Network Analysis
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Lecture – 42
Basics of Signals in Brief

Welcome to lecture number 42, so let us now try to understand how this course is progressing you know, we started with our main motto is okay, there will be a network given with energy storing elements, sources maybe, may not be sinusoidal for general case, it will be any arbitrary signal and we would like to find out the currents, power, these that of the circuit that is the thing.

So to do that, we have seen that if the circuit does not contain any energy storing elements, then it will be pretty easy, only the distance, no differential equation nothing will come but energy storing elements is bound to be present in a circuit in general, therefore we have to solve differential equations.

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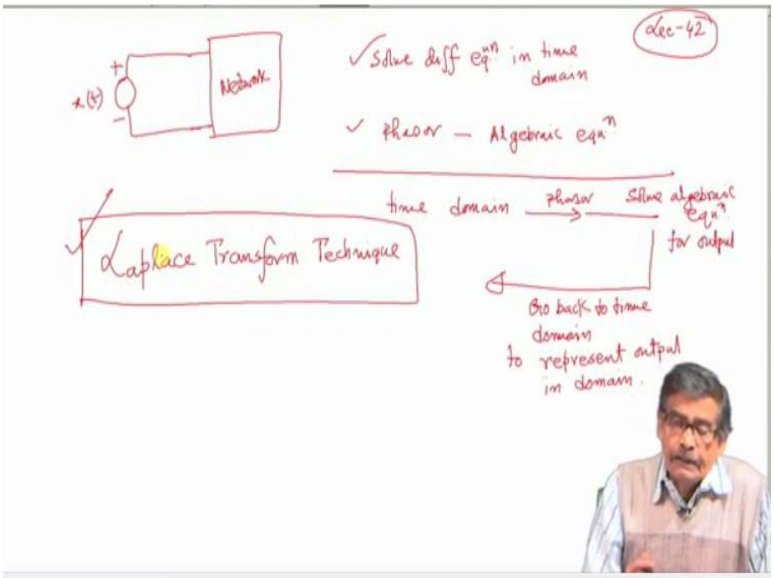

And we have gone extensively how to tackle those circuit problems which is excited with some any arbitrary signal xt; xt is any arbitrary signal and this is your say, network, so you have to write down the KVL equations, KCL equation correctly and got the current response in the various branches of the network and one can solve any circuit as it is. The next phase was okay, so classical way solve the differential equation in time domain, everything is real, this that.

This is the thing we have covered and then also this xt could be anything by that I mean that xt even could be a delta function that is what we have found out but however, to find out to solve the network, when the excitation is an impulse, you have to be careful, the usual rule will not be operating because initial currents at 0 minus suddenly changes to some other value, okay.

For an inductor for example, I0 plus will be I0 minus plus 1 by L, if it is an impulse applied to a series RLC circuit like that. Then, I also told you there will be, these solutions will generally consist of a natural response and a response due to forcing function that is called also called steady state response. Now, the natural responses will generally die down with the time constant of the circuit.

So, after of the order of milliseconds that natural will be; natural response will be born when you do some switching in the circuit and after some time it will die down and finally, the steady state current will be only existing therefore, the question is whether we can find out the steady state response quickly for some very definite type of input signal for example, people are electrical engineers at least will be often using a DC signal, constant value signals as input, DC supply or an AC supply which is sinusoidally varying with time.

Then, we told that if it is sinusoidal varying with time, then this there is no need of solving differential equation because you will ignore the transient part, we are only interested to know what will be the response of the circuit after quite a time has elapsed because that transient will not be there therefore, what is the point. Therefore to find out the steady state solutions, we told that phasor method can be applied, phasor notations.

And there you do not solve differential equation, gives algebraic equation, these what we did, so go to phasor, so time domain, go to phasor domain, solve algebraic equation, then come back, go back to time domain for solve algebraic equation for output and go back to time domain to write to represent output in time domain because after all we have applied in time domain, that part we have done.

Now, there is another method whereby a network can be solved by what is called the by Laplace transform technique, it is one of the most powerful method of solving network problem, we will see that Laplace transform technique once again is a technique where the differential equations will be converted to some algebraic equation in what is known as S domain.

And then you solve for the output and you will get Laplace transform of the output and finally, take the inverse transform to get the time domain expression and there, also the initial conditions can be comprehensively taken out and whatever solution you will get in contrast with the phasor approach there, both the transient part as well as steady state part of the solutions will be obtained.

So, this is the thing now, therefore we plan to tell you how to apply this Laplace transform technique to solve circuit problem or in other words to solve some differential equation that is the whole idea.

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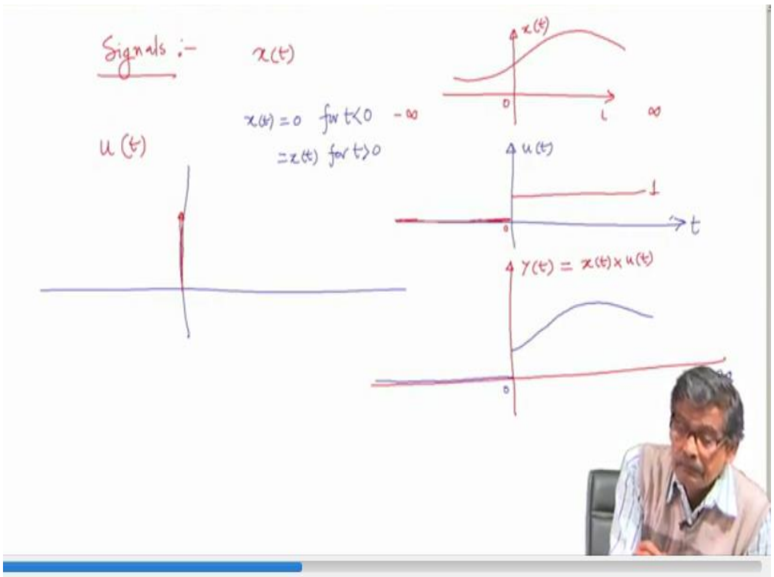

Now, before we go to this Laplace transform, I will tell you something about signals, okay signals, very basic things will not go much deep into it, any signal I will denote it by xt, it is the time descriptions of the signals for example, xt could be anything like this and generally, t ranging from minus infinity to plus infinity, this is t and this is xt, this is how we show it. Now, we have earlier defined 2 functions, which are very important; one is called unit step functions, which is I am drawing below this against time.

And I am sketching ut; ut will be a function whose amplitude will be equal to 1 for, t > 0 and it was 0 earlier like this, this is the thing. So, we will just tell you something what happens if you multiply a signal xt with ut, so if I want to sketch a signal yt which I suppose, I say it is equal to xt into ut, unit step function, if you multiply any function xt, which is otherwise the existing like this.

Then, what happens is this for t < 0, 0 into all these things so, it will be a flat 0 there, it will be 0 but for t > 0 this ordinates gets multiplied with 1 and this signal will be just like this, is it not, so any signal if you multiply with unit step function, the value of the function is drops down to 0 before t < 0 that is I am only taking the value of the function xt for, t > 0. In fact, any signals which will last between 0 to infinity, generally called causal signal okay.

We will; in our circuit, we will always try to find out at t equal to 0, something I am trying to do with some input signal xt and this is the input signal okay, so this is xt. Similarly, you recall that if you have an xt like this so, we will be dealing with functions now, which exist for t > 0 that is xt, I will say is equal to 0 for, t < 0 and xt is equal to xt for t > 0 that is for signals which starts beyond t equal to 0.

Before that I will presume the signal is not present, so unit step function and this one also, we have seen that one function we have introduced is called an unit impulse function, Acha, before that unit impulse function, let us play with this unit step function okay.

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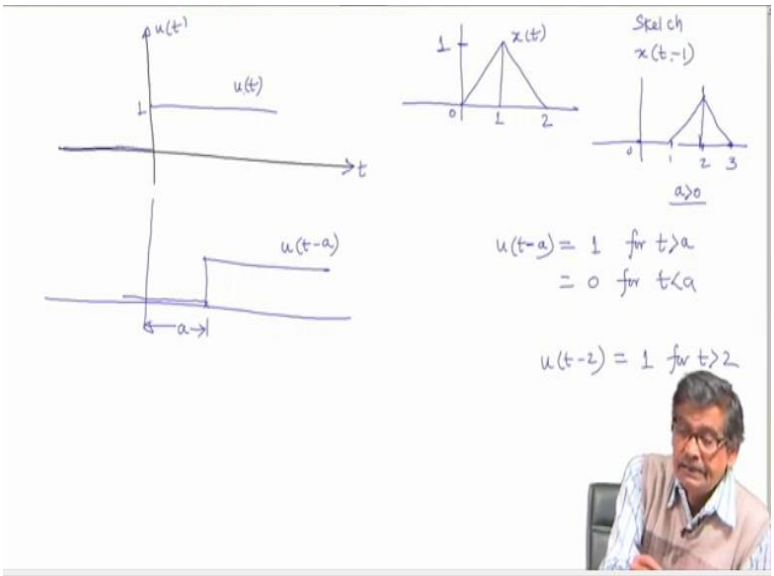

Suppose, you have an unit step function like this, ut, this is 1, this is time and this is ut, this was 0 before, now if I asked to sketch you what will be ut minus a, where a is a positive number it means, that this will be equal to 1 for t greater than a and will be equal to 0 for t less than a, this much I must know that is your unit step function. If this is a; a is a positive number greater than 0, then your unit step function will look like this one.

And I will write it as ut - a that is this unit step function gets shifted by an amount a, for example u minus; and why it is, because see if you put any number t, let us take an example, ut – 2, if t is > 1, t is > 2, then only this argument will become non zero and by definition of ut that will be equal to 1, that is why ut - 2 will be equal to 1 for, t > 2 why? For, t < 2 for example, choose any value say, minus 1 so, it is u – 3.

But from u - 3 I know, it will be 0, very easy to interpret this so, any time this one if you replace by some t minus a, where a is a positive number this function gets shifted to right and it is not only true for ut, it is true for any function for example, if you have a function like this, xt if I say and this is suppose 0, this is suppose 1, this is suppose 2 and this is 1, a pulse; triangular pulse.

If I ask you to sketch xt - 1, what should I do; this function, this is 0, this is 1, this is 2, this is 3, this function bodily shifts towards right by an amount 1 to the right, so this function xt - 2 will be just like that. So, for a given function in general xt, I will be able to sketch shifted version of the function by any amount you like, if you make this plus; ut + 2 that function gets shifted to the left and you know that.

And I presume that you know some basics of signals where I will just make a cursory reference, so that you can recall those important ideas about signals. So, shifted versions of signals will be there okay.

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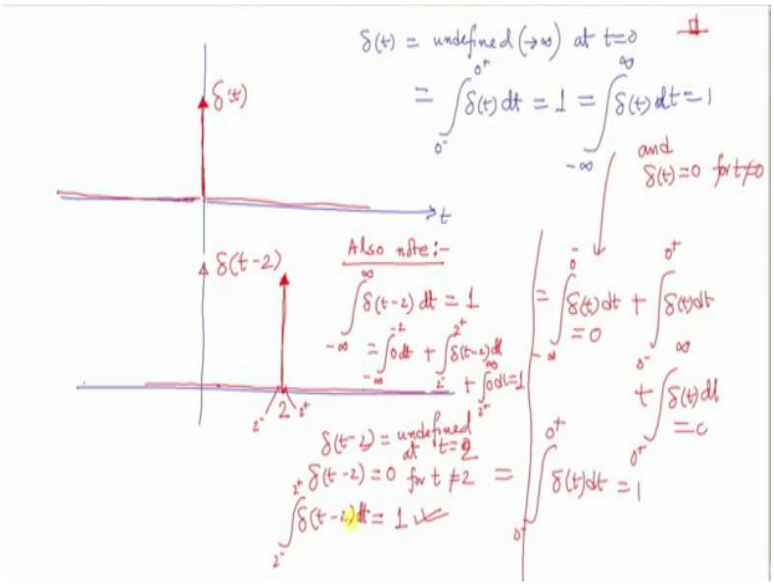

Then, I say that another function which is called an unit impulse function suppose, unit impulse function I used to show by an arrow like this ut and unit impulse function, it is an even function because there is a square pulse around point 0, you recall this collapsing anyway, so this is not ut, this is impulse function, this is delta t, it was written like this. Similarly, this where will be and recall that delta t is equal to undefined going to infinity at t equal to 0.

No point in asking what is the value of the function at t equal to 0 but what is important is these calculations 0 minus to 0 plus, delta t dt, this we have discussed somewhat detail earlier, this is equal to 1 and obviously, this integral is minus infinity to infinity delta t ut; delta t dt, these also 1, why? Because up to 0 minus elsewhere, the function is 0 so, this is 1 and delta t is equal to 0 for t not equal to 0 other than t equal to 0, the function is 0, very simple function otherwise, everywhere 0 with the constraint that this one.

So, minus infinity to infinity if you integrate, that will be 0 up to this point, 0 minus, then 0 minus to 0 plus you have to integrate, so this one is nothing but minus infinity to 0 minus delta t dt + 0 minus to 0 plus delta t dt, then plus; 0 plus to infinity delta t dt, now this integration is 0 because delta t is 0 between minus infinity to 0 minus, this 0, this one; 0 plus to plus infinity, it was 0 so, this will be also 0, so this is 0, this is 0.

So, this is equal to 0 minus to 0 plus delta t dt and that is equal to 1, so minus infinity to infinity delta t dt is equal to 1. Now, let us see the shifted version of delta t, suppose I say that sketch delta t – 2, so delta t – 2, will be this is suppose, t equal to 2, so the impulse will be sitting at t equal to 2, elsewhere it will be 0, so impulse is occurring, so how do I describe this function?

I will say, delta t – 2 is equal to undefined at t equal to 2; at t equal to 2, undefined infinite, same impulse but occurring at t equal to 2 and then I will say delta t - 2 is equal to 0 for t other than 2, anywhere for t less than to 0, for t greater than to 0 and delta t - 2 ; 2 minus to 2 plus, this will be a finite value 1 that is what it means, impulse area under this impulse at t equal to 2 will be 1.

Therefore, shifted signal we have been able to sketch and not only that also, note that integral minus infinity to plus infinity delta t - 2 dt will be equal to 1, why? Because minus infinity to 2 minus; 2 minus is here, 2 plus is here, so minus infinity if you break it up to minus 2, the value of the function is 0 dt, then you will write minus 2 to sorry, not minus 2, 2 minus is the number very close to 2 but less than 2, to 2 + delta t - 2 dt.

And finally, 2 plus to infinity the value of the function is 0, this is 1 but integration of 2 - to 2 + delta t - 2 dt that is equal to 1, so this will be 1. So, remember that any delta function if you integrate from minus infinity to plus infinity, the area under the curve dt will be equal to 1 at wherever that delta function is sitting, do you like this.

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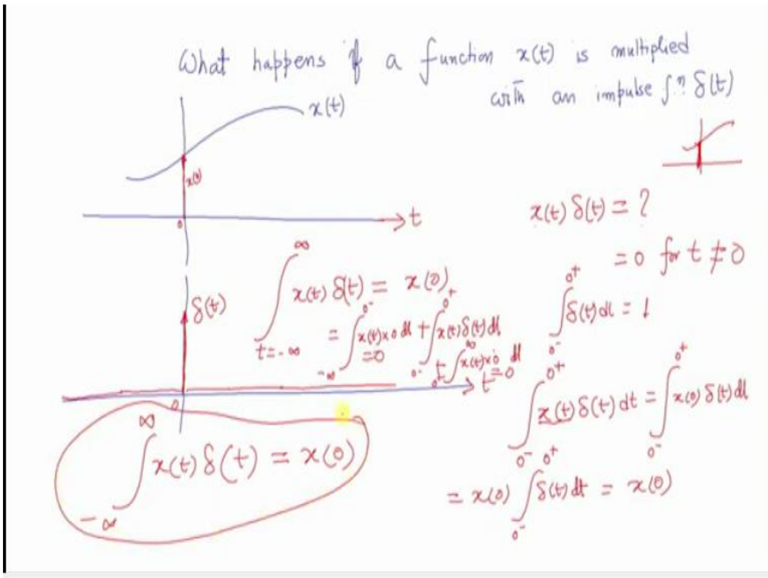

Last comment about the delta function is this; what happens, if a function xt is multiplied with an impulse function that is suppose, you have a function xt and I have a delta function sitting at origin, so delta t not shifted at origin, so these axis are time axis, so I multiply xt with delta t, we have seen that if xt is multiplied with ut, this negative values for t negative whatever value of xt was there that was discarded.

And if I multiply this with delta t, xt into delta t, I want to is what, that is the question we are asking; now the thing is this xt delta t function, we know it was 0 for, t less than 0 and it is once again 0 for t greater than 0, so when you are multiplying xt into delta t, what you will be getting; there is some finite value of x at 0 here, this is the value of x0, for other values it has no meaning.

Because this function into 0, this function into 0, functional values so nothing will; only at t equal to 0, something it may give you some values so, xt into delta t is a relevant extend to delta t, I will say this is equal to 0 for, t not equal to 0 that is certain because you are multiplying this with 0, this with 0 for, t less than 0 as well as for t greater than 0, so that will be 0, I know for certain.

Then, what is the value of xt into delta t at y equal to 0, well that cannot be said because xt has got a finite value but delta t as I told you at t equal to 0, it blows up towards infinity, nothing is disturbing like that or what is the value of this function at t equal to 0, we do not know but what we know is this one about delta t, I am certain about this, delta t dt is equal to 1, this much I know.

Therefore, let us try to see that xt multiplied by delta t dt, this integral let us concentrate upon, what it will give, now definitely the value of xt at t equal to 0 minus and at t equal to 0 plus and at t equal to 0, they are one and the same thing, any well behaved function, if I ask you to find out you know the equation of this curve, I ask you to find out what is the value of the function at t equal to 0, no problem.

You calculate the value of the function at t equal to 0 minus and 0 minus is what number; any small number, whatever you can think of, very, very small number, so do you think the value of the function is going to change at 0 minus 0 plus 0 plus? No, it will be close to 0, so this integral will become equal to x0, I can over this range and delta t dt, what is x0; x0 is this one, finite value.

And once it is there, you can take this x0 outside and you can say 0 minus to 0 plus and you say, it is delta t dt, so but this integral is 1, so this will give you x0, got the point therefore, if you multiply a function xt with delta t from t equal to minus infinity to plus infinity, this is going to give you x0 only, this integral, why? Because minus infinity to 0 minus delta t is 0, xt may have some values but that does not matter.

Then 0 minus to; so this I write down it is like this minus infinity to 0 minus, xt may be there but delta t is 0, dt + 0 minus to 0 + xt delta t dt, this whole integral minus and then plus the third term is 0 plus to infinity xt delta t dt, of which this part xt delta t for, t greater than 0 plus, this is 0 so, this will be 0 here, so these 2 vanishes 0, this is also 0, so you are left with; so we say that even if you integrate it from minus infinity to plus infinity xt delta T, this will be nothing but x0, okay. We will continue with this in the next class, thank you.