Behavioral and Personal Finance
Prof. Abhijeet Chandra
Vinod Gupta School of Management
Indian Institute of Technology-Kharagpur

Lecture # 40
Energy Stored in Mutually Coupled Coils

Welcome to lecture number 40. And as you know we have been discussing about circuit analysis when mutually coupled coils will be also present apart from normal RL circuit etc. So, we are concentrating upon majorly coupled coils and let us do just quickly write down.

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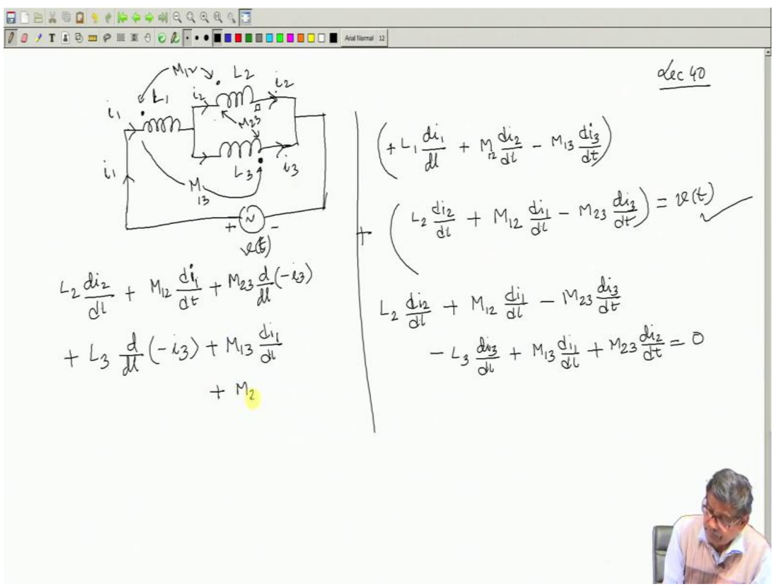

So, that you can review the other things for example, I say there are 3 coils very quickly I will do so, that so, that you also this time I will do rather easily without to drawing those circles these that it is like suppose there are three coins here there is AC voltage connected vt plus minus once you can write down without going that detail. What I am telling is suppose this is L1 and this is the, this is L2 and this L3 will have mutual inductance between these 2, L1 L2 is supposed I want to got the point.

And this coil say for this one that this coil is shown to be here and this is M 13 similarly, these 2 will have a mutual inductance of M 23. So, all the 3 coils are coupled here. Now how to write down the KVL here. So to do this and suppose I say this current is I1 and this current is supposed to this I2 my priority to choose the direction of the current as I have been telling many times, so, I2 I3 and I1 are the currents in the coils and I have to write down the voltage equation KVL equations how many members are there to messes 1 and 2.

So the KVL equation in this mess I1 and I2 write down got the point with this mess which I will try to do it in one way so, this is I1 see the voltage drop between these 2 points will be L1 di / dt definitely when another to EMF one because of I2 they have mutual coupling. So, that term will be a M di / dt and between these 2 points there will be another term which is M 13 this M 12 They want to this M13, di3 / dt this will be the thing these will be the drops here.

Now, the question is L1 at the drop across AB these terminals L1 di / dt this will be plus whatever the other times plus minus or minus plus how do I decide, you see I will do is flowing like this through it is entering, therefore, mutual induced voltage in L1 will be left side. So plus for this coil it is leaving dot therefore, right hand side will become plus that is, so this will be minus I am now I have been avoiding are those small steps you should be you should your understanding should be at this level now quickly you should write it or do you think in this way that for these 2 are dots.

Then these 2 are also dots at best you can do this with square like this I am writing. So, through this square it is entering, so, this square will become like that. So, it will be like inside and this must be equal to vt. This is first loop, in this equation in this loop, this equation will be you start from anywhere. For example, here in this room, it will be L2 di / dt L1 di / dt I have taken the M12 di2 / dt have taken then M13 di3 / dt and this is across it. So, this only is across L1.

So, this is only L1 then I have to write down the voltage drop between these 2 points to get vt. So, what will be those terms? So, this will be plus L2 di2 / dt then because of one there will be induced voltage. So, plus we side could be plus this left hand side because through dot I1 entering. So, I1 to M 12 di1 / dt and for this if these 2 are dots these 2 are also dot one square, so, through squared it is entering it this squared will become plus. So, it will be - M 23 di3 / dt and this will be equal to your vt now it is correct.

So, this similarly in this loop if you go by you will see that L2 di2 / dt and for this one this will be plus so, this will be M 12 di1 / dt then you reach this point then from this to this you are coming. So it will be - L3 di3 / d3 in this loop I am writing. So L2 di2 / dt plus 2 minus, then once again plus 2 minus then L1 di / dt because left hand side is plus for this fellow it is you can write it best way is to there are so many ways you form your own mechanism.

This always I will write plus, interest I mean d / dl to this side plus this side minus it is entering minus it is entering. So, this side plus the L2 di / dt plus M12 and M13 is there, this then M 23 because of that also there will be voltage. So, 3 sources and for 2 mutual these 2 these so, what should we the polarity of this voltage I2 is entering here, this must be minus because the right hand side is plus through the dot the square so, this side will be plus or minus M 23 di3 / dt then 3 term here.

I have reached from this to this point, then from this to this point it will be minus L3 di3 / dt. Then another 2 terms this and because of this, is dot so, this left hand side is plus so, this will be also minus M13 di1 / dt and through the dot it was entering. So, this will be plus M23 di2 / dt And that was lead to 0 so, this is the thing correct. Now, what I am telling if somebody says that this is that current is entering like this, you devise your own method.

And if any problem then go to the basics, but you have your own ways of currently writing that. You can always imagine that current is entering through dot for example, here, the dot terminals were mentioned what I am trying to tell, he had the dot terminal is mentioned. But I3 direction is living the dot you imagine minus it is entering that is the variable M. This M13 di1 / dt or another way of writing it what I am telling you always pretend that current is entering through dot, then no confusion.

For example, in this 1 current is entering through dot is I2 so L2 in this loop I am writing. So, L2 di2 / dt be at this side plus this side through this dot current is entering I1 so plus M12 di / dt here once again I will write class because of this coil and I will write M23 d / dt left hand side will be already plus, but then you have to write – i3, what is entering through dot minus it that we bring that minus sign here got the point.

Similarly come here, here you imagine through the dot currents are entering, so, plus L3 d / dt of minus i3. So, right hand side to us so, through the dots so, plus M13 di / dt, and through this dot it has come di / dt. And finally, these 2 through the dot entering. So right hand side, there M 23 to this negative sign is this. Anyway you will, so we will be able to write down the KVL equation correctly provided I know the dot conventions (FL: 14:29).

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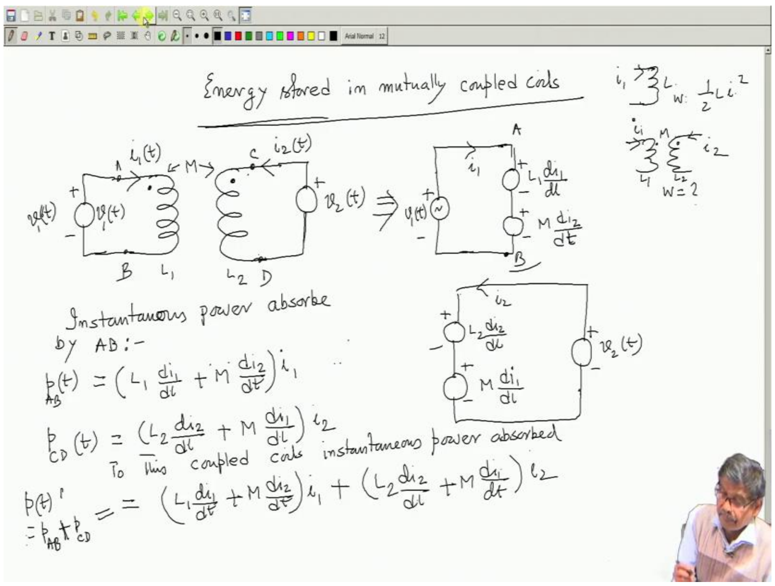

I will tell you about the energy exploration, energy stored in mutually coupled coils only 2 coils we can find out for many more coils. So, that is what I want to find out that is what I want to say suppose there are 2 coins 1 inductances there anyone it carries a current I1 at any instant how much is the energy stored we have seen up in I1. Now suppose there are 2 coils there we can reach a recovered and one is carrying a current I1 and that is carrying a current how much is the energy stored we want to know.

This is L1 this is L2 these are given and this is mutual inductance between these the problem what is W suppose at a given instant they are carrying I1 and right now, this is an interesting way we can do it. Suppose these are the 2 coils. And suppose here I have connected a voltage source v1 and let us assume this inductances having no resistance so, this is the 1 vt and here also I have connected a both the source, v2 t and the direction of the current I will assume.

Suppose these 2 are dots given without dot you cannot proceed, so, I want i2 and this current on the other coil. Let it be assumed to be direct in this direction these the things. So, this is L1, this is L2 and this is the mutual inductance between them and this circuit is equivalent to v1 t is there and across which 2 sources are connected plus minus, which is L1 di1 / dt and there is another source through the dot current is entering this will become plus so, plus minus M di2 / dt is that this will be the primary site mutual inductance between them is M.

Similarly, in the second coil the voltage applied is v2 t with this polarity T and you have two sources of emf in series. One is obviously I2 is entering, so L2 di2 / dt with this plus this minus because I2 is I should not like that mind this is I1. And through the dot is I1 is entering these dots will become plus. So, upper one is plus minus M di1 / dt and let the things be time varying not at all on both the sides sources are connected or not.

So, I have drawn the KVL I mean the voltage instantaneous voltage polarities hopefully correctly like this (FL: 19:20) at any time t these are the 2 terminals of the cord A B and this is suppose C D, this is A B. Now, supply voltage has got a polarity like this and A B across we have this mutual coils comes in therefore this one is absorbing power instantaneous power absorbed by absorbed by V AB this element every element will be v1 i1 instantaneous power.

And instantaneous power in the same way absorbed by coil CD will be easy the voltage between these 2 points plus minus and current is i2. So it is absorbing power. So, this will be equal to be I2. If you please allow me to write in this way, instantaneous power absorbed by A B will be the voltage V AB, which happens to be called to I1 write that but I know now in terms of L1 I2 I have to find out. So, this will be L1 di1 / dt voltage between these 2 points A B plus M di2 / dt. Similarly, same thing which am not writing in language power absorbed by CD.

Instead instantaneous power will be this voltage that L2 di2 / dt plus M di / dt here each must be multiplied by I1 and here it must be multiplied by I2 voltage into instantaneous current, this will be the thing. Therefore, to this system ABCD to the coupled coil instantaneous power absorbed, will be equal to some of these 2, so, L1 di1 / dt plus M di2 / dt into I1 plus off L2 di2 / dt plus M di1 / dt into i t. This will be the instantaneous power, total power that is pt. Let me Pt will be like this which is equal to be a P AB + B CD equal to this I can write therefore, energy supplied to the system in time dt will be.

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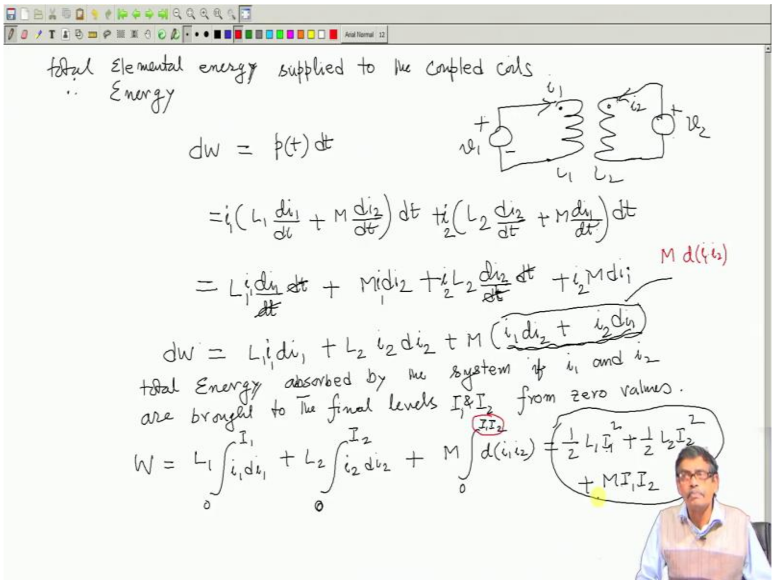

Therefore, energy, elemental energy supplied to the coupled coil supplied total elemental energy supplied to coupled coils. Coils will be dw equal to Pt into dt is it redraw the circuit here for ready reference. So, this was a L1 and this was L2, this is v1 minus this is v2 and assumed current to be flowing like this. And this is so, Pt into dt is the total energy supplied. So, it was equal to L1 di1 / dt plus M di2 / dt into dt plus L2 di2 / dt plus M di1 / dt and this is also in multiplied by dt.

This will be the thing, which can be written as L1, open the brackets di / dt into dt, push it inside plus M di2 plus L2 di2 / dt into dt plus M di1 into dt, dt cancels out. Similarly, dt, dt cancels out so this will be the thing. So, this one then becomes L1 di1 another important thing I missed, what is that, that is voltage into current I hopefully I wrote it here the previous section into I1 into I2. So, this was into I1 and this was into I2, that is, so, this was I1 will come here and also here I1 will come and also here I2 will come this will be the thing.

Now, this dt goes cancel them out. So, it will be a L1 I1 di1 plus L2 I2 plus M i1 di2 plus M i2 di1. This will be the energy and not energy supplied in time dt elemental element so this term will be twice so this will be the thing. Now you see if there were total energy stored total energy absorbed by the system if i 1 and i 2 brought to the level, to the final levels I1 and I2 from 0 values. Energy total energy absorbed by the system should be L1. This is i1 di1. So, I must write the limit of 0 to I1 similarly plus L2 i2 di2 I have to integrate and suppose the current is 0 to I2 and for this one these 2 terms I1 and I2 can be written as d i1 i2 the product of differentiation.

And then I should write the limit of I1 and I2 on it, which was 0 and finally, I1 and I2. So, this will give us the familiar terms L1 I1 square plus 2 terms are very familiar plus 1/2 L2 I2 squared then plus in M I1 I2. In other words after getting this I will tell if at any instant of time I know the value of I1 and I2 and time reading at any instant of time this current then energy stored in the inductor is 1/2 L1 I1 square second inductor will store energy 1/2 L1 I2 square and then the energy stored because of that mutual coupling is M I1 I2 all these 3 we go.

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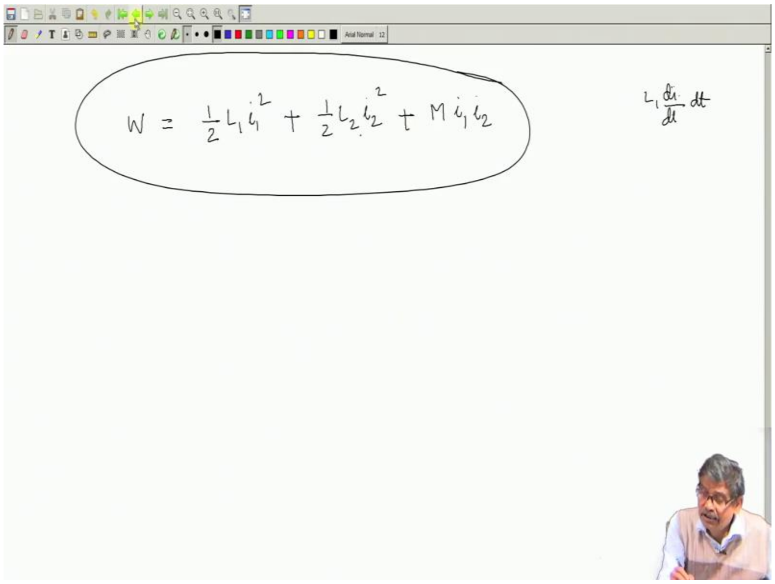

In general I should say the energy stored should be at any instant if the current in the first coil is smaller one plus 1/2 L2 I2 squared is the current in the second then M I1 I2 will be the total energy stored in a couple of coils got the point because after all these are coils lines of forces will be there and this therefore, will be the energy stored in a single inductor L apply one will did the same thing voltage across the inductance current flowing into it and then L1 di / dt in time dt how much energy you are supplying then you are integrating from 0 to I1 a similarly second term.

And fortunately these 2 terms comes in such a beautiful fashion that i1 di1 and i2 di2 is nothing but this M d of i1 i2 differential part differential equation 2 variables and then you assume at this instant current is I1 and I2 and this is started from 0 this changing so limit of this 1 should be productive this to because the integral of this one will be I1 I2. So 0 to I1 I2 so always remember that in mutually coupled coils, the energy stored will be 1/2 L1 I1 squared plus 1/2 L2 I2 squared plus M I1 I2. Thank you.