Network Analysis
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Lecture – 47
Numerical Examples – 1

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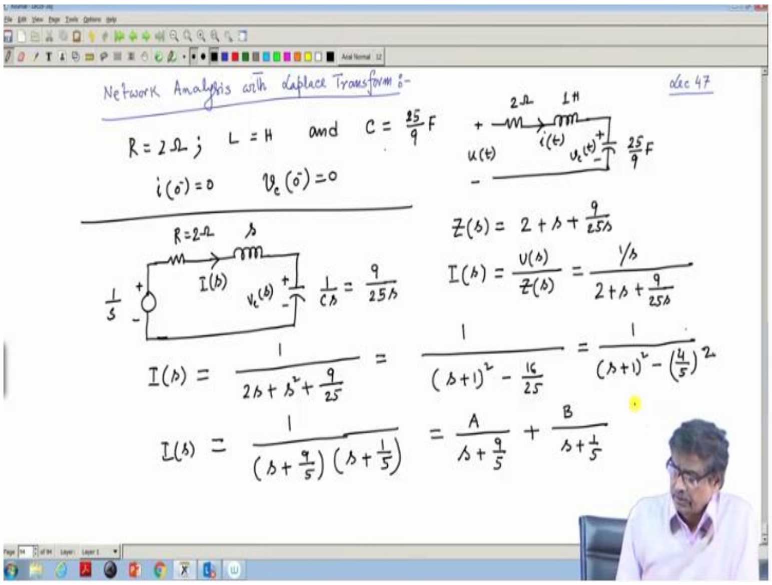

Welcome to 47th lecture and we have been discussing with the network analysis using Laplace transformation and if you remember our circuit will be in time domain with RLC values provided. Then next step will be to replace this redraw the circuit in each domain replacing R/R by R that is no replacement for our resistance replace l/sL and replace any capacitance by 1/cs and if you assume that the initial conditions are relaxed that is all the capacitor voltages are 0 initially.

And all the currents through the inductors are 0 then you do not have to write down the differential equation even in time domain straight away draw the circuit in s domain and solve for it. So I will tell you one example for example you have a series RLC circuit with R = say 2 ohm L equal to say 1 Henry and C = say 25/9 farad. So the circuit in time domain is like this R 2 ohm then an inductance of value 1 Henry and a capacitor series RLC circuit.

Let us apply this technique whatever we have learnt, and this is C = 25/9 farad and our plan is to find out what will be the response of these are it if a in step voltage is applied between these two. So this circuit is in time domain and it is given that all initial condition is relaxed. So this will be some it and this will be the voltage across the capacitor at any time t,vct. So io- = 0 because current through the inductor was 0 and voltage across the capacitor at 0 minus was equal to 0 these are the things given to me.

Now I can solve this writing down differential equation classically that gives you better insight those things we know but let us apply Laplace transform method to solve this circuit. So first step is redraw these circuit this remains 2 ohm no change this inductance will become s into L value of L is 1 Henry. So it will become s in s domain and then this capacitance it will become 1/cs you recall so this will become 9/25 s this is the circuit and here you have applied a voltage Laplace transform of that voltage.

So ut Laplace transform of that is 1/s so we want to know what is the current in this circuit and what will be the voltage across the capacitor plate. So everything you show as a function of s, so total impedance of the circuit is Zs all are series. So 2 + s + 9/25 into s therefore Is just like this is circuit v/Zs so supply voltage that is us divided by Zs this will be the current in the circuit and this will become 1/s us is 1/s divided by 2 + s + 9/25 s.

This s you bring let us first solve for Is so Is will be equal to this s you bring below, and it will be 2s+ s square +9/25 it will become okay now I have to take the Laplace inverse of Is to get the time domain description of the current this how to do it. So I must bring this right hand side in some familiar Laplace transform. So that the inverse can be easily identified and written that is the whole idea.

So general rule is you one thing is clear as you can see this one can be written as s + 1 whole square s square + 2s add plus 1 and subtract another one. So that this will become -16/25 I think you have got this step add plus 1 subtract minus 1. So that it becomes a perfect square because it is so easy to identify that, and this one can be written as any mistake you point out. So this divided by s + 1 whole square minus so -4/5 whole square this will be this.

So it can be easily factorized A+B into A-B everything is real here no Z terms so this I will write it as A+B if you do it will be s + 1 + 9/5 into s -1 +1/5 it will be like. So this is a very happy situation two simple factors of their these roots are real. So this can be broken up into parts these are the steps you have to follow that is why I am solving this problem so this can be written as A by this plus B by this.

So Is = this now if I can find out where A and B are constants to be determined how to determine these constants, I will follow this rule there may be several other rules which you can also adopt but nonetheless you have to break this up into what is known as expand this in partial fraction is the word. So this thing will be so Is so this thing now you just copy it and go to next page.

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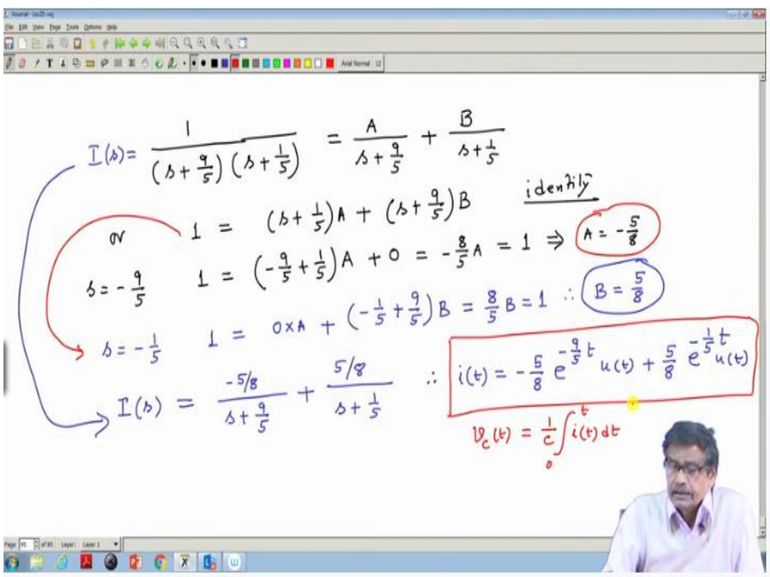

And paste it so this is the thing so I am assumed this to be this one is it not how to determine this constants A and B this is B mind you this will be an identity. So for any value of s left-hand side and right-hand side will be equal what you do is this you multiply both sides with the with this s+ 9/5 into s+ 1/5 so that it will become 1 on the left hand side on the right hand side you will have s + 1/5 into A and s +1 5 goes for this second term is s+ 9/5 into B this will be the thing multiply both sides.

Then this is true this is an identity mind you identity true for all values of s. So to put a value of s such that B will be eliminated so s = - 9 /5 if you put on both sides but on the left hand side there is no s term so it remains 1 and this will then become -9/5 +1/5 into A and this term becomes 0 is it not. So this becomes = -8/5 into A and that = 1 which means that A = -5/8 we leave it this is the important.

Similarly you put two these once again come back to this equation and put s = -1/5 it is easily you can see so left hand side is 1 the right hand side will be 0 into A it will be and this will be 1 – 1/5+ 9/5 into B which will be equal to 8/5 + into B = 1 therefore B will be equal to +5/8. So this is another thing, so we have almost solved the problem therefore you recall that Is then remember Is was this so this was equal to Is this itself was Is.

So Is now written as a like this -5/8 that is A divided by s+9/5 and B is 5/8 + 5/8 divided s+1/5. So from this we substitute the values of a and B now we are in a very happy situation therefore take Laplace inverse of both sides. So that it will become it on the left hand side and on the right hand side 1/s+A its Laplace inverse is e to the power minus at so it will be -5/8 is of course this constant into e to the power -9/55 into t into ut mind you.

Our all Laplace transforms our causal functions one-sided and this will be +5/8 this one into e to the power of e to the power of -1/5 into t into ut. So this will be the answer for the current so I have solved for the current so I dealt with only algebraic equations then express the variable I want to get in terms of s domain and then you expand it in partial fractions and then try to identify this while taking the inverse I did not apply any formula just some standard tables are there and I got the result.

Now my next thing was what was vcst for example coming here if I want to solve this vcst one way of doing this since you have got it then remain in time domain and say that vct capacitor voltage will be 1/C use this formula 0 to t because capacitor had no voltage for t>0 into it,dt you have to integrate these and get the capacitance voltage but this is this it can be easily done. But nonetheless I will still apply Laplace transform to get vcs for example here vcs will be Is into 1/

CS voltage across this Is into 1/cs its impedance capacitance that way we will try to see. Is is already is this one okay.

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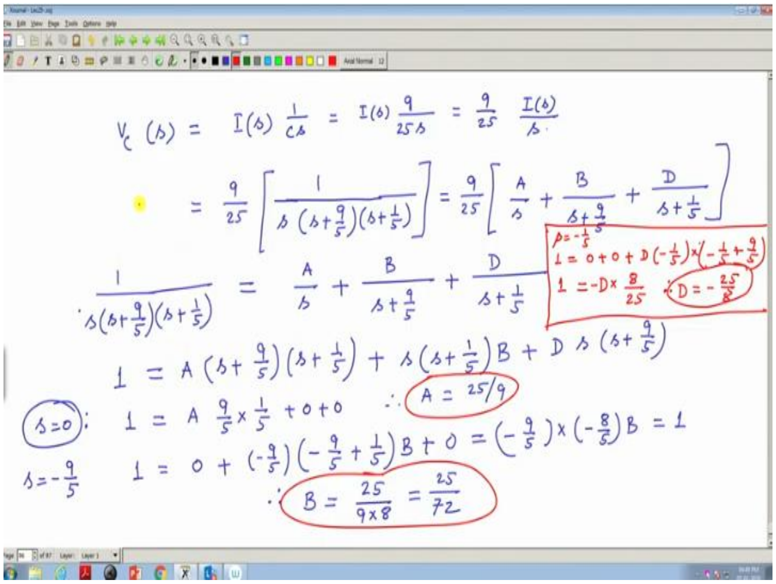

So I will say now I am interested to know capacitor voltage. So I will say capacitor voltage will be vcs will be nothing but Is into 1/cs now 1/cs c value is 4/25 capacitance value c value was so much 25/9 c value so I will say Is into 1/25/9 so it will be 9/25 into s which = 9/25 into Is/s then you put already I have calculated Is that = 9/25 will be there Is was s + 9/5 into s +1/5 and there is 1/s another so this will be vcs here also I have to expand it to partial fraction.

So I will write this quantity as 9/25 let it be this side and here it will be A/s + B/s+9/5 C/ but C I will not write because you may get confused with this C say D/s+1/5 these are the three constant to be determined. Once again, I will determine these constants considering this thing 1/s into let me repeat the earlier stage so that you know what exactly you have to do on pen and paper to get the correct solution.

This I have assumed it to be a by s + B/s + 9/5 + D/s+1/5 going to calculation let us go back. So now you multiply both the sides by this s into s+9/5 into s+1/5 so that it will become 1 and you will be left with A into s+9/5 into s +1/5 this will be the first term second will be s into s +1/5, s+9/5 goes and into B correct and +D into s + 1/5 will not be there this two only will be there so s into s +9/5.

Now here I have to determine 3 constants so first thing is you put s =0 you can put any value of s no problem if you put s = 0 then this term and this term will become 0 and you will straightaway get the value of A. So A into 9/5 into 1/5 this will be the thing other things will become 0 + 0 + 0. Therefore A will be 25/9 that is all similarly put s is equal to, so A value is now known is it not.

So put s = -9/5 let us put s = -9/5 if you put s =-9/5 so this will be 0 and this term will be 0 and you will be left with 1 = this is 0 first term and this term will be + -9/5 that is s into s+1/5 so -9/5 +1/5 and that is equal to B and then this term is 0 because s = -9/5 so how much it will become so this will become -9/5 into -8/5 is it not and this is equal to B this into B so -9/5 and -8/5.

Therefore and this is equal to 1 therefore the value of B will be equal to this will become +25/9 into 8 ,72 anyway put 25 by this one is B that is 25/72 this is the value of B so put a red mark along this so A I have got B I have got finally I have to calculate D to calculate D what you have to do put s = -1,5 if you put this term will vanish first term this term will vanish only this term will remain therefore if you put here I am writing on this page only okay this space let us use it.

So put s =-1,5 on both the sides in this equation so it will be once again 1 = this will be 0 s = 1-5 this also will be 0 s = -1/5 and this thing will remain it will be equal to 0 + 0 + D into -1/5 s into – 1/5+9/5 which is equal to 1 = D into it will be 8/5 so 8/25 and this will be minus negative sign will there. So this is minus therefore D will be equal to -25/8. So I have got the values of A,B and B in this expression vcs. So vcs was this and where ABD unknown so this will be let me write it okay A/s,B/s +9/25.

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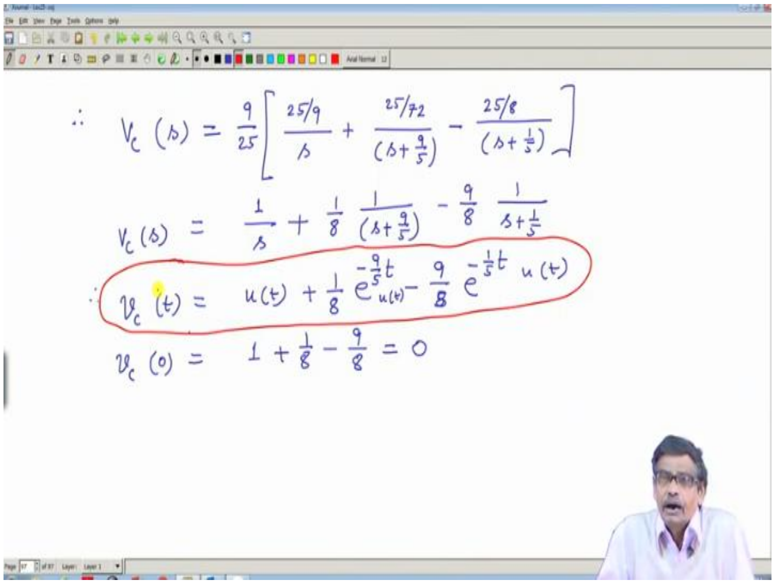

So we come here and say therefore vcs = A/s 25/9 before this one this factor 9/25 was there already 9/25 then there is a bracket there A value just tell 25/9 divided by s + B value, B value was obtained to be 25/72+ so + 25/72 and this is s + 9/5 and finally the value of D which was obtained to be -25/8. So this is -25/8 and this is s +1/5 I am now very happy because it is so this 9/25 you push it inside so this will become 1/s, and this will become 25 goes 1/8 into 1/s+9/5 this 9/25 and here it will be -9/8 into 1/s +1/5 hopefully it is correct this is vcs.

Therefore in time domain taking Laplace inverse it will be equal to Laplace inverse of 1/s is ut only you know +1/8 into e to the power -9/5 into t and this is -9/8 e to the power -1/5 into t into ut 9/8 only thing let us cross shake because capacitor voltage cannot change instantaneously so vc0 should be equal to 0 if that is not happening then something is wrong see vco if you calculate this will be ut is 1 +1/8t=0 this is 1 and this is -9/8 so this is 0.

So this is the expression of the capacitor voltage therefore you see how easy it is now at least understanding this circuit redrawing the circuit in s domain and whatever quantity you want to find out just apply your DC rules okay v= i into R here it is vs = Is and Rs this sort of thing.

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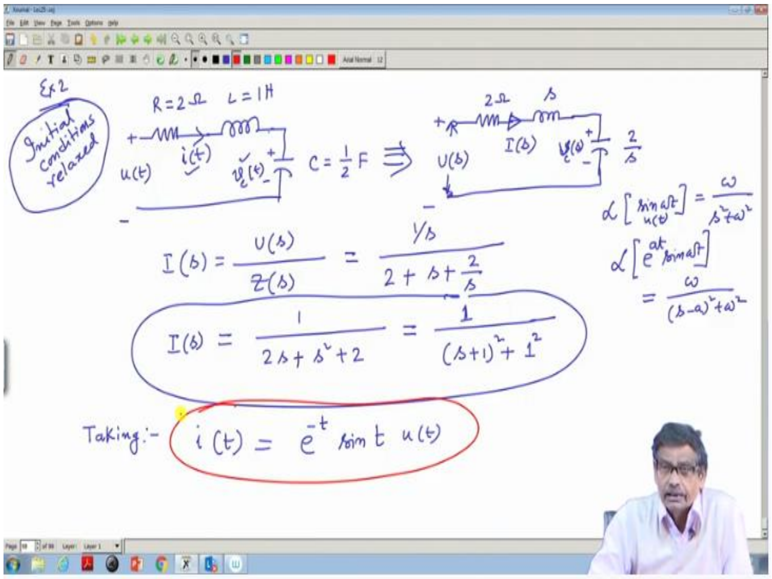

The third part is this one the same problem we will now do with another value of capacitance same circuit. So I just draw the circuit and try to solve it so what I do is this R = 2 ohm example 2 R= 2 ohm inductance let it be 1 Henry only and this capacitance value let us change it suppose capacitance value is suppose now 1/2 farad C = 1/2 farad and let me do it now very quickly here the voltage applied is unit step voltage.

So redraw the circuit in s domain which will be hard and initial condition relaxed that means the current in the inductor at t=0- was 0 and voltage across the capacitor at t=0- is 0 these are the time domain description of this circuit. So I will avoid writing KVL equations because I know initial condition relaxed. So I will simply write this is 2 ohm R remains R inductance remains facing to L so it will be s only and this capacitance is 1/cs so that will become 2/s is it not and this voltage is your us Laplace transform of unit step voltage do not forget to show this sign and this current I want to find out.

So in the same way this current in the circuit will be Is = us divided by the impedance of the circuit which = us is 1/s divided by 2 + s+ 2/s this will be the thing. Now let us solve for the current first and also, I would like to know what is the capacitor voltage capital VCS these two things I want to know. So first let us calculate Is so Is is this which = 1/2s+ s square + 2 and this will then become equal to 1 over here also you add +1 and subtract another one.

So that it will become equal to s +1 whole square + 1 is it not then it remains this one this way you can solve. Now this is Is you see in this case should I go for partial fraction you know that I know this looks like this Is itself is in the form of some standard Laplace transform formula what is that for example we know that Laplace transform of sine omega t = omega/s square + omega square mind you this is ut always there even if I do not write I mean that so omega/s square + omega square.

And if you Laplace transform of e to the power some at sine omega t will be omega wherever s is there s-A whole square + omega square is it not. So it is already in this form only thing this is s+A so taking Laplace inverse straightaway because the moment I know it is in a known form therefore I will say oh it will be equal to the Laplace inverse of this quantity is going to be e to the power -t here plus term is there -t into sine of what is the value of omega 1 means 1 square and here also 1 is there.

So omega/omega square sine omega t 1 into t into ut this will be the solution for the current you remember that I assumed the initial conditions to be relaxed let us see whether it satisfies at t =0 e to the power -t is 1 no doubt but sine t will make it 0 therefore i0 = 0 is verified i0+ will be 0 because i0 minus because of this initial conditions relaxed is i0-. Anyway in the next class we will calculate vcs for this voltage across the capacitor thank you.