Network Analysis
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Lecture - 49
General Second Order Circuit Analysis with L.T - I

Welcome to 49th lecture and today, we have discussed how circuit analysis, in general, can be done by redrawing the circuit in s domain and then just apply the circuit rules in terms of impedance and supply voltage. All things will be Laplace transform and get the desired response. Finally take the inverse. While taking inverse, only thing is that you should be very conversant with the partial fraction expansion of a fraction.

Today I will just conclude that. Of course, those things can be applied as I told you, you can avoid writing the differential equation, provided there is no initial condition. Today I will also tell you how to take that initial conditions into account. That will be the second part, but first what I will do, I will formally tell you how to analyze a second order system using Laplace transforms.

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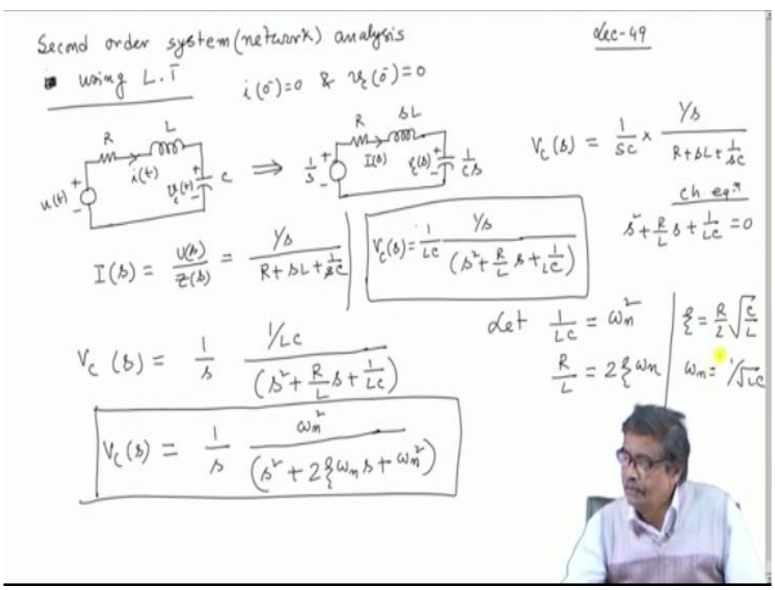

We have in fact done specific problem in my previous lectures, but in general a general treatment of second order system analysis. System means, in our case, it is network, second order network analysis in using Laplace transform. That is the thing, we will be doing, a general treatment. So a second order system in circuit analysis is like this R, L and C and we have done this many times and this is a time domain circuit. This is your ut step response and this is L and this is C.

And this is it and this voltage across the capacitor is Vct okay. Now this thing first of all, with all initial condition 0 means i 0 minus current through the inductor must have been 0 and Vc 0 minus is also 0. So this then can be written like this, redrawn, the circuit is redrawn like this R sL and then you have 1 over cs and here the Laplace transform of the voltage, which is unit step, so 1 over s and then I will show this to be my is and this to be my VCs okay.

Now so is will be step response of second order system we were doing. So I will say that is is equal to us by zs and us is 1 over s by zs is R plus sL plus 1 over sc. This is the impedance matrix and this can be written as 1 over Lc. It can be simplified to 1 over Lc. If you take this c common etcetera, manipulate this, then it will be 1 over s will be on the top and on the bottom it will be s square plus R by L into s plus 1 over c. This will be your, in fact this is not, this is is.

This is fine. Then, this is actually if you get Vcs will be 1 over sc into the current. So current is 1 over s divided by R plus sL plus 1 over sc. This if you simplify, it will become this way. So Vcs will be 1 over Lc on the top 1 over s and this is the thing. The voltage across the capacitor can be written in this fashion. It is a general statement. Earlier I took some values of R, L, C specific and tried to see the response. Now in terms of general expression, it will look like that.

Now this is can be written as Vcs is equal to 1 over s and 1 over LC same expression by R by L into s plus 1 over Lc. It will be like this. I missed one L earlier here. Now this one is square plus R by L in to s plus 1 by Lc, from this it is the actually the characteristic equation we know from the time domain analysis of differential equation and depending upon the nature of the roots of this characteristic equation.

So s square plus R by L into s plus 1 by Lc equal to 0 is the characteristic equation of the system. So it will have two roots, etc. Now what is done, after this, we will let 1 over Lc, I will call a variable called omega n square. We do not know what omega n signifies right now, but we can do this and we will define R by L as 2 zeta omega n. That is these two things are the unknowns. I mean 1 by Lc and R by L.

So I have translated those variables in terms of another two quantities, which is omega and z. In fact, if I think it should not disturb you, because if you solve these two, you can see that zeta will become equal to R by 2 into root over c by L and omega n is equal to root over L by c. That is given a circuit 1, this one sorry, omega n is 1 over root over Lc. So if I know the value of RL and c, I will be able to calculate zeta n omega n. If you put it, it will be like this.

So under this scenario, this assumptions, it will then become on the top I will write omega n square on the bottom I will write s square plus 2 zeta omega ns plus omega n square. We will see the significance of zeta and omega n, as we proceed with our calculations. Omega n, we will see it will give you the frequency of the natural response. Zeta will be called dumping ratio. So this is Vcs. So if I know R, L, c, I know zeta and omega n.

This is how this equation gets transformed into, okay, like that. So I will take three conditions, see zeta is equal to R by 2 into root over c by L. If R equal to 0 , eta will be equal to 0. So the first case, there will be 3 cases when zeta is 0, when zeta is between 0 to 1 and another is zeta is greater than 1. So case 1, let us assume the values of R, L, c are such that zeta becomes equal to 0. When zeta will become 0, when R equal to 0.

Now one may say if c is 0, then also zeta will be 0, but then it will not be a second order system, got the point. So we will take one at a time.

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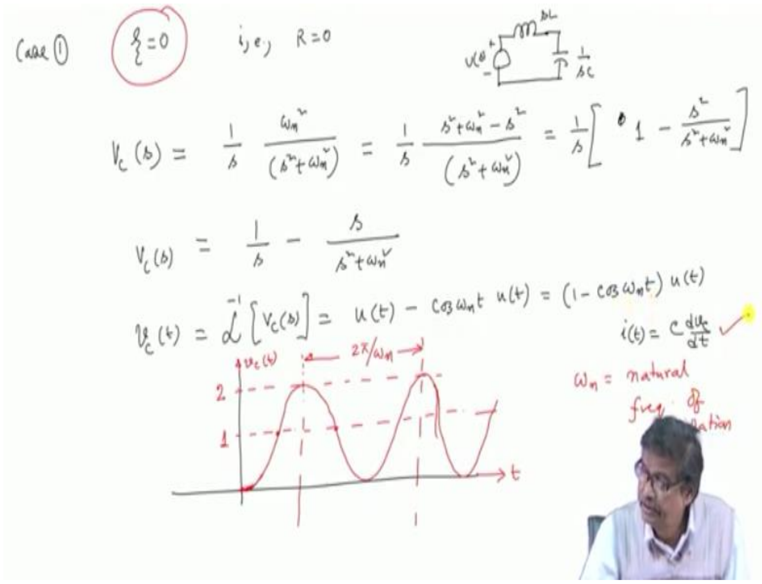

So first case is case 1, zeta is equal to 0 that is R, resistance of the circuit is 0. In that case, I could do separately that is only sL and 1 over sc and then this is is. But I am not going to do that, because I have already done, but I will put R equal to 0 or zeta equal to 0. So voltage across the plate of the capacitor under this condition will be equal to 1 over s into omega n square by s square plus omega n square, because 2 zeta omega n term goes and you will be left to with this one.

Now this can be once again partial fractions can be carried out. So the easiest way for this particular case is, I can do partial fraction, but mentally it can be really done. What you do is this 1 by s and then you add one s square and subtract another square and this is s square plus omega n square. This way, I will write. If you do like that, then it will become 1 over s. I am essentially doing partial fractions.

One can write A by s plus omega, s square plus omega n square A by s plus As plus B by s square plus omega n square, but here it is not necessary, because it is so simple, you mentally calculate. This way if you write, you see, it will become 1 minus s square by s square plus omega s square, is not? It will be like this, where it will be 1 over s minus a by s square plus omega n square. One s goes, it will be like this. So it is partial factor.

In whichever way you do, you have to convert it to this. So after getting this Vcs, Vct will be Laplace inverse of Vcs, which will be for this term it is ut and for this term, it is cosine omega nt ut, that is all. So this is equal to 1 minus cosine omega nt into ut. Therefore, voltage across the capacitor will vary like this and if you sketch this capacitor voltage and after you have got Vct, it will be simply, it also one can calculate and sketch. This will be C dVc dt.

One need not once again go to is and take inverse it. It is so simple. Now this waveform if you sketch, it will be something like this. This is the time axis and this is Vct. So capacitor voltage has to start from 0 and also current is 0, so it must be cosine. So it has come cosine. So at t equal to 0, you see 1 minus 1 0. It starts from there. This will be 1 and this will be 2 in this plot. You can easily verify if this is correct.

After all there is 1, go to 1, then draw the minus cosine omega t with this horizontal axis, you will get this. Amplitude of this one is 1, so 1 plus 1 2. So capacitor voltage will be oscillating in nature with a DC component. It is equal to a DC constant across the plate of the capacitor over which there is a cosine terms, of what frequency, of frequency omega n. So what is omega n? Omega n is this is the time period, so peak to peak positive.

This will be 2 pi by omega n or negative to negative 2 pi by omega n. That is the time period of this waveform. So this omega n is called the natural frequency of oscillation and for obvious reasons, there is no exponentially decaying term multiplied with cosine omega t. This will be sustained oscillations. Energy transfer will take place between capacitor and inductor as well as with the supply.

So when you switch on a DC voltage occurs and series or LC combination without any initial current, it will be like this, over. So this is case one, that is damping is 0. So the significance of defining this as omega n square, now becoming clear. Why people write like that is because of that, okay some cosine omega nt term you have got. So this is the case 1 and I am not sketching it. This can be sketched okay. Only I can write down, okay not necessary you complete this part.

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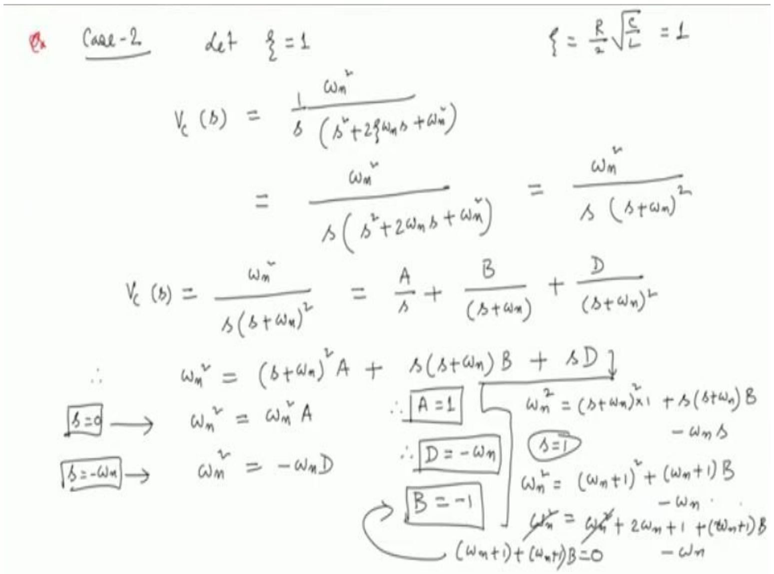

Now the second part case 2 let us take, case 2 will be suppose zeta equal to 1, I take. Let zeta equal to 1 and the expression of Vcs of course to remain same as omega n square by s square plus 2 zeta omega ns plus omega n square into 1 over s, step response. So same circuit zeta 1. Zeta is what? Once again do not forget zeta is equal to R by 2L, R by 2 into root over C by L. Zeta is equal to R by 2 into root over C by L.

So I can choose values of C and L, and R, such that this product becomes equal to 1, I mean that clear. Now if zeta equal to 1, then I will straightaway write omega n square divided by s into s square plus 2 omega ns plus of omega n square. Put zeta equal to 1 and this is a perfect square. So this can be written as omega n square divided by s, I am sorry, it is okay s plus omega n whole square you know. This will be a perfect square situation.

This also can be manipulated rather easily in the same way as we have done earlier, no. So this of course, I have to find out the partial fraction expansion, maybe if you can mentally mentally find out, this is fine. So let us do in that way that omega n square divided by s and s plus omega n whole square. This is the thing. Let this be equal to A by s plus B by s plus omega n plus D by 3 constants s plus omega n whole square. Let us redo this very quickly, how to do it?

So this can be written as Omega n square is equal to this you bring it to this side. So this will be s plus omega n whole square into A plus it will be s into 1 s plus omega n will go. When you come here, into B, any mistake you point out, plus s into D. This will be the thing and I have to find out A, B, t so three equations you have to form. So one thing is, if you put s equal to 0, then it will be omega n square is equal to omega n square into A. These two terms not there.

Therefore, immediately I know A is equal to 1. A is equal to 1 is known okay. Then, put s equal to minus omega n. s equal to minus omega n if you put, left hand side remains omega n square. Here A goes, A is already known, anyway it goes, B goes and you are left with minus omega n into D. Therefore D is equal to minus omega n. This will be D. Then, so my equation now is A and D known, I will write, same equation I am writing here omega n squared is equal to s plus omega n square.

A is 1, I know I put that plus B is unknown, s into s plus omega n into B and D is known minus omega n into s. So in this expression, put some suitable value, maybe s equal to 1 simple suitable value, put s equal to 1 in this expression. If you put, it will be omega n square plus omega n plus 1 whole square plus omega n plus 1 into B, this is 1 and this is minus omega n and this has to be equal to omega n square. Omega n square I have already written is it.

Omega n square is equal to, I have written right hand side first. Omega n square, this is equal to is it. I have already written that, so it will be omega n square equal to this and this may not be written. Is that clear? So these are the three terms. Now it can be easily solved for that this one can be broken up and solved for and are ultimately what you will get is, it is better I work it out. It is equal to, if you expand this it will be omega n square plus 2 omega n plus 1 plus omega n plus 1 into B minus omega n.

Now this omega n square goes 1 and this is once again omega n plus 1. So it will be omega n plus 1 plus omega n plus 1 into B is equal to 0, from which I will get B is equal to minus 1, got the point. Those who are very fast at your age, they can quickly get these three constants, A, B, D. I have not assumed this to be C. It may confuse with the capacitance value. So this is the thing. Once I get this, then I will say under this condition to meet with A D value level.

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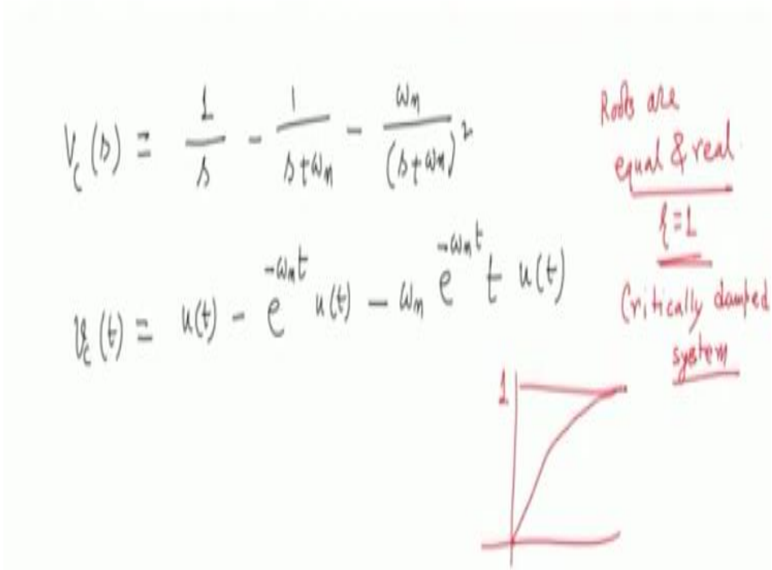

Then I will write that Vcs is equal to A by s, A was 1 plus B by s plus, so B was minus 1. So s plus omega n and finally D is equal to omega n by s plus omega n whole square. Now the Laplace inverse if you take, you will get Vct which will be equal to ut standard form. This will be minus e to the power minus omega nt ut and this one 1 by s square that Means t into ut something, but s plus omega n.

So it means, it should be multiplied by omega n was there and then e to the power minus omega nt into t into ut. This will be the thing. See it also reminds me, what are the roots of the characteristic equation here? They are real roots and repeating roots, repeated is not. The characteristic equation was this part in this case. What is the characteristic equation? It is this one. This is 1 by s is the input signal. So this is the characteristic equation.

Roots are s1 s2, is not. They will be real plus minus 1 not equal to 0, s plus oega n whole square that means minus omega n minus omega n roots are minus omega n and minus omega , they are real root, but repeated. So I know from our differential equation fundamentals that then the solution will be t plus some constant into t, A plus Bt into e to the power minus omega nt. Anyway this will be the response. So all the comments you can write here.

Roots are equal and real with zeta equal to 1, when theta is equal to 1. This is called critically damped case okay. So that that is the thing and you can plot this. Mind ut equal to 0, this term is

0, this is 0 capacitor voltage. Similarly you can find out it by differentiating this expression as simple as that. Now case 3 is the last case, so this one is called critically damped system. It will be almost like exponential. There will be no oscillations with frequency omega n okay.

It will be finally, the solution will be something like this once again. Add these two terms at different times, so you have to add one exponential term growing from ut and so on, this is 1 okay. Now the last case that is the case three.

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Case three, when zeta is present but its value is neither equal to 0 nor equal to 1, but in between. That is zeta is a number less than 1. Zeta, of course, will be always positive okay. Suppose a circuit has got RLC parameter, such that this is the case. This system we will see soon is under damped system and we will continue this in the following class. Thank you.