Behavioral and Personal Finance
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Lecture # 39
Steady State Response with Sinusoidal Excitation when the Coils are Mutually Coupled

So, welcome to lecture 41. And we have been discussing in the last couple of lectures about the mutually coupled coils they exist in SRP how to tackle them and also we found out the energy stored in a system of coupled coils. So, for example, just to summarize this, I must tell that you should not forget this.

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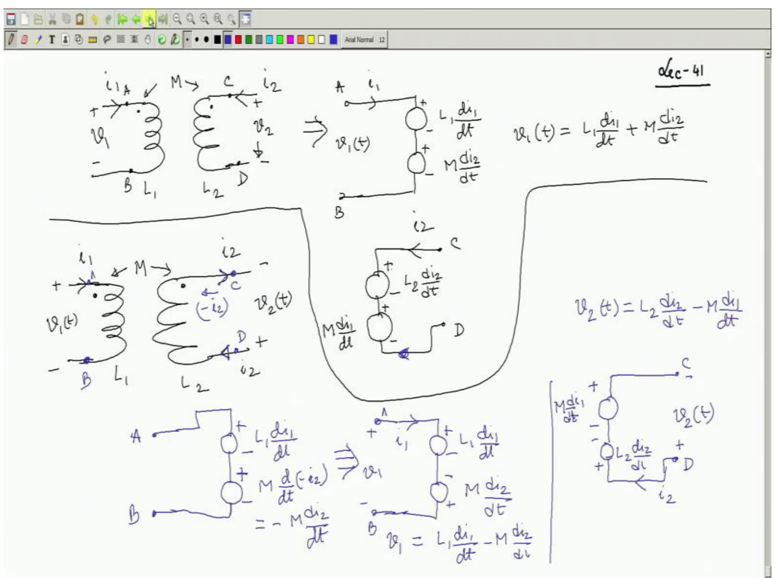

That easy there are 2 coils and dot markings should be there. There are some problems so, where dot Mark independent dot marking the problem can be solved. So, if a problem is giving like L 1 L 2 and mutual inductance between the coils to be M and if the door markings are not provided then you are free to choose your own dot markings. That is what you do and then you can suppose the current in the circuit is I won and current in the circuit is I do like this. Then I told that clause AB am essentially reviewing summarizing the thing in a contact form, Compact form and it should be supposed these 2 terminals A and B and these 2 terminals are C and D and suppose they are connected to some other parts of the network. Towards this side and that side and suppose the current i 1 and i 2 respectively, then the point I want to make is this 1, there will be 2 voltage source here across suppose this voltage across the coil is v1 these voltage polarities are important, you cannot ride just v1 or some i 1 without showing any directions of the currents or any voltage without any polarity never do that.

So, this is the thing. Now, then I told that, look at the direction of the current for L 1 part it will be always + or - here because I 1 is entering that is fine L 1 di dt that is there and here there will be another source emf coming in because second coil is also carrying current and deep through the dot current enters then the dot this side, the polarity of the instantaneous voltage will be also this is + this is - and this will be M di dt that is the essence of the thing like this and these will be all points and we got the point.

So, I will right we want it to be is equal to L 1 di 1 di dt dt + m di 2 dt like that KVL on the other hand a the second coil if you see the CD terminal C and D terminal and have shown the current to be i 2 so, there will be once again to emf coming in Which are connected in series and 1 thing is about tail to it will be + - this is wherever current is entering that becomes + self-inductance L 2 di 2 dt but first coil is also getting current.

So, there will be another emf M di 1 dt here and through the dot current is entering. So, this dot that is C towards sightsee, it will become classical become -, if suppose, the same equations I want to do it in this way suppose, somebody says that these are the 2 mutually coupled coils and this is a low L1 L2 not to like this L 1 L 2 like this mutual inductance between them is M that is also fine. And suppose somebody says e has a few current i 1 in this direction and the current in this circuit is shown like this 2 by 2.

And suppose the voltage here it is shown to be V1 V2 suppose the voltage here shown to be V 2 t I am just telling situations when we like this also because current elections I can assume in any by 8 is my choice it can be this way that way, but after I write down the decide upon the current elections, then I must be very consistent about the polarities of the voltages. So, in this case once again If you see this is suppose a.

This is supposed to be and this is suppose 3 and this is supposed D it is another example same mutually coupled coil, but somebody has extreme current directions this way voltage polarities this way like this, then what I am going to do now, the across A B first let me write down there is self-inductance L 1 and this will be of course, + - L 1 di 1 dt self-inductance in whichever terminals current is entering that will be + it has nothing to do with dot like that and there will be another voltage here which will be M di 2 dt.

Now, what I am telling if I remember this rule through the dot current entering will cause a voltage with dot + here. So, I can say he pi 2 is flowing it is coming out from the DOD, it is as good as telling - i 2 flowing this way is the current entering through DOD. Therefore, I will through the DOD this current is entering this blue current - i 2 and they are four dot will be + so, yeah, you are absolutely correct in writing like this m d dt of the new right - i got the point.

This is the point I want to make. Therefore, I will remember only 1 thing through the dot if current enters the other dot terminal will become + Provided these 2 coils have got mutual inductance M and then this is the circuit, but here you show it like this which is exactly same as this is equivalent to writing because this is – M this is what - M di 2 dt it will be like this treat this but polarity right like that, and then you can say.

This is then + L 1 di 1 dt and then this is - this potential of this point with respect to this is - this quantity. Therefore, you can also reverse this polarity and tell that this is M di 2 dt So, either of them a screw you know, and this is for this problem I am doing. So, this is your A B, A and B then you say the 1 which is where we want you say that it is equal to L on di 1 di - m - because + 2 - the - 2 + therefore, - M the i 2 get what about CD terminals here.

Let us draw the diagram across CD this is your C this is your D and I have already pointed out, this is my voltage polarity applied voltage and then this coil will we will have 2 year M. 1 is because of self-inductance and that is because of mutual inductance. Now, the current direction has been assumed to be linked the site to their goal. So far as the self-inductance is concerned it will be L 2 di 2 dt without any problem through whichever terminal that is this way, no not this 1, this way i 2 is flowing.

Therefore, it will be L 2 di 2 dt and then because of mutual inductance the polarity of this voltage through the dot - i 2 is entering. Therefore, dot here will become + that idaite + - and the new right here M di d dt of - 2 - site this is - i 1 through the door i 1 is entering. So I am So M through the door i 1 is entering. So, it will remain m di 1 di di 1 the MC matter and the KVL equation if you apply B to D will be equal to L 2 di 2 dt di di 2 dt - M di 1 dt.

Once you are accustomed with this 1, you need not go to all these tapes mentally you can write down correctly, the polarity of the voltage is induced due to mutual inductance self-inductance whatever be the direction of current chosen and things like that, that is what I told and next time.

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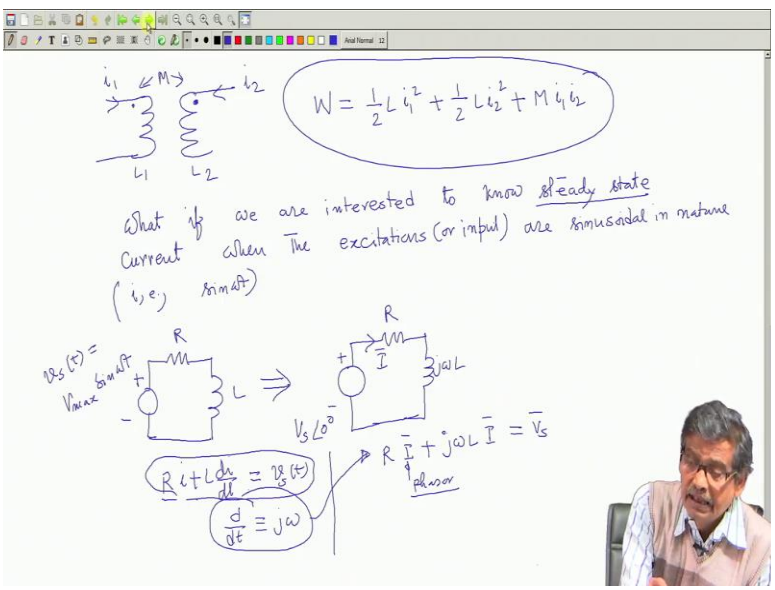

next part what I did is if there are 2 mutually coupled kinds of being mutual inductance between them we with for a change let me write this way mutual inductance and this is 1 this is L 2 and if at any time the current through the coils are known, then energy stored will be equal to instantaneous energy storage will be = half l 1 square + half l i to square + M I want this should be remembered energy stored in a corporate culture will be like. So, this will be the magnitude of the energy stored in the system, we have found out last time.

Now, as I was always trying to tell you that these equations whatever we have written is a general equation and in time domain for any voltage we want to apply to both the coils in time domain I will be able to find out the currency the voltages are known, because I know how to solve differential equations that was there. Now, there is another source as I am telling you do I get and if the sources are DC of course, there will be no induced emf either in terms of self-inductance or In terms of mutual inductance and these 2 points can be shortened for DC It is very simple.

Although cosmic eddy current and they will just old energy that will be there stored energy will be there, but there will be no voltage drop across a B and C D if I want it happens to be constant values because that di even dt di 2 dt all things vanish to kilo. So, that was there but another important source if you recall what if we are interested if we are interested to know Steady current when the excitations Wendy excitations are suicidal in nature excitations or input synthesize sinusoidal.

For example, that is suppose some voltage is sine omega t, this sort of voltage is applied then What is the thing you will recall that we have with single energy storing elements or multiple energy storing elements having no mutual coupling, we have seen that for example, and RLC circuit recall that if you have excited with a suicidal voltage source + - l some BMX sine omega t we have done this problem earlier in that case it can be transformed into it can be solved as a feather notations + - the supply voltage will have some RMS value 0 degree.

We find out that and then our is to be replaced by our no change and this omega L you write it as you omega and find out the phasor current and from there you can come back to the time domain to find out the steady state current entanglement that can be always done we have seen that whether such a thing can also be applied to me surely coupled coils the answer disease, why I will just tell you like this see in time domain the equation is L di dt is a call to your supply voltage Vst is not supposed.

This is your Vst these the circuit and in this case the equation is our into IB + j omega L into i 1 is called to Vst Fair that is not this. the question now If you see from this site to the this is the steady state solutions mind steady state if you are only interested in you can do so, it is you can easily see this time domain expression if you put replace d dt this operated by j omega you get this equation and this small letters which are real quantities in time domain they are the replaced by feathers.

So, any differential equation can be translated into corresponding algebraic equation when the sources are suicidal in netcare very easily wherever d dt is there g omega that way you can always think the Omega you replace d dt by omega and be asleep. Therefore, in majorly coupled coil so also this can be done, why not.

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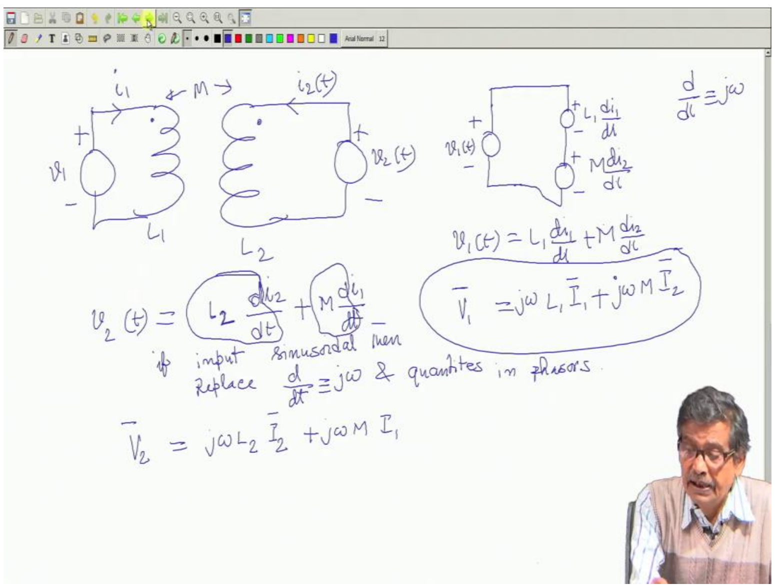

Suppose you have got to meet your likability say with mutual inductance M between them and this is 1 this is 2, this is v 1 this is oppose v 2 and suppose these are the currents are you anti and this is it and we have seen that across v1 t right now, we have seen + - vivante will be equal to will be equal to L 1 da 1 DT and then here also it is + - m di to DT through the dot entering that is + this is the equation which vivante ease.

So, we want t = L 1 di 1 dt + M di 2 dt, it is loads. Therefore, the corresponding phasor relationship will be 1 feathers will be equal to L 1 d dt u t placed by j omega Through j omega L

1 and current is i 1 said that + d dt into M j omega M into current feather right that is what I want to tell you. So, if the excitations are suicidal and you are interested only to know the steady state current, then you can straight away go to frequency domain like this present domain and write down the equation solve for the current feathers and get the current. Similarly, for the second side it will be equal to v 2 t will be equal to L 2 di 2 dt.

the ITU dt + which I am not drawing that equation We have come all the way several times with that thing m d dt and if input is suicidal if input is suicidal you cannot do for all the any type of signals we want to be today must be suicidal inputs in a suicidal then then replace d dt by Gail mega and quantities in feathers so that he will get me to where that is a call to the This d dt operated into L 2 becomes the NGA omega L 2 and small I 2.

I will not right for that + similarly it will be j omega m into current federal is like this. Therefore, all the coupled coils if the inputs suicidal we can write in the same way replacing d dt by j omega. Therefore, this finally, what.

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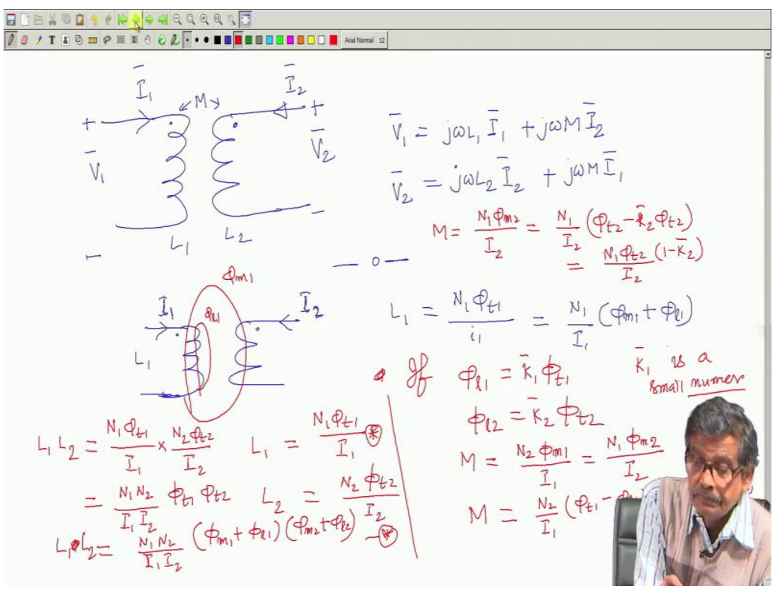

essentially i mean is this if suppose you say here the voltage applied these feathers + - and this current you have shown to be current for that d dt and the other side this is also dark this is mutual inductance and this current you have shown to be current phase 2 and this voltage is + -

suppose we too bad all our feathers, then I will say that v 1 bar straight away IE it is not true that you write down in time domain then come to phase domain not necessary.

If you are only interested in steady state current I will draw like this all the quantities in federal forms and then I will say be 1 bar will be the self-inductance j omega L 1 into Ivan says that 2 sources will be there. Then I 2 is entering through dot all other polarity those constraints will be there. Through the dot i 2 bodies entering, so, the upper 1 will become class, but for d dt it is j omega only and then M and then do not try small light to light this 1. Similarly, the KVL equation on the secondary side this coil it will be to bar is equal to j omega L 2 l 2 l 1 or the self and mutual inductances Yama guide to into I to buy and then it will be j omega am into it 1 by 1.

But we will have a say in the induced voltage in the second coil because these 2 have a mutual coupling what where it should B + R -. e has the upper 1 through the door i 1 is entering it will be + this is the thing I wanted to tell their hold steady state analysis when the input signals are suicidal, the corresponding phase equations also can be written straight away and please practice it to write it correctly. And when you will be able to write the equations correctly.

When polarities of the voltages have been shown very clearly which 1 is + which 1 is -, similarly, directions of the current feathers shown with these arrows, then only you will be in a situation to write down correctly the KVL equations in different loops. Then of course, you solve the circuit to get the currents and if you wish from phasor to come back to the time domain service we already know it is nothing you can immediately do it.

Now, only 1 last point I want to make with this mutually coupled coils which till now I have not pointed out, but it can be shown like that. For example, I have told you that when you pass a this information 1 should know in time domain It was like this I suppose, this is I 2 this is a was dot all these things are fine. I told you that l 1 l 1 is equal to n 1 fighter 1 total flux linkage per unit current i 1.

So, this is the self-inductance in time domain ivities it will be that instant value of current, but t v value will be like that. Now, suppose you assume constant values are flowing this this is it this is it. So, this is the total flux linkage of which in 1 by 1 and this I broke up into 2 components mutual flux + the leakage flux leakage flux is not going to link the second coil. So, this is what we have done this is the mutual flux by a moron and this is the leakage flux file 1 only links the primary.

So, that way I have done now this leakage flux if the coupling of the coil is very tight, no leakage flux then file 1 will be 0 impact file 1 will become only a percentage of the total flux created. So, 1 can say that l 1, if liquid flux, suppose some percentage of total flux created, then k 1 bar is a small number is a small our liquid flux can be reduced. If you bring these 2 coils in close proximity and things like that. Then Liggett flux can be reduced, but anyway suppose it is some k 1 bar will be a number say point 01 10%, maybe less than that five person point 05.

And similarly going to the second coil I can say delicate flux of the second coil will be some percentage of the total flux created by the second coil fighty 2 is the total flux. So, this way it can be written. Now, if you recall that l 1 is called to is a call to N 1 fighty 1 that is what I have written here divided by i 1 I have written this. Now, this can be therefore written as in terms of this leakage flux at some key 1 but no 1 into Fighting. Fighting 1 is the total flux. So, the this is the thing, I am.

What I wanted to tell you l 1 is in 1 fight 1 by 1 and it can be written like that. Similarly let us write like this. Similarly, you can write l 2 l to is a call to end to fi t to buy it, this is that and what is this this mutual inductance ease into i 1 divided by i 1, which is also same as mutual inductance calculated from the other Say that is no 1 by m 2 divided by both all are correct yet. Now, this is the thing now, if you multiply these 2 that is L 1 into L 2, if you multiply these 2 equations this and this equation you will get no 1 fighting 1 by 1 into fi t 2 buy i t.

This is what you will be getting. And this can be written as anyone into divided by i 1, I can right should I assume this is it will correct now. So anyone into by Iowa night to into fi T 1 into piety to like this. Now this can be written as in 1 into by I won I 2 and fighty 1 is nothing but fire mon fighty 1 is fire mon + the Liggett flux is 1 And fighty 2 is 5am to + the liquid flux of the second coil like that I can write . Now, this this competition what I wanted to show is I will rather To make matters simpler leakage flux will be a certain percentage also for mutual flux is 1 can be a this is 1 - k 1.

Suppose this factor I say that I'm sorry you please this is 1 - k 1 it will be doing so, K 1 bar. So, this becomes this is the mutual inductance term. Now, in terms of the scale 1 it can be easily written as as you can see that this aim can be written as into by Ivan m 2 by Ivan and fire 1 is fighting 1 - five l 1, this is this will be your fire 1 for that but file 1 as I told you, it is nothing but into by i 1 and this is then 1 - 1 bar into fighter 1. Similarly, from the other equation, that is aim is also equal to aim is also equal to n 1 five m 2 m 1 5am 2 divided by 2.

And this can be written as n 1 by 2 and 5 m 2 is total flux fighty 2 miners delicate flux which is a fraction of the total flux fighty and this will be equal to anyone fi T 2 divided by i 2 into 1 - k 2, but this will be the thing. Now, what I will be doing is this this Cape you just forget it is not now this is equal to M is equal to this as well as this that is concentrate on these 2 equations. 1 is. So M, can be written as, m can be written as in 1 fighty 2 by 2.

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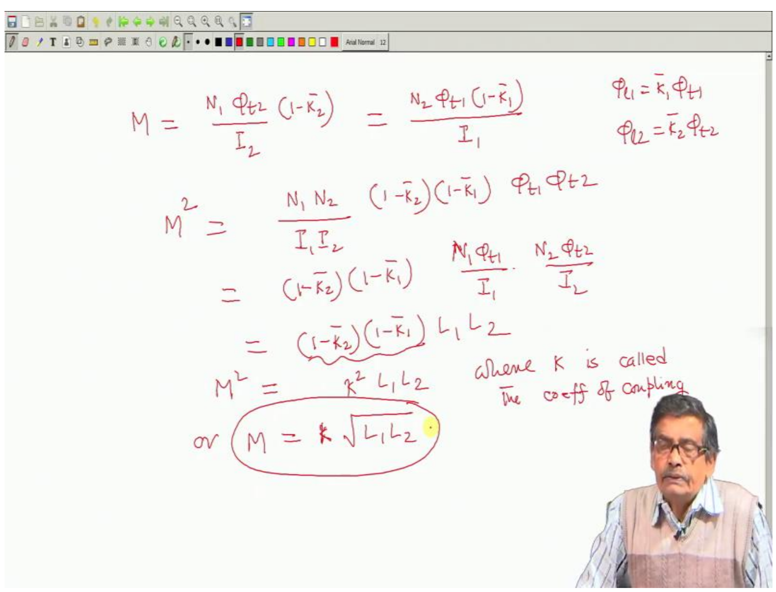

N 1, 5 t 2 by i 2 into L - K 2 by also, this is equal to end to fight it 1 by 1 - 1 bar what is k 1 bar and K to bar they are the percentage of the total flux, which becomes liquid flux recall that file 1 is equal to k 1 bar into fi T 1 and liquids flux of the second coil is ketubah in to fight it. Now, what I will be doing therefore, I can write m squared multiply this too and you will get it as no 1 no into by i 1 night to then you will get L - K 2 bar L - K 2 work into L - K 1 bar And then you will get it 1 by the 2. That is what you will get.

Which can be easily written as 1 - k 2 into 1 - k 1 bar is equal to no into, multiplied by, multiplied by anyone. No 1 fighting 1 by 1 into fighting 2 by 2 and these are nothing but your 1 - k 1 bar into hell 1 into L 2 Now, this particular thing, these are numbers less than 1 and this can be witness some k square into L 1, l 2 or we say that mutual inductance will be tech square root is equal to L 1 l 2,

Where were case called the coefficient of coupling where k is the coefficient of coupling, it will be a positive number, and K should be for a tightly coupled coil. They should be close to 1 mind you k 1 is very small. So, 1 - k 1 will be close to 1 this will be close to 1 of course, you are multiplying 1 second it becomes less than 1, but this is the thing I wanted to point out. Therefore, what we have done is mutually coupled coils if they are present.

If you know the coefficient of coupling and the self-inductances which by measurement it can be found out then m can be evaluated like that. And not only that, they such mutually coupled coils are part of any circuit I will be able to write down the KVL equations correctly provided I know the conventions both in time domain as well as when the inputs are suicidal in terms of phasors, and we can solve the subject. Thank you.