Network Analysis
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Lecture - 48
Numerical Examples - II

Welcome to this lecture. We have started solving some example 2 in my last class and we got the expression of it.

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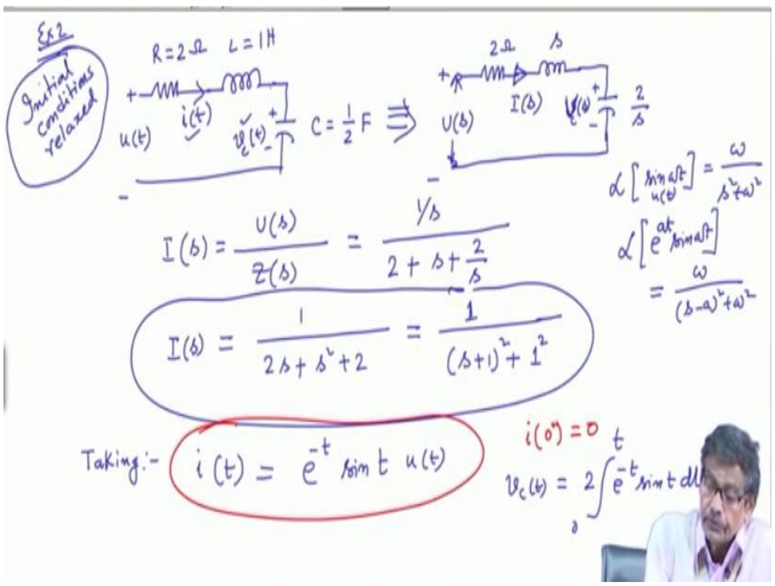

You see this is the example 2 and where capacitance value is now half farad, R, L values are same. I want to get the unit step response. So usual step is calculate zs. This is the series. So 2 plus, do this then take the Laplace inverse, get the value of it. Then, what I am telling vct at this level can be also calculated in this way 1 by C that is 2, half is the capacitance value into 0 to t and e to the power minus t sine t dt if you do, you will get the time domain description of the capacitance voltage.

Nonetheless, this integration is integration by parts or some standard formula is there e to the power x sine bx dx, integration is e to the power x by root over a square plus b square sine bx minus tan inverse b by a. Those way you can get it, but nonetheless let us still apply the Laplace transform to get these Vcs okay. So is I have already got.

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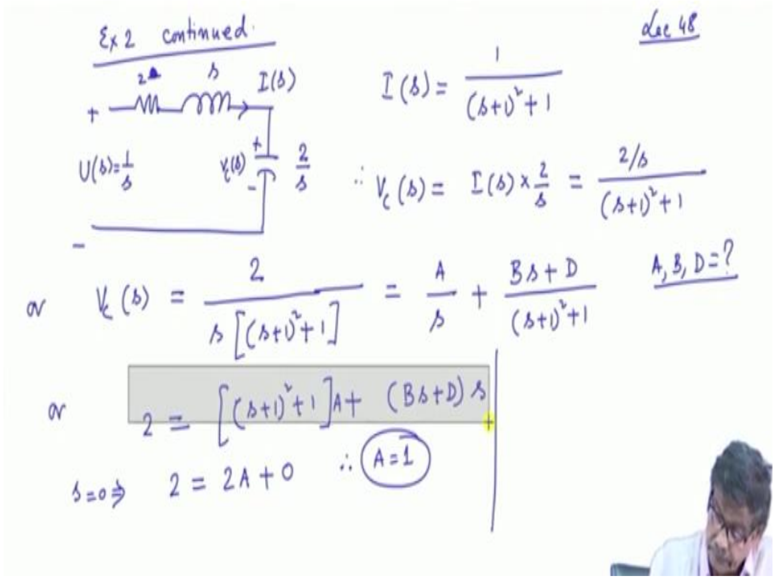

So better draw the circuit, so that we understand what is what. This is the thing so 2 ohm, 2 right 2, this is sl, it was s and this was 2 by s capacitance and this is the voltage, I want to find out, and this is Is, current is and voltage Us, which is equal to 1 over s. So we have already got this Is. We have solved for it, Is was this one 1 by s plus 1 whole square plus 1 square. So Is was 1 over s plus 1 whole square plus 1. This was Is.

Therefore, now Vcs will be equal to Is into 2 by s, this is what 1 by Cs and this will then become 2 by s into s plus 1 whole square plus 1, where Vcs, r Vcs is equal to 2 over s into s plus 1 whole square plus 1. This is the thing. Now here of course, I cannot directly find out this one. Earlier for current solving, it was very simple. It was in standard form. This s was not there, but here it partial fraction you have to do. How to do it? This will be A by s and plus this one.

On the denominator A square is there, so standard form will be Bs plus D, say divided by s plus 1 whole square plus 1. This will be the description in partial fraction ABD are to be determined, what. So here once again, we will say that this is an identity. So you have to form three equations, so that you can find out ABD by putting different, different suitable values of s. So you multiply both sides with this denominator or you will get two is equal to this thing.

You bring it, multiply it this side, so that it will be equal to s plus 1 whole square plus 1. This s s cancels out plus this will be Bs plus D into s only, because this factor and this factor will go. So this is this equation, A into A thank you. So this is the thing now the value of As can be easily calculated by putting s equal to 0. This BD will go. So from this if you put s equal to 0 on both the sides, so you will get 2 is equal to this s is 0, so 1 square 1 plus 1.

That is 2 into A plus this term will be 0, because whole thing is multiplied by s, so 0. Therefore your A will be equal to 1. If you practice it you can do it in a faster way. So A is 1, one constant is obtained. Once this constant is obtained, I rewrite this in the next page, so that you copy this one. This is what we are copy and next page we paste it.

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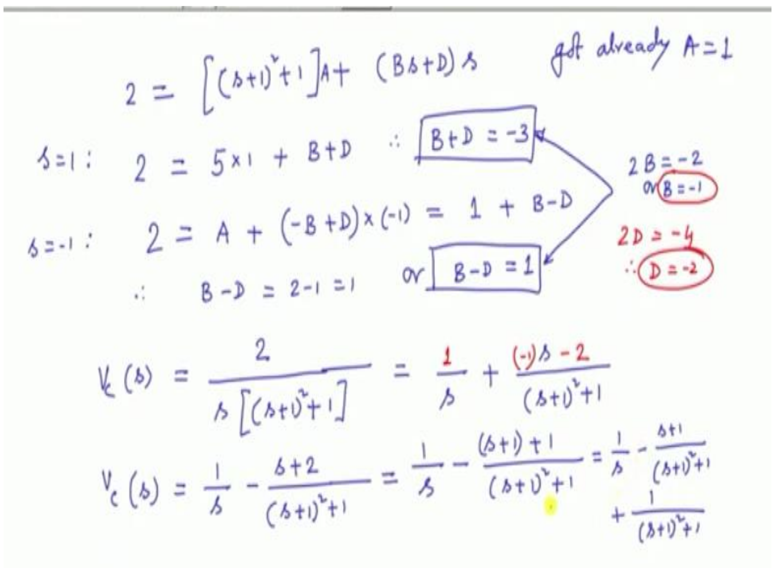

So we are dealing with this equation and you know this is s plus 1 whole square plus everything nothing got cut, know. So this is the thing and we have already got A is equal to 1 got, already A to be 1. So now, I have to put some values of A, so that it becomes suitable values of s any values you decide. Say, I will put s equal to plus 1, let us put on both sides, because it is an identity. So if you put s equal to 1, this will be 2 is equal to 1 1, 2 2 square, 4 plus 1 5 into A.

But A is already 1, so A1 plus you will get B plus T. B into s 1, therefore your one equation will be B plus D is equal to minus 3. This is one equation. Another equation you put A is equal to say minus 1 you put on both the sides., so that left hand side is 2. This will become only A. This goes 0, A into 1 A then plus. This will be minus B plus D and once again minus 1 into s and this is the thing, but a is equal to 1, so this will be equal to 1 plus B minus D.

Two is equal to 1 plus B minus D. This minus will make it plus and minus D. Therefore I will say that B minus D is equal to 2 minus 1 is equal to 1 or I will write here B minus D is equal to 1. This is another equation. Now these two equations can be easily solved to obtain the value of B and D. Add these two, this two from this two you will get 2B is equal to minus 2 or B is equal to minus 1. So B is equal to minus 1 you will get.

Subtract these two or once you got B, anyway subtract these two you will get 2D. This minus this 2D is equal to minus 4 minus 3 minus 1. Therefore, D is equal to minus 2. So fully it is correct. Let us see. So all constants, I have got now, then I will say that is this thing Vcs is this one, because so this I will copy. It is now necessary. So this is the thing, got the point. So Vcs now I have got the values of A, B and D. So I will put it here.

So A value of A was obtained to be 1. So I write that a is equal to 1. B and D also I have got, B is equal to minus 1. So this B is minus 1 into s plus D only. D is equal to minus 2. So this will become minus 2. So this can be written as Vcs, I will say, is equal to 1 over s minus s plus 2. This minus can be taken outside s plus 2 into s plus 1 whole square plus 1. This way I can write and this can be separated as 1 over s.

See after you do you will become very fast in your dedications maybe faster than me. So do not worry about that. So these thing only, you see this numerator should be written as s plus 1 plus 1 by s plus 1 whole square plus 1 square, then this can be written as 1 over s plus s plus 1 divided by s plus 1 whole square plus 1 and then plus 1 over, that is you group this one, 1 over s plus 1 whole square plus 1. Both of them will become minus, correct. So this is minus and this is also minus, got the point. Therefore, we have got next page let me do it slowly.

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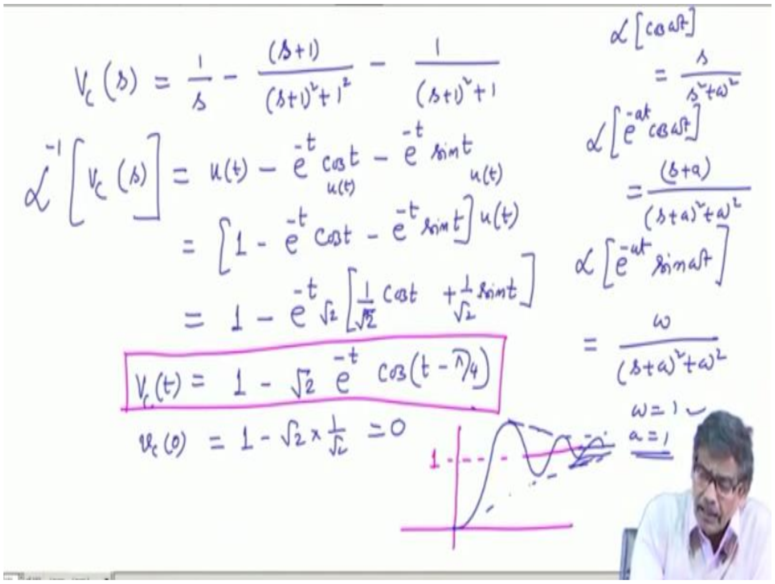

So I have got Vcs to be equal to, you just see, one over s then minus this term s plus 1 whole squared divided by s plus 1 whole squared plus 1 1 by s minus s plus 1 by s plus 1 whole square by 2. This is no square, is not? Minus 1 over s plus 1 whole square plus 1. Now here you see you have to, here your practice with Laplace transform will tell you, okay Laplace inverse of Vcs will be 1 by s Laplace inverse is ut. Can you tell me what is the Laplace transform of this?

Laplace transform of cosine omega t, if you recall, it is equal to s by s square plus omega squared unit and Laplace transform of e to the power minus at, cosine omega t will be the Laplace transform. Replace s by s plus a, that is s plus a divided by s plus a whole square plus omega square. Similarly Laplace transform of e to the power minus at sine omega t is equal to omega by s plus a whole square plus omega square. So look at these two expressions.

Omega, we can easily recognize this to be 1. In our example, omega is equal to 1, a is equal to 1 in our example. So if you take the Laplace inverse, I will straight away write it to be e to the power minus at, a is 1. So e to the power minus t into cos omega t, cos omega is 1, cos t. The second term is e to the power, once again minus t into sine t. That is all and whole into, of course, this is multiplied by ut, this is multiplied by ut.

And so this can be written as 1 minus e to the power minus t cos t plus e to the power, not plus minus only, plus minus e to the power minus t sine t whole into ut. If you wish this can be further simplified as 1 minus e to the power minus t, you take common and this is cos t plus sine t and then multiply with root 2, divide by root 2 and then say this will be equal to 1 minus root 2 into e to the power minus t. This is nothing but cos a minus b, t minus pi by 4.

That way also you can write it. Therefore this is the solution Vct after taking Laplace inverse, has it come correctly, cross-check. Vc0 let us calculate whether we have got the expression correctly. Put t equal to 0. This is indeed equal to 1 minus root 2 and this is cos pi by 4 1 by root 2. So this is 0 satisfying. So this will be the final result of this capacitance voltage and if you sketch the capacitor voltage for example 1 and example 2, which I have not done for the first example.

In the first example, that is example 1 we have got this to be the solution it and then Vct I got to be like this, where is Vct for this example to continue though Vct, capacitor voltage, it is the first example capacitor voltage I did not get know, where it is? This is the capacitor voltage in the first example. If you sketch this waveform, it will be exponentially increasing to 1, got the point? But in this case the capacitance voltage will be equal to this one.

If I sketch, I will sketch it like this as a function of time if you sketch it. So final value of the capacitance, it is 1 and initial value is 0, so it will be changing like this. This some cosine function is there, there will be some oscillations which will decrease with time and this sine function is modulated by this damped sinusoidal e to the power minus at. So it is called oscillatory response of the system.

So capacitor voltage will of course be equal to 1 final value, similar in the previous example, but in the previous example, there was no oscillation. It will continue, as a general thing will do that, but for these two examples, as you can see you can find out the voltage of the capacitor for these things. So here the highlight was given a circuit, you first draw the circuit in s domain. Draw the circuit, so that you can avoid writing the differential equations.

That of course you can do it, provided you do not have the initial conditions. All these initial condition relaxed, then it is very straight forward, then find out anything you like, voltage across any element of the circuit. After you have found out is, multiply with respective impedance to get the Laplace transform of the voltage across that element. Then once you do that you will find, in most of the cases, we have to do some partial fraction expansion.

That you somehow have mastery over that very quickly. I followed one sequence, so that same things here applying and get the partial fraction constants and once you get that partial fraction expansions taking Laplace inverse is to correlate with the known Laplace transform table, consult in any table in the Laplace transform and get the solution.

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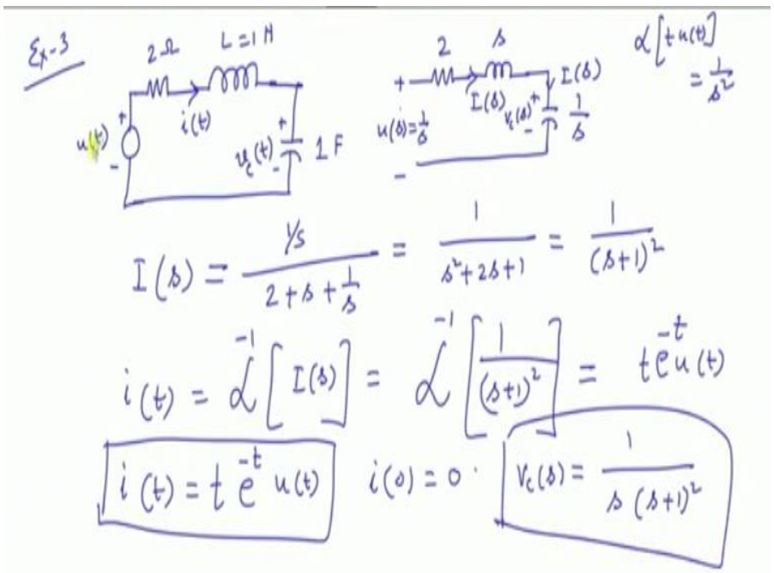

The third problem, example 3, this I will not solve. I will only give hints, but same problem with r equal to 2 ohm. This is the time domain circuit, let me say you, inductance is 1 Henry, L equal to 1 Henry, but this time the capacitance value is 1 Farad, say 1 Farad and I will energize the circuit with a unit step function ut and we would like to know what is the time domain description of the current in this circuit as well as voltage across the capacitor.

So in this case, the transform diagram in this domain will be this is 2, this is sL s into 1 s and this is 1 by Cs that is how much 1 by s only and this is your us, which is equal to 1 over s, because the unit step I am applying. Therefore, I will once again calculate is. This time let me go faster. So is will be equal to 1 over s voltage divided by the impedance, it is like this and this one you bring this s down, so it will be s square plus 2s plus 1, it will be this.

Now here you see, this is nothing but is plus 1 whole square, is is this, what will be the Laplace inverse of this? Whose Laplace transform is 1 by s square, t? You see go to some previous pages where we discussed, I mean some more previous pages, this page perhaps no. I think here you will get. This is impedance thing, some standard formulas were there, that page I forgot to bring. See there is a Laplace transform of ut, this, that.

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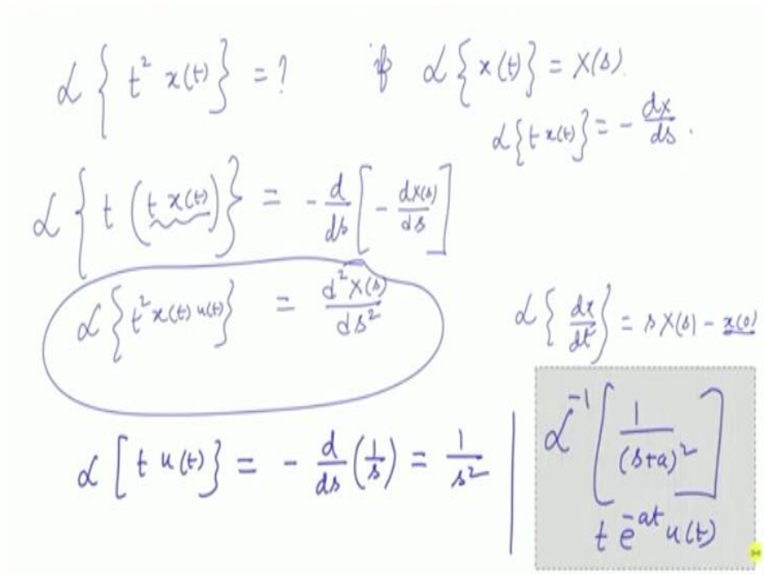

Then previous to that, you see if you have this function t into xt Laplace transform of t into ut will be how much? If you apply this formula, if Laplace transform of xt is xs, then Laplace transform of txt is minus dx ds. So in this case, this will be then minus of d ds of 1 by s and that is equal to 1 by plus s square. So whenever s square term is there, that means the time domain function is to be multiplied with.

It is the t into ut Laplace transform is 1 by s square, is not? So what will be, if it is s plus a whole square, Laplace inverse of this will be then how much? It will be wherever t is there t but multiplied by e to the power minus at into ut, got this. So this is the thing. These results we will be using now. So copy it, go to the last page and paste it here, copy, delete. So what did I write? So this is this thing.

Therefore it will be equal to Laplace inverse of is is equal to Laplace inverse of 1 by s plus 1 whole square and what I am telling, this Laplace 1 by s square it is tut, t into ut. Let me write it, Laplace transform we know t into ut to be 1 over s square. This is known, but here it is s is replaced by s plus 1. So it must be multiplied by e to the power minus t. This will be the solution. Therefore the it, therefore will be simply t into e to the power minus t into ut.

Does it satisfy the initial condition? Yes at t equal to 0 current is 0, because it is 1 but t is 0, i0 is 0 fine and similarly the final current in the circuit has to be 0, because after all you have applied a DC voltage, finally there is no DC current can exist, because of the open circuit behavior of the capacitor and final voltage of the capacitor will be once again 1 volt. So this is the thing. Now let us write down the voltage across the capacitor. What will be Vcs? Vcs will be 1 by s into is.

So this is Vcs. Vcs will be 1 over s. We have to multiply with is, which is s plus 1 whole square. This is the thing. So we write it.

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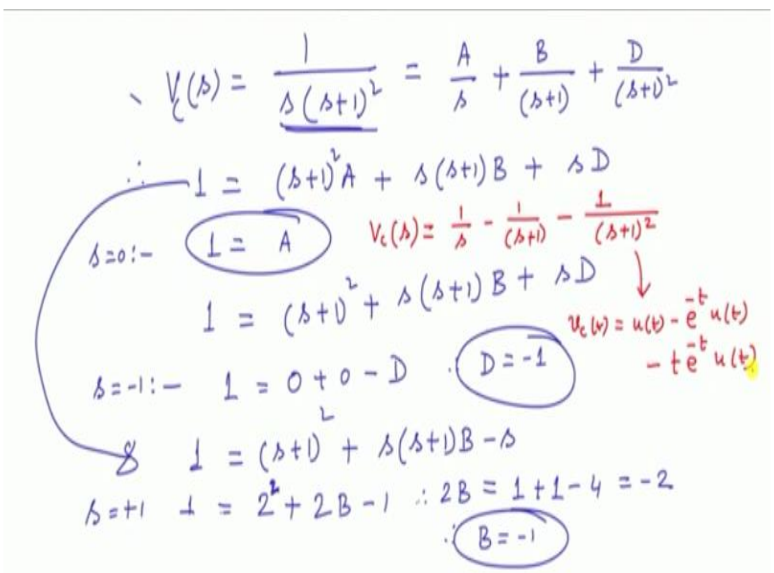

So Vcs, we have got 1 by s into s plus 1 whole square. Now in this case of course, you have to do partial fractions. These are the finer things you must know. Here the partial fraction will be A by s plus B by s plus 1 and say plus D by s plus 1 whole square. This will be the partial fraction and so multiply with this thing. So very quickly let me try to do, so that something goes to your head.

So this is s s goes, multiply with this factor both the sides plus it will be s into 1 s plus 1 into B plus s into D.

This will be the thing, we have to find out A, B and D. So s equal to 0 is the natural choice. As you can see, it will straight away give you the value of A. This, this term vanishes so A is 1 obtained. Therefore, I will write 1 is equal to A now is known. So this is s plus 1 I rewrite that plus As into s plus 1 into B plus s into D, this is the thing, correct. Now what you do? You put s equal to minus 1 on both the sides, s is equal to minus 1.

It is identity so 1 equal to this will be 0, s is equal to minus 1, this will be also 0 and you will be left with minus D. Therefore, you find D is equal to minus 1. D is also obtained. Therefore, you can write this identity as 1 is equal to s plus 1 whole square into A is known into s plus s into s plus 1 into B, which is yet to be known and this is minus s. So put any values of s to get the value of B. Do not put any value such that this term disappears, s is equal to 0 will not do.

So maybe s equal to plus 1, you can put. So 1 put s equal to plus 1. So 1 equal to this is s plus 1 2. This is also 2, 1 into 2, 2B and minus 1, thank you. So this is 2 square plus s equal to 1, this term is okay, 2 into 1B and s equal to plus 1, so minus 1. Therefore I will say 2B is equal to this is 2B is equal to 1, this 1 you bring to this side, then minus 4 and this will be equal to minus 2. Therefore, B will be equal to minus 1.

If this is correctly obtained, then I will say that Vcs, let me write on this page only. Then VCS will become A is equal to 1, 1 over s, B is equal to minus 1, minus 1 by s plus 1 and plus D, D is equal to also minus 1, minus 1 by s plus 1 whole square, you know this will be Vcs. Therefore, from this I will say Vct is equal to unit step, this one. This one is minus e to the power minus t into ut and this one right now we have seen it is equal to minus t into e to the power minus t into ut.

So this will be the capacitance voltage t equal to 0, does it satisfy this boundary condition? Yes, if you see t equal to 0, this is 0, this is 1 minus 1 0 and if you differentiate this, you should get the current expression. This it you should get back, if you differentiate that. There are several ways of cross-checking. So in these lectures, what my primary focus was to tell you that if there are no initial conditions, then you do not even have to write down the differential equation.

Straight away write down, redraw the circuit in s domain and write some general impedance zs. So Bs by zS is equal to is. Even if the circuit is complicated, there are several series parallel branch, do not worry, go on doing like this. Whatever output you have to calculate just like, this is circuit you do the algebraic manipulations and then of course before taking the Laplace inverse, you must bring the right-hand side in such a form that you can easily identify what will be the Laplace inverse of that.

Hopefully you have understood these steps and we will in the next class tell you, okay when the initial conditions are relaxed, this is a very nice way of doing things, but what happens if in a circuit, there are also initial conditions and I also do not want to write down the differential equation and how those initial conditions can then be incorporated in this Laplace transform domain circuit. So some modification in the circuit then needs to be done. That we will see in the next class. Thank you.