**Network Analysis**
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**Lecture – 45**
**Laplace Transform Applied to Circuit Analysis – 1**

Welcome to lecture number 45 and we have been discussing about how to solve system equation or rather differential equations. When the excitation is any function xt using Laplace transformation so in our last class, I introduced with the concept of Laplace transform and Laplace transform of various useful functions okay.

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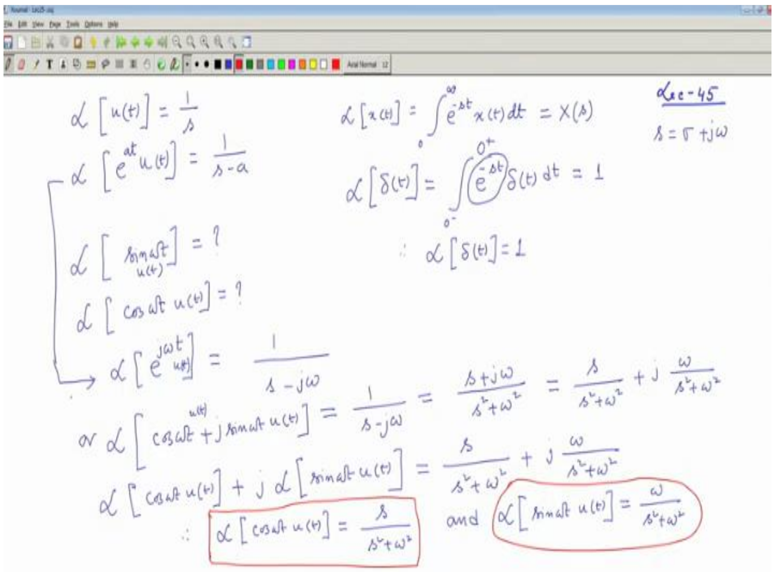

Today we will continue with that for example we will try to find out for example we have seen that Laplace transform of ut is equal to I mean some results I am just writing 1/2 we have seen. Similarly we have seen that Laplace transform of e to the power at into x into ut this is 1/s-a is it not that is what we have seen so we are discussing with unilateral Laplace transform that is from 0 to infinity.

We will integrate the function and the basic definition of Laplace transform of xt as we have seen is equal to this integral 0 to infinity e to the power -st into xt,dt and this is we were writing as because it will be a function of s where s is a complex number sigma + omega. So this is putting these values integrating it we got some results of Laplace transform of some useful functions.

First, let us today do another Laplace transform of another function which is delta t unit impulse function which is just very simple but only thing is this delta t function exists from 0- exist between 0 - to 0 +. So this integration I will then carry out from 0 - to 0 + because 0 to infinity instead of that it is like this and then e to the power - st into delta t, dt. Because beyond 0 + the value of the delta t is 0.

So this integral 0 to infinity is essentially 0 - to 0 + but when you integrate it this function it is a function multiplied by delta t. So this will be actually the functional value of this at t = 0 that is this will be equal to then 1 therefore mind you that Laplace transform of delta t = 1. This is another useful results but from that we have found out the Laplace transform of differential of a function integral of a function and so on.

Now today we would like to know what will be the Laplace transform of say sine omega t what is the Laplace transform of this and what is the Laplace transform of cosine omega t . Of course sine omega t into ut mind you that is important in to ut. Now these things can be derived very easily, and I will take records of this particular formula that is Laplace transform of e to the power at, ut=1/s-a where a could be also imaginary it does not matter is a some constant number.

Therefore what you do is this we will first try to find out Laplace transform of this is the easiest way to find out the Laplace transform of e to the power j omega t. So using that result I can say that where j omega is the imaginary constant number, and this is t. Therefore by virtue of this formula we will say that it is = 1/s-a is it not s- j omega it will be. Because the value of a is j omega in this particular case.

Now or this Laplace transform of e to the power j omega t into ut of course is nothing, but Laplace transform of cosine omega t + j of cosine omega t, ut + j of sine omega t into ut is it not and this Laplace transform is 1/s – j omega. Now this thing I can multiply with s + j omega both numerator and denominator and if you do that then you will get here omega square mind you j is root over – 1.

So this can be written as s/a square + omega square + j omega/s square + omega square. So Laplace transform of cos omega t, ut + Laplace transform. So this can be written as Laplace transform of cos omega t, ut + j Laplace transform of sine omega t into ut and this we have got this to be = s square + omega square + j omega/s square + omega square therefore the real parts and real parts will be same and imaginary parts if you equate you will get the same results.

Therefore equating the real parts I will say Laplace transform of cosine omega t into ut is nothing but s/s square + omega square and Laplace transform of sine omega t, ut is nothing but omega/ a square + omega square. So these two results are very useful in circuit analysis because our inputs may be sinusoidal functions which is quite common and so these two results are very important so easier way to remember it cos omega t.

So cos that s is in the numerator denominator it is s square + omega square in both cos and sine Laplace transform and for sine omega t on the numerator it will be omega/s square + omega square that I must remember correctly okay. Therefore these are the Laplace transform of these 2 important sinusoidal functions sine and cos okay.

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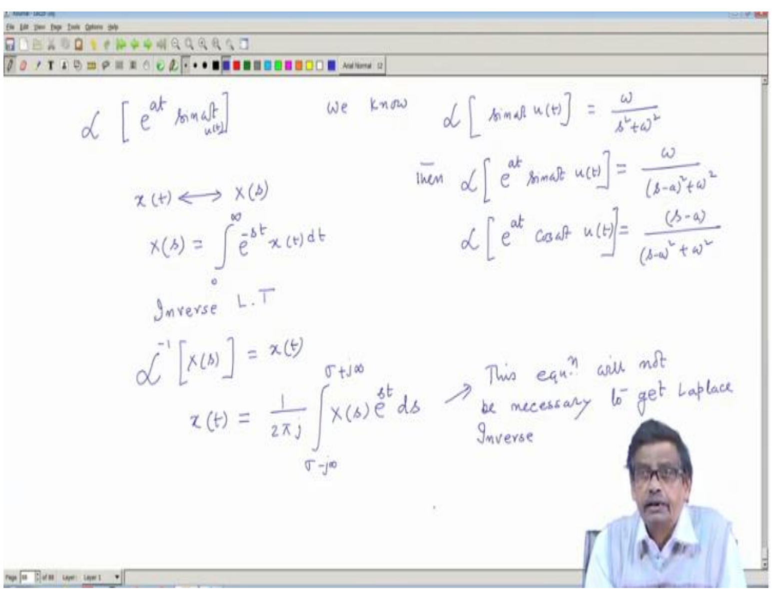

Now for example if I say let us try to now write down some asking you something for example if I say Laplace transform of e to the power at into sine omega T what will be its Laplace transform, we know we will write like this argue like this we know Laplace transform I mean into ut of course. We know Laplace transform of sine omega t, ut = omega/ s square + omega square. If this is true then then Laplace transform of e to the power at sine omega t, ut it will be same thing, but s is replaced by s - a this is the idea got the point.

Therefore the earlier results that we got if we know the Laplace transform of xt which is capital XSS then if you multiply that function with and exponential e to the power at is same Laplace transform but s is replaced by s – a. Similarly for cosine omega t Laplace transform of e to the power at cosine omega t into ut. For cosine omega t Laplace transform is s/s square + omega square that is what we have seen but since it is multiplied by e to the power at, s should be replaced by s – a.

Wherever s occurs on the right hand side should be replaced by s - a like that. Now what is the idea, idea is I will take the Laplace transform of a given differential equations and it will result into an algebraic equations and then whatever output I am interested in I will take the Laplace inverse of that. So mind you xt and xs form a pair if you know xt then xs we have written several times it is equal to 0 to infinity for once I take Laplace transform e to the power - st into xt into dt in that that is what we have been writing.

What is the inverse relationship that is if I know xs what should be xt I will simply write down without proving anything that xt inverse Laplace transform that is Laplace inverse of xs = x t it will give you xt. Now what is this operation Laplace inverse it will be shown it can be shown that xt = 1/2 pi j then sigma - infinity to sigma + j infinity into xs into e to the power st, ds. So if you integrate this function which looks very much complex and but anyway it can be done, and you can get the corresponding time domain description of xs.

Then it looks like it may be a very difficult task when I want to apply this to find out the response of some circuit in time domain, I go to Laplace domain no doubt but then I have to come back to time domain taking the inverse. But the easiest way to do it we will not use this the this equation will not be necessary to get Laplace inverse. In fact we will try to avoid the use of this rather complex looking equations to xs is known multiplied with e to the power st integrate it get the time domain description. Instead what people do is this for some standard function xt and xs.

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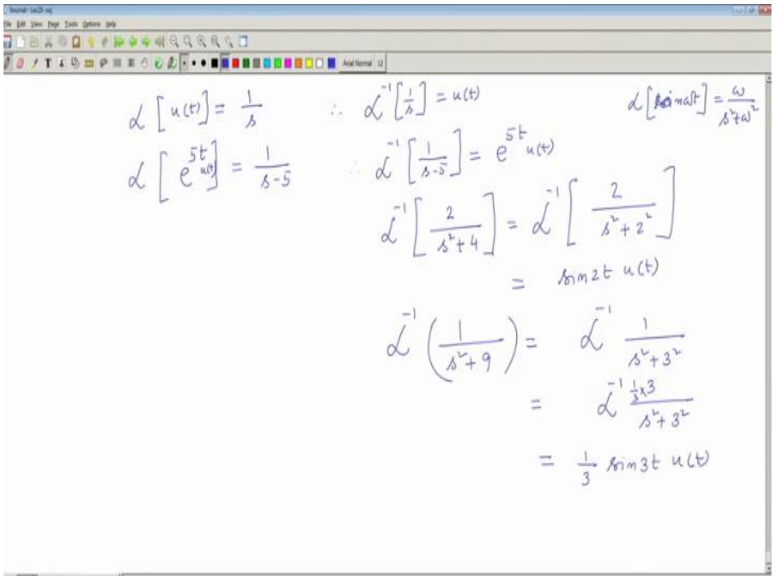


For example I will better write it in this way for example we have seen that Laplace transform of say unit step = 1/s then I will say this table I have to remember this is I have evaluated this I have already evaluated. So I will say Laplace inverse of 1/s = unit step so look at the description on the right hand side try to fit it with some transformed formula then you are done.

For example Laplace transform of e to the power 5t into ut = 1/s-5 correct therefore I will say that Laplace inverse of 1/s - 5 is nothing but e to the power 5t into ut like that got the point. Similarly Laplace inverse of this function for example I am just writing some function Laplace inverse of 2/s square + 4 suppose I have been asked to find out what is the Laplace inverse of this this s square + 4 means Laplace inverse of 2 you know s square + 2 square is it not . Now I know Laplace transform of cosine omega t or sine omega t = omega/ s square + omega square.

Therefore, you just match these two and say that the Laplace inverse of this quantity is nothing but sine 2t into ut that is all got the point. Similarly suppose somebody says Laplace inverse of 1/ s square + 9 what I am going to do is this I will say that Laplace inverse of 1/s square + 3 square but you see there is no 3 here. So what you have to do is this you have to multiply with a constant one third and say that Laplace inverse of 3/s square + 3 square.

That is what you do you what I am doing here I must bring this 3 here on the top and so I will say okay multiply with 3 and divide with 3 in the numerator. Then i will say this is the Laplace transform of one third sine 3t into ut got the point. So I have to looking at this standard transformed formula or looking at some Laplace transform table I should cost my Laplace transform equation in such a fashion that it is inverse can be readily understood by me and that way we will be applying this formula to take the inverse.

So while taking the inverse we are not going to use this so-called complex equation then the all the advantages goes every time you take the inverse of this complex equation. So beforehand we make some table and our input signals after knowing several properties of Laplace transform can be easily formed out. So this is the scenario there are other properties of Laplace transform very interesting properties. But since our focus will be how to apply this in network analysis, I will just take a simple example and try to first tell you what it is all about okay.

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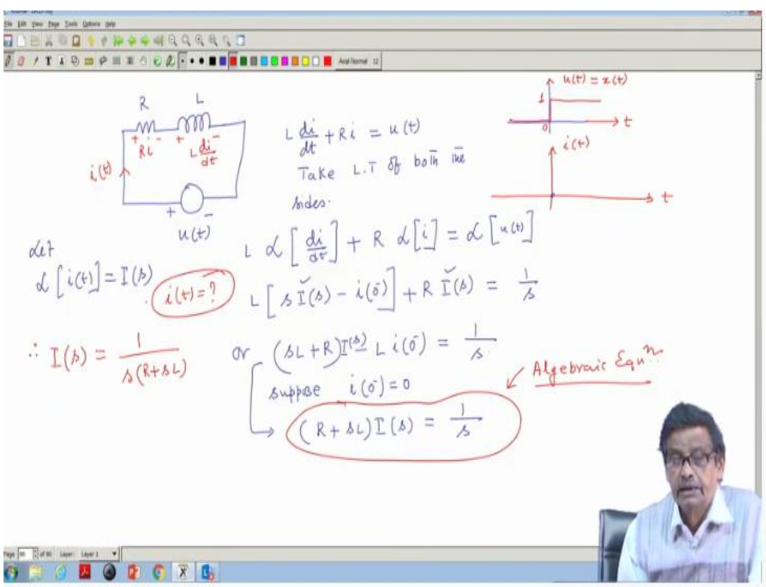


Now let us come to an example for example the standard problem that we have already done suppose you have an r and an inductor L and here you apply a voltage which is suppose ut unit step voltage. What is the nature of the voltage? Nature of the voltage as you know it was 0 all along up to $t = 0$ then there is a step jump of 1. So you are applying a dc voltage across the network from $t = 0$ is it not this is ut and this happens to be your excitation xt.

I want to find out what will be the current in this circuit now try to follow me very carefully. So I want to sketch it. This result is already known by classical method but let us try to apply the rule in the Laplace transform domain okay. So what I am going to do is this I will write down first the differential equation and this we know Ldi dt and this we know ri with the assumed direction of the current.

Then i will say the equation is Ldi dt + ri = ut. So in the time domain the differential equation is like this then you can take Laplace transform of both the sides. Take these points you just try to note take Laplace transform of both the sides. If you take Laplace transform of both the sides, you will get Laplace transform of Ldi dt l can be taken outside L being a constant Laplace transform of di dt + R Laplace transform of this variable time domain variable I which is still not known and on the right hand side I will get Laplace transform of unit step ut.

Now let Laplace transform of it which is the unknown which I want to find out be denoted by this capital letter IS. So this one L into what will be their Laplace transform it is SIs - i0 - you recall the differentiation formula of a time domain function its Laplace transform is Sis - i0 - 1 + R into Laplace transform of this one is Is and this is = 1/s and we want to get solution for $t < 0$.

So this is our whatever time function you will get there for $t < 0$ so this is the thing all I can say that SL plus this term and this term you group together - Li0 - Is = 1/s here this will be the thing into Is. Suppose i0 - = 0 in there was no initial current then this equation would have been R + sL into Is = 1/s is it not. So I get an algebraic equation, so this is an algebraic equation our goal is to find out Is.

So what next, I will do is this therefore I will write Is is nothing, but 1/s was there this thing will come down here R + sL is it not that will be the thing. Then of course my goal is to find out it what is it that is my final goal in real time what will be the value of the current when such a voltage has been applied across an series RL combination. Next step will be there for it what we have to do is you have to take Laplace inverse of Is and if you take Laplace inverse you will get the it.

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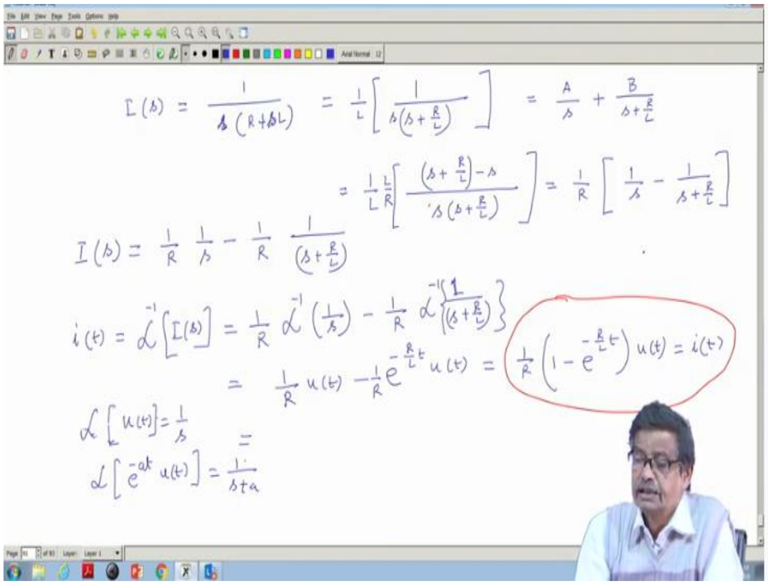

Now let us do this so I have got now Is I have got it to be how much here s into R + sL. So write here s into this is s + R + sL is it not this is the thing and by 1. Now this thing see in Laplace transform the formulas we know is s plus something not to a so this L you take common so I will take common of L so 1/L you take common and then it will look like 1/s + R/L into s is it not this will be the thing.

Now what you have to do is this you have to take does it fit into any of the known formulas really not. So right now I am not sure so what I will be doing is you break it up into some partial fraction try to write it in this form a/s plus some b/s + R/L and try to find out the constants A and B. In this case it is so simple that is why I will not I can do mentally so also you what you do is this on the top it is s into s+ R/L it is there.

So on the top there is a constant what you do you add this then -s there is no s term, so you do it like this and then there is no R/L also on the top. So multiply this thing with L/R got the point. So this and this are one and the same if you just manipulate this s goes R/L multiplied with L/R will give rise to 0 here got the point. Therefore this will simply become = 1/R into this thing . So you separate these two terms.

So it will become 1/s - ss goes and it will be s + R/L got the point. So Is will be = 1/R into 1/s- 1/R into 1/s+ R/L is it not this is the thing now you look at these things it = Laplace inverse of Is which means 1/R I am writing elaborately so that you do not miss these steps later it will not be necessary when you become habituated with it. So Laplace inverse of these – 1/R Laplace inverse of 1/s + R/L this is one.

Now what is the Laplace inverse of 1/s it is ut so 1/R into ut and what is the Laplace transform by s + RL it is nothing but e to the power this is the value of A Laplace inverse. So e to the power - of R/L into t into ut and by 1/R. So the result will be 1/R into 1 - e to the power – R/L into t into ut and that is equal to your it.

So interesting point to be noted that if you want to solve a differential equation using Laplace transform you know that it will give you the complete solution in one stroke including even if there is some initial condition here of course I have taken i0 - = 0 you could also carry on with this i0 - no problem and take the Laplace transform write down the differential equation take Laplace transform of both these sides and I will write down differential equation only.

So this formula is to be remembered Laplace transform of dx,dt is x into xs - x0. So using that formula only thing here instead of x it is I you take, and Laplace transform of the input signal must be known an own signal so 1/s and then manipulate this terms this is algebraic equation. So as if you are solving a dc circuit Is = some this by this. Therefore finally you arrange this term appropriately.

So that you can connect you can easily make up your mind that what will be the Laplace inverse of 1/s in that form you have to break this two term by partial fraction you can also do it, but it is so simple a case I did it just mentally you can also do it. So do it like this then take Laplace inverse Laplace inverse of 1/s is a standard thing unit step Laplace inverse by of s + 1/s + R/L is also standard e to the power – at, ut.

So what are the formulas we have used here I have used two things two results Laplace inverse of ut is 1/s and Laplace inverse of e to the power - at into ut = 1/s + a. I think you have got the idea who will talk on this in the next class thank you.