Network Analysis

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Lecture – 46
Laplace Transform Applied to Circuit Analysis – II

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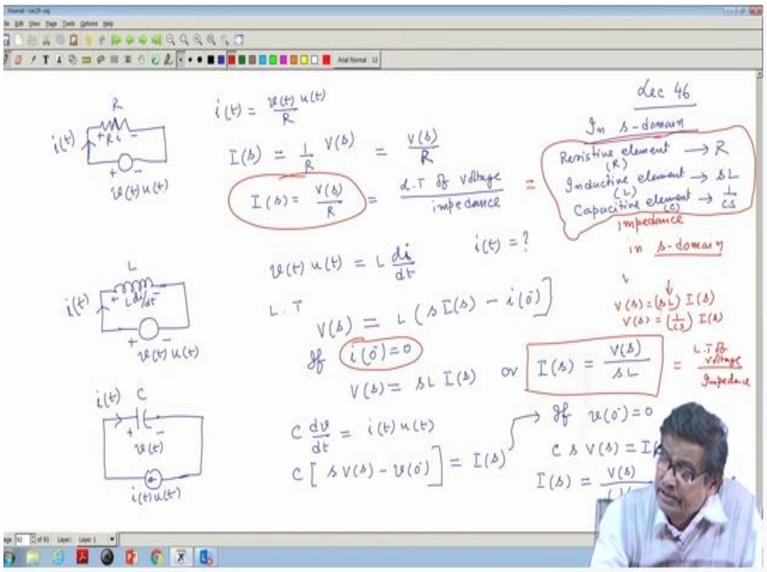

Okay welcome to lecture number 46 and let us continue to have play Laplace transform technique to solve electrical network problems. In my earlier class I solved a simple RL circuit how to do it. Now I will be a little bit more formal with each of the elements and their Laplace transform. For example, I will say that I have a resistance R pure resistance and it is excited with the voltage any voltage it could be.

But only thing is this voltage is applied at t =0 before that vt was 0 that was that will be always true. This voltage I have applied I am interested to find out the current of course in a resistive circuit one need not go to a Laplace domain to solve it but to make the idea clear let us proceed like this. So in this case it =vt/R straight straightaway vt into ut/R that is known. But what I will be doing is here the, this equation itself is a; what is called an algebraic equation.

No differential term but nonetheless let us take Laplace transform of both the sides. So Laplace transform of both the sides if you take it will become Is Laplace transform which I want to know and 1/R is there R and Laplace transform of vt, ut is vs. Therefore, you see the Laplace transform of the current is equal to Laplace transform of the voltage divided by R that way we should start interpreting the thing okay.

And resistance is a memory less device therefore its initial current does not matter. So the present value of the current is decided by the present value of the impedance that is all. So nature of vs and Is will be same so Is =vs/R that is I will say Laplace transform of voltage because voltage by some impedance is current, we know. So Laplace transform of voltage input voltage divided by the impedance, impedance in general here it is only R no inductance is there.

Let us take inductance separately for example here L here you have applied voltage vt and there will be some current it what is vt, vt into ut I have applied. Therefore here of course the voltage current relationship will be I vt, ut applied equal to voltage drop here that is Ldi, dt and this will be equal to Ldi, dt is it not. I want to find out it is what? So it does not appear here explicitly in the form of differential I take Laplace transform of both the sides Laplace transform we will take about the sides.

So left hand side will be vs and right hand side will be L into sIs – i0- this will be the thing if i0- mind you this point is important if i0- is 0 then I will say vs will be simply equal to sL into Is or I will say the current what is the excitation here? The input voltage current will be simply vs Laplace transform of the input voltage divided by sL. So with the initial condition relaxed the current in an inductor can also be expressed in the same way as we have done for resistance.

Some voltage divided by some impedance voltage means Laplace transform of the voltage divided by impedance which was R. Similarly, here I should say LT of voltage Laplace transform of voltage divided by impedance. Now this impedance in this case if you write it is like a impendence in s domain impedance when I will tell about this impedance s domain instrument that is how it is. So if it is an inductance is present the impedance offered will be sL not LL or like resistance only R.

Therefore, with initial condition relaxed this is true. Similarly take capacitance for example a capacitance is there C this is the voltage and suppose you have excited the capacitance with the current source it like this, so it is flowing like this. Now, voltage at any time across the capacitor is integral idt/c but as I told you earlier, I will be always try to write down the terminal voltage current relationship of a particular element in the form of some differential equation.

And I know that I want I know c dv dt is nothing but it no this this is the thing c dv dt into it take Laplace transform of both sides so it will be c svs-vo- initial voltage across the plate of the capacitor is equal to your is then I say if v0- is 0 if the voltage across the plate of the capacitor prior to the this is it into ut I have applied mind you. So it into ut so if v0- = 0 then I will say cs, vs = Is and I will say that okay Is can written in this form that vs over 1/cs this manipulation you just note down.

This the current in the circuit is the Laplace transform of the voltage across that element divided by its impedance which happens to be now 1/cs in his domain. Therefore, impedance of in s domain I summarize in s domain resistive circuit resistive element R will appear as R only no change for inductive element that is L in time domain will appear to be sL capacitive element in time domain it will be c it will appear to be 1/cs this is the crux of the matter. These three things one has to remember so in time domain if the circuit contains several R, L C in s domain you just replace them.

If it there is some s you replace it by sL if there are some capacitances replace it by 1/cs provided this inductor and capacitors have no initial conditions that is the if it was an inductor i0- must have been you 0 then only this linear relationship holds good. Therefore voltage Laplace transform of voltage and current they fall they maintain a linear relationship because vs = sL into Is is it not these are constant sL.

Similarly, for capacitance it will be vs = 1/cs into s so this is the impedance of the capacitive development and for resistive element of course it remains as R, R into Is got the point but this will behave as an sL and 1/cs provided the initial conditions are 0 that must be understood okay. I will tell you shortly how to take into account this initial conditions. So one way you take the

Laplace transform write down the differential equation write i0- here then take Laplace inverse of both the terms you will get the solution that is fine, but I will tell you something else also you can do it.

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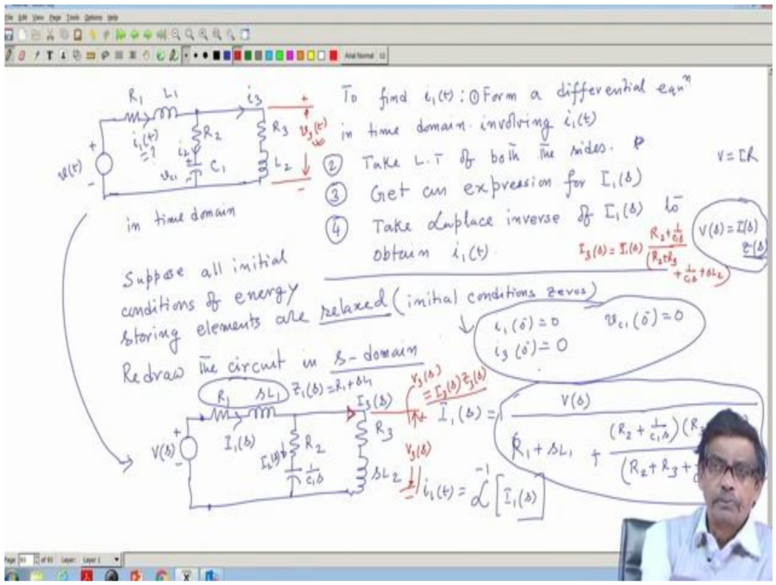

What is the discussion I had in the previous page? how it can be now translated into for example, I have a circuit like this now first let me tell what I am trying to do is okay given a circuit in time domains circuit will be given for example you are given a circuit like this rather complicated circuit I am taking R capacitance and there is another resistance and another inductance is there and suppose I am going to apply a voltage and this voltage I will put now.

I will remove all the restrictions. These voltage could be anything including your impulse no problem because the Laplace transform of impulses also known which is 1. So everything is so nice now suppose this is in time domain this is the circuit given to me this is suppose R2 this is c1 and this is suppose R3 and this is L2 and this is R1 this is the circuit given I have been asked to find out the currents.

Now if I had been asked to calculate the current and I want to remain in time domain see the involvement that would have been necessary you have to assume this current i1 in time domain i2, i3 apply KCL here suppose I want to know only this current what is this current I want to know? Then I have to nonetheless write down the time domain differential equation will there be a differential equation? Yes, because there are energy storing elements apart from resistances then write down form a differential equation even using a Laplace transform.

I am telling what I have to do is this if I want to find ilt what is the thing form a differential equation in time domain this is step number 1 step number 2 will be form a differential equation in time domain involving i1 because I want it1 to know. So I must form a differential equation out of these 2 loops these that do something get me one differential equation and that can be done.

Now second step would have been that take Laplace transform or both the sides then step 3 then get an expression for I1s and finally take Laplace inverse of i1s to obtain i1t these are the steps they have to follow nothing doing. So formally if you want to apply Laplace transform first write down the differential equations get this no voltage do something to finally get a differential equation involving i1t alone no i2, i3 should be there.

Okay then take Laplace transform get ils etc. But this looks quite a some sort of a heavy load I mean following these steps even with Laplace transform I am telling. Now can I do something better? Suppose I have been told that suppose all initial conditions I also tell this much this information suppose all initial conditions of the energy storing elements are relaxed. What do I mean by this term relaxed?

It means that these are 0 initial conditions are 0 all are 0s what does that mean? i10- will be 0 for this inductor if you call this to be your i2 and this to be your i3 then I will say so this means for this problem it means that i10- = 0, i30- there is an inductor here its initial current must be known that is 0 and here is the capacitor the voltage across the plate of the capacitor vc1 0- =0 these things are given.

Suppose if that is given if this is known then I will say for this circuit to find out i1t you need no start from differential equation What do you first do this is that draw the circuit redraw the circuit rather in s domain this you try to understand. So circuit structure will remain same so it will be like this there is a voltage source there is a resistance there is a inductance here and there

is resistance here, capacitance there and there is another resistance, another inductance is there is it not.
Then what you do all the time quantity there replace it by its transform. So vt Laplace transform of vt is vs write vs all the element values for example resistance R1 will remain R1, R1 resistance will remain as it is no change R3 then the inductances this was L1 then you write sL1 no j into sL1 no s into L1 no imaginary number like that , that imaginary thing sigma + 0 omega within s it is there but we do not care sL1 and this capacitance so I will write 1/c1s is its impedance in s domain.

So write all the impedances element wise in s domain and here I will write sL2 once I do that then I have gone to s domain and in s domain what is the thing vs = Is into some Zs and once you can write the equation involving voltage and current in this form what I told you, you can revoke all the things we have learned in DC circuit in this circuit as well in fact we did this in case of while finding out the steady state currents of a sinusoidally excited currents or voltage in RLRC circuit is it not.

V farad = I farad into z so here also similar things happens not only algebraic but the relationship in terms of this equation Z has been constant v=iR whatever I did in resistive circuit I can now do this here as well for the point. Therefore this current will be i1s this current i2s and this current I will say i3s. Now everything in his domain so from these to these you just sketch it what I am discussing about I am telling you need not write the differential equations if initial conditions are relaxed just redraw the circuit with the parameter values replaced in terms of as appropriate if it is an inductor it should be s into that inductor.

If I is an capacitance impedance will be 1/c1s that is what we saw in the previous slide. Therefore you do this then you know vs -Is into Zs therefore I will be able to apply series parallel rule of finding out equivalent impedance things like that. Therefore I will say now your i1s look follow me very interesting this will be equal to applied voltage vs divided by equivalent impedance between these 2 points.

What is that equivalent impedance these two will be in parallel so it will be R2 + 1/c1s into R3+ sL2 this Z into this Z by sum of these 2 Z divided by R2+R3+ 1/c1s+sL2 and this is with series with this . So it will be = R1+sL1 that is all. So in one stroke I will get the expression of i1s then you and everything is algebraic no differential so manipulate this do partial fractions and write down the right hand side which will be function of s only in such a fashion that you can easily identify if you take so finally what you have to do.

So i1t you have to take the Laplace inverse of i1s that is this big thing its Laplace inverse with numbers it looks quite a challenging task okay in general I have written if the numbers are there I can always simplify this to some handy expressions and find out the inverse and get the value of the current time domain expression of the current so easily. So one way of handling this is go on writing down the differential equations apply KCL, KVL form a differential equation involving i1t alone then take Laplace transform from both sides get an expression of i1s.

So both these are Laplace transform method only but what I am telling after knowing that in circuit this R1, L1 will get translated into R1,Sl1. So it is an impedance Z1 so people may write Z1s= R1+sL1. Similarly there is an impedance at Z2 at 2+ 1/c1s ,Z3 at 3+ J no J at 3 +sL2 and since vs= Is, Zs for each of the elements therefore I can invoke all the things I learned in DC circuit v=Ri and that was instrumental from which I got the QL and impedance expression as R1,R2/R1+R2 all these things can be nicely applied here.

But in this second method in this method here in this method there is a no differential equation happened. So I will translate this circuit in terms of s and as if now it is a DC circuit sort of thing like that, I will manipulate this circuit get the Laplace form of a particular element and do it. Similarly just last one point about this. Suppose I say that in this circuit also get this voltage v3t got the point this voltage I want to know also in this circuit how do I do it? You have known i3s look these steps.

So I know i3 just I am telling verbally i1 I know then I will calculate what is the voltage between these 2 points it is v3s is it not and what is the value of this voltage? The value of this voltage v3s will be i3s into Z3s what is Z3s it is i3+sL2 that will be this voltage. Therefore if somebody tells me calculate the this voltage as well with v3ts then I have to calculate v3s first because i1s I know if I know i1s what will be i3s, i3s will be equal to the total current i1s into impedance of the other that is at 2+ 1/c1s and then sum of these 2 impedances these 2 impedances what is that?

That is R2 +R3 +1/c1s+sL2 this will be i3s then i3s into Z3s will be this voltage then take Laplace inverse of this one to get with v3t. So it is almost like analyzing your DC circuit. These are the greatest advantages of using Laplace transform. In my next class I will tell you okay that is fine I will give in a circuit I will redraw the circuit in s domain replace inductance by s into L. Replace wherever capacitance is there 1/c into s then some sort of impedance thing I will get and once I get impedance thing everything is vs, is etc. I will be able to find out the QL and impedance or I can even apply nodal method this that to get the currents.

But the all is fine, and you can avoid writing down the differential equation that is another greatest advantage, but these things will be true provided there is no initial condition. In the next class I will tell you how to still retain this advantages that is avoiding writing differential equation at the same time take the initial conditions into account. Thank you.