this case, current is entering through this terminal not through dot. Therefore, this will become class and this will become -. So, through whichever terminal current is entering this current which is causing an induced voltage here that side it should be +.

So, I think I you must understand this point, suppose there are 2 mutually coupled coils which are shown dot and this is a mutual inductance, suppose this is A this is B This is C in general I am telling the now, what you do these 2 dots are given it is also equally current. This 2 has been given other 2 terminals will also have like polarities when this is + this is +, at that time this was - this was -, but to some extent this will become - this will become - this will become + this will become +.

In other words what I am telling if you say it is these 2 dots these 2 things are absolutely same pair of coils with this dot markings is same as same pair of coils with this dot markings no problem that is you can use some other symbol This is not some which I when you are very expert in do not need to do that square you like they also represent like this therefore, if this is square this is also square need not to be done, I mean I am telling you the logic behind it, so through so this can be the square also can be treated as dot forget about this.

So, through the dot current is entering and this side will become class that is how I think you have got deployed. So, once this is done I will go to the second coil that is C D and terminal D and between these 2 also there will be 2 sources is not. 1 sources the current due to self-inductance for this coil current is entering I am so Please forgive me, this is the connection, I should draw correctly the exactly the same way. That is your this thing now goes to this and then your C here then your surface where V_t connected spinors people are divided.

So, here current is flowing like this. So, first the self-induced voltage current is entering through this 1. So, this is + this is - and this is $L \frac{di}{dt}$ and then the fast coil is carrying current then it will have another induced voltage because of mutual terms, which will be $M \frac{di}{dt}$ and through the dot current is entering therefore, each dot terminal here will have +. So, this will have class, this will have - got the point then right down the KVL V_t from this to this - 2 + then from this to this - $L_1 \frac{di}{dt}$ then - 2 + $M \frac{di}{dt}$,

Then probably a + to - so - $L_2 \frac{di}{dt}$ and then - 2 + $M \frac{di}{dt}$ and this must be 0 that is what KVL last tells. Therefore, this can be written as $v_t = L_1 + L_2$ take everything on the right hand side - 2 M into $\frac{di}{dt}$. And we say that $L_{\text{equivalent}} = L_1 L_2 - 2M$. Therefore the few connect 2 coils in

series the equivalent inductance between those 2 coils we get having mutual coupling between them will depend upon the connections it can for example, 2 resistance I connect it in R1 R2 whether you connected like this equivalent ratio $R_1 + R_2$.

the way that you connect like that it does not matter equivalent ratio is once again $R_1 R_2$ but we inductances or coils which has got mutual coupling you should be careful. The series connected no doubt they are, but equivalent inductance between the terminals will depend upon whether they are deeply connected like $L_1 + L_2 + 2M$ are in subtraction mode they are connected you should be careful. The next problem which I will not solve completely, but I will leave.

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$$v(t) = L_1 \frac{di_1}{dt} + M \frac{d(i-i_1)}{dt}$$

$$v(t) = L_2 \frac{d(i-i_1)}{dt} + M \frac{di_1}{dt}$$

Equating the R.H.S

$$L_1 \frac{di_1}{dt} + M \frac{d(i-i_1)}{dt} = L_2 \frac{d(i-i_1)}{dt} + M \frac{di_1}{dt}$$

$$(L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} + L_2 \frac{di_1}{dt} - M \frac{di_1}{dt}) = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$\text{OR } (L_1 + L_2 - 2M) \frac{di_1}{dt} = (L_2 - M) \frac{di}{dt} \therefore \frac{di_1}{dt} = \frac{L_2 - M}{L_1 + L_2 - 2M} \frac{di}{dt}$$

It as an exercise to you is suppose these are the coils 2 coils just it will hopefully as many exercises you do you will understand it better that is what I think suppose, these are the 2 coils and these 2 coil as got self-inductance $L_1 L_2$ and mutual inductance between them is like this like that I have connected like this. Now, what I will do I connected these 2 coils in series earlier now These 2 coils now connected in parallel connect them in parallel. And once again tell that this terminal is A this terminology is B and the quiet this coil term in C this coil term in addition to of course A and C are same point B and D are same point.

I want to find out what is L equivalent to this parallel combination, I am assuming the coils are having node existences inductance, what I will do is this parallel combination across this parallel

combination, I will connect a AC voltage source v_t With this polarity is important in solving circuit whenever you have to write down the KVL equations, the polarity of each element function known voltage polarity, then only you can correctly add them up otherwise not. So, anyway here first thing is first.

Suppose somebody has drawn a circuit like this, no doubt nothing L_1 L_2 and telling mutual inductance between them then I will I have every right to ask, Who has whoever has given me the problem, unless you give me the dots, I cannot do it because I do not know the sense of the whining otherwise I could find out if you give in totality the magnetic structure and the sense of the winding then I could Be able to find out, but here I cannot proceed unless you tell me because dot convention is essential to know the correct polarity of the induced voltage due to self-inductance as well as due to mutual inductance.

So, that is why dots are given these 2 are the dots that is.(FL: 22:22) Now across A B what is happening let us see (FL: 22:28) first of all, I will say that let the current ground from the supply is i_t let the current and let to this current B i_1 then I will apply KCL at this point and say that current flowing in this coil is $i - i_1$ so, $i - i_1$ is flowing here and the direction I have mentioned and this is if that be the case, let us try to see the voltage drop across A B in the first coil L_1 current is entering i_1 . So, we tell 1 there is no problem + - it has to be this is 1 and this i will write $L \frac{di}{dt}$.

But I know second coil is carrying current and there is a mutual coupling between these 2 So, there will be another source of here what will be the value of that $M \frac{di}{dt}$ of current in the second coil which is $i - i_1$ that is what it came out to me. Now, the big question is I must write down the polarity of the race our Can I do anything what will be the polarity of the industry voltage through the dot $i - i_1$ is entering and because of this current time calculating the induced voltage here $M \frac{di}{dt}$.

So, through the dot it is it enters, this dot will become +. So, this will become + this will become -, this is a story of me coil A b what about coil C this coil has got 2 terminals C and D there will be induced voltage in this coil, which is having a self-inductance currently shown to be flowing

like this, this is $i - i_1$ therefore, it should have $L \frac{d(i - i_1)}{dt}$ the straightforward and then there will be another emf here. The value of which will be because of current in the first coil, which is i_1 and this has to be $M \frac{di_1}{dt}$.

Now, the next thing is, this is $+$ this is $-$ this is $+$ this is $-$ who tells me that this convention through the dot current is entering in coil AV and that current is causing a induced voltage here where the dot is known, so, the half should B $+$ or should B $-$ and then these are the days these 2 are a parallel connected now I draw the circuit This is parallel connected and here is the voltage source V_t Is the voltage source and this current I called i_t so, $i_t = i - i_1$ then this is $i - i_1$ current in the second coil is $i - i_1$.

So, this current was $i - i_1$ - suppose, I have been asked to calculate L equivalent of this circuit. Now, L equivalent means, what my strategy will be to get a differential equation which will have a constant down here into $\frac{di}{dt}$ if I can write it like that, I will say this constant is L equivalent that is the idea that is how I will try to go. So, now you see these 2 are parallel circuit. So, first thing is V_t applied voltage here will be equal to in this branch it is $L_1 \frac{d(i - i_1)}{dt} + M \frac{di_1}{dt}$ of $i - i_1$ also V_t in this loop, it will be equal to $L_2 \frac{d(i - i_1)}{dt} + M \frac{di_1}{dt}$ then once again $+$ to $-$ $+ M \frac{di_1}{dt}$ this will be the thing.

So, what is to be done from these 2 equations, you have to eliminate i_1 right to eliminate i_1 now equating the right hand side equity the right hand side because these 2 are same as V_t , I will say $L_1 \frac{d(i - i_1)}{dt} + M \frac{di_1}{dt}$ of $i - i_1$ is same as $L_2 \frac{d(i - i_1)}{dt} + M \frac{di_1}{dt}$ by this, I will get. So, what you do you collect the term we can having i_1 bar. So, it will be L_1 this term i_1 and this term is $i_1 - L_1 \frac{di_1}{dt} - M \frac{di_1}{dt}$ and if you bring this term with this side, it will be $+ L_2 \frac{di_1}{dt}$ another term minus and $-$ Are you getting I will write it first. It is let us know tightly.

So collect the terms $\frac{di_1}{dt}$ on the left hand side so it will be $L_1 \frac{di_1}{dt}$ will let me right in longhand, $- M \frac{di_1}{dt}$, that is the second term, then this 1 will be $+ L_2 \frac{di_1}{dt}$. This term I have taken and this term is also i_1 . So, $- M \frac{di_1}{dt}$ this is the thing I will get on the left hand side right hand side bring all the terms having $\frac{di_1}{dt}$. So, $\frac{di_1}{dt}$ there is $L_2 \frac{di_1}{dt}$ is already present on the right

hand side that is this term and another $\frac{di_1}{dt}$ will come only this i_1 that is this term. So, $-M \frac{di_1}{dt}$, this is the thing.

So, I will write $L_1 + L_2 - 2M$ into $\frac{di_1}{dt} = \frac{L_2 - M \frac{di_1}{dt}}{L_1 + L_2 - 2M}$ fortunately here in this particular example, all i_1 terms are coming in terms of $\frac{di_1}{dt}$ are $\frac{di_1}{dt}$. Therefore, my objective was to eliminate i_1 means I know now therefore, $\frac{di_1}{dt}$ is nothing but $\frac{L_2 - M}{L_1 + L_2 - 2M}$ into $\frac{di_1}{dt}$ this is $\frac{di_1}{dt}$ so, put it back in any of these equation to M . So, this i_1 I will now suppose take this equation and I will add this I will not complete I leave it to you, but only thing I will say then $V_t = L_1 \frac{di_1}{dt}$ and $\frac{di_1}{dt}$ is this i_1 is $\frac{L_2 - M}{L_1 + L_2 - 2M}$. this is $\frac{di_1}{dt}$.

This is the first term then $-M \frac{di_1}{dt}$ $-M \frac{di_1}{dt}$ this you open the bracket I could do it in 1 line anyway and then $-$ this is $+$ and then $-M \frac{di_1}{dt}$ and once again for $\frac{di_1}{dt}$ put these numbers. So, everything is now some constant into $\frac{di_1}{dt}$. From that you please try to find out a liquid got the idea so, first thing fast. Do not get confused about the convention that is 1 thing. Unless you do not know the convention, you will not be able to write down this you will not be able to show the correct polarity of the voltages. Which is essential in writing KVL equation, and why should I write KVL equation.

Because I want to solve a circuit Unless I can write down the KVL equation, how can I for that reason coupled coils can be shown just like that, but they are dots must be properly put correctly given, then only I can proceed with a problem write equations this that and do whatever is necessary. Similarly, last thing is I write it also another exercise if you suppose these 2 coils and these 2 you connect them in parallel, what will be the equivalent of all this L_1 this is L_2 and mutual inductances them can be also found out that that expression will be slightly different, but the steps should be same, you should be very logical in calculating that thank you.