Network Analysis
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Lecture – 51
Network Theorem - I

Welcome to 51st lecture on Network Theorems, Network Analysis, and let me just very briefly review what we have done so far. We started with simple KVL/KCL with DC supply and then there will be energy-storing elements present in general in the network, inductance and capacitances and they invariably give you differential equations when you write KVL/KCL etc. and to solve for currents, classically, we solved those differential equations and got the current response and so on.

Then, I covered another important topic that is apart from being in time domain, solving differential equations to get the responses, you can alternatively solve the network problems by going to s-domain. The advantage of that is those differential equations get transformed into some algebraic equations and we can invoke whatever laws KVL/KCL or nodal analysis, all those techniques which we applied in DC circuit analysis, can now also be applied to s-domain circuits straightway. Why because Vs=Isx Zs.

In the same way, V=Ir. Therefore, all those rules can be applied and this can be applied to Linear Time-Invariant Networks, and of course, we assumed, when we did s-domain that everything starts at equal to zero, your input signal you are applying is equal to zero, so causal signals are there, system is causal and so on. Now, we are going to start some network theorem, and hence therefore and Laplace transform, when you get say Is, then you should take the time inverse of that Laplace transform to get the time-domain expression.

Because our goal after all everything is real and time dependent. I applied so much voltage s=0, what will be the expression of the current. Then you have to take inverse transform to get the solution and this solution will be the same as the classical differential equation. I mean comparing these two methods, it looks like okay, Laplace transform method does not give, I mean, very straight forward steps are known.

But only problem is when you take the inverse, you have to be a little bit careful, properly expand the Is. Suppose I am interested to know the current in some branch, that Is will be equal to some function of s x Vs. We have to take the inverse of that. To do this, we have to correctly expand that expression in terms of partial fractions and they are sometime gets consumed, but in classical way, the beauty is you can after writing down the differential equation, you will know what are the terms which will be present.

Okay, constants are to be determined, that is fine. Depending upon repeated rules, equal rules, this that, I can looking at the differential equations, straightaway write down the nature of solution. Then some constants are to be determined from initial conditions or so. So both these approaches are important. Particularly, Laplace transform method assumes importance, because you will learn also control system theory next time.

They extensively use Laplace transform. Laplace transform straightway gives you the transient as well as steady-state response. So, with this background up to that we have got, now I will start some network theorems, interesting network theorems. There are several.

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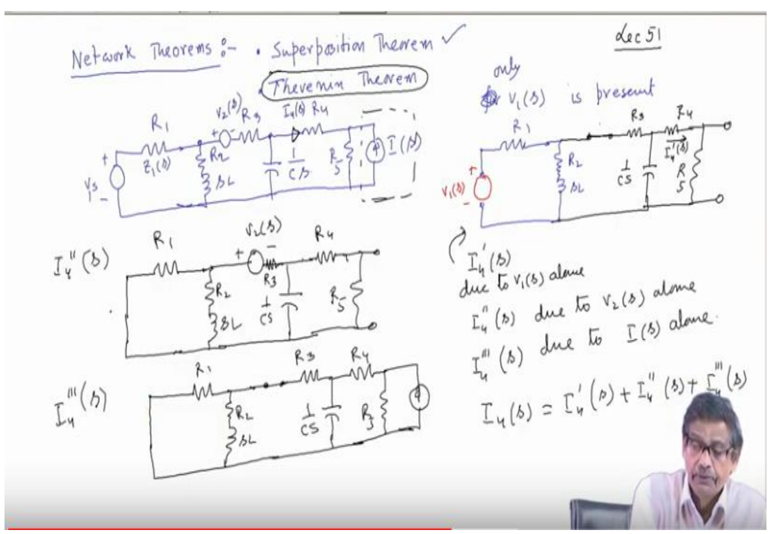

One is called Superposition theorem, then you have Thevenin theorem, and then there will be several others. I am just listing these two for the time being.

Superposition theorem can be applied only to linear networks, that we must know, and henceforth, this network always I will write something like this, in s-domain. What is the point, see Vs is there, there is R. If R is there, there is R1, this is R2, then SL, and there is R3. There may be a capacitance also present. I do not mind, because I know what is to be done, 1/Cs. Like that, the networks, henceforth, I will draw.

That is, I will treat this is as Z1s, of course, pure resistance is R1, this is R2+ SL, Z2s, Z3s and so on, and then whatever method you want to apply, you can apply, and suppose there is another voltage source here. This is suppose V1, there is another voltage source, which may be V2s. Okay, if in a circuit more than two sources are present, and there may be another suppose, I say that there is a current source also present like this, Is.

So a network can consist of several voltage and current sources. Here, two voltage sources and one current sources, I have drawn. Now Superposition theorem tells me, okay, you can solve this network considering all sources present at a time by maybe if you adopt Mesh analysis, then assume the currents all for them or nodal analysis, assume the node voltages and solve for them. This is suppose R4, this is R5, that is fine. But superposition theorem tells you, that suppose you are interested to find out the current in this branch, what is I4s.

Then Superposition theorem tells that, you draw the circuit with one source at a time, that is, and replace the other source, by its internal impedance. For example, in this case, a voltage source V1s is there, for V1s, only V1s is present, like that you treat, only V1s is present. Then redraw the circuit. So it is R1, it is there, now this should be replaced by V1s is here, should be replaced by its internal resistance. If nothing is there, short them, and then this is R2, very quickly I will tell, you know this thing, R2, SL, and then only V1s is present. Okay. Then, V1s should be there.

I have assumed V1s is present. Only V1s and the other source V2s and IAs should be replaced by their internal impedances. So here, the internal impedances will be these two points where V2s is present. I will replace it by short circuit and then R3 is of course there. R3 I will draw, and then 1/Cs, these are impedances. So 1/Cs that is fine. Then R4 will be there, R5 will be there and these current source should be replaced by its internal impedance. Between these two points, it is an ideal current source.

It is internal impedance is infinity, so it should be replaced by open circuit. Now, in this network, only one source is present. Therefore, I will calculate the current flowing through R4, that is this current, I4, I will find out. Due to V1s alone, say this is I4 dash s, I say. Similarly, I will calculate I4 double dash s, due to V2s alone, replacing other sources by their internal impedances. I am not redrawing, you understand, and this will be shorted, here V2s will be present.

This IAs is also absent, so that we have to solve once again. Then, find out I4 triple dash s, due to the current source Is alone. Only this Is will be present and this should be shorted, and then when all of them are present, I4s will be equal to I4 dash s +I4 double dash s+ I4 triple dash s. This is called Superposition. So Linear Network Superposition Theorem is an important tool, sometimes may not be for solving the networks, because you see this network, if you solve all the sources are present.

Compared to this, for each of the sources network is slightly simplified. For example, this source is not there, okay, but you have to calculate, each time to solve this network. So, for example, this was for calculating I4 dash s. Let me redraw. The second one, this is this, to calculate I4 double dash s. Very quickly, I will draw. It should be only due to V2s, so V1s is present. So that you get visually what is happening, how it gets a bit simplified. R2, this is SL. Impedances should be present, same impedances.

Here V2s is present and then 1/Cs is there. Very quickly, let me redraw, so that you do not miss any point. Here R4 and here R5. Because these two are open circuited. This is for I4 double dash s. Then, the third one is to calculate I4 triple dash s. What I should do is this, only Is is present. So voltage sources should be replaced by a short circuit R1, this is R2 SL, and then here was some resistance, R3 was there.

Correctly draw this circuit. Here also, it is shorted between the source, then R3 and then some 1/Cs is there. It should be shorted, and 1/Cs, and then R4, R5, and only the current source is present, and then I have to add, this plus this plus this, to get the current flowing through I4 in this network where all the sources are present. But nonetheless, you have to solve the network for three times, although you have to handle with one source at a time. That is there. But the idea of Superposition theorem is to understand many circuits, to establish many theorems of network analysis, this is adopted quite often.

Okay, understood this point. So, that is what Superposition theorem tells us. Therefore, a network may consist of several sources including both voltage and current sources, and I know how to solve it by applying Superposition theorem. It is slightly lengthier process, it looks like, compared to if you consider this whole network as it is. That will be slightly complicated to each one of them, that is fine, but here you have to apply three times to solve the networks. I think you have got the point. So that is the idea.

So superposition theorem, it is also true that, it is needless to say, but still I am telling, that okay, for two sources present, third source is absent, you find out the currents and then only third source is present, and other two sources are absent, the result will be the same. Add this two, you will get I4s, got the point. One will say that okay, so Superposition theorem tells that one source at a time, that is fine, but what I am telling, two sources, I will take V1, V2, Is is absent. I will find out the current.

These two together only will be present. Similarly, when Is is present, these two should be replaced by their internal impedances, calculate the currents, add this up, and you will get same I4s. Here, I showed you how to find out currents in a specific branch, by applying Superposition theorem and also another disadvantage, should I say disadvantage I am not sure, but you try to understand this point. This is only for this branch I am doing, to find out this current, I have to solve.

Similarly, to find out another branch current, all the branch current, when one source is present is to be solved, you know. Okay, it looks like if you are interested to find out one branch current, but superposition with two to three sources, maybe a good choice. That you have to work it out. But if I ask you, in this network, solve current for all the branches, then for each of this network, when V1s is present, you have to solve not only for I4s, but also I1s, I2s. In this branch also like that.

So computational overhead will become, it looks like hired. More complicated the network is, more time consuming it will be. But nonetheless, superposition theorem helps us understand several other interesting theorems. That is what we are going to do it. The next thing is Thevenin's theorem. One of the most important theorems in network analysis, and before doing Thevenin's theorem, I will first tell you about one very interesting observation about networks. Some people call it Substitution theorem.

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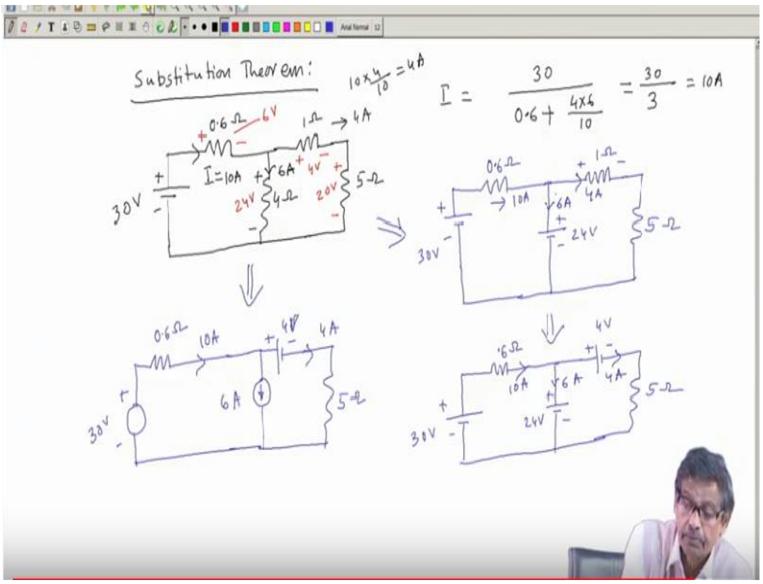

This theorem, what is the statement or what it is actually to make you understand this, I will take a simple network to tell you about that. Suppose, you have a network like this, simple network. I have taken the values that the calculations become easy. Suppose, you have a network like this, 5 ohm, and this is 4 ohm, and say this is 0.6 ohm, and this is say 30 volt. Okay. This network can be easily solved, because this is 6 ohm parallel with 4 ohm.

So this current in this circuit straightaway, I = 30 /0.6 +(4*6)/10 = 30/3= 10 amperes. If this 10 amperes, the other currents in this branch, will be this 10 x 4, other resistances divided by some of this resistance, 4 amperes. This will be 4 amperes, this is 10, so this must be 6 amperes. Okay, now consider this network, what I am telling, I can solve it by any method, nodal or Mesh, because the values are so simple, series and parallel things are there.

Why should I go for those methods at all? So I have solved numerically. Now, what this particular substitution theorem tells that, this network, what will be the voltage of this resistance, 6 x 4 = 24 volts, let me write it with other color, 24 volt. What will be the voltage drop between this two color? Current will be flowing this, so this is 6 volt, 10 x 0.6. What will be the voltage drop here, +/- 4 volt, so what will be the voltage drop here, this is 20 volts.

This is the thing. I know, I have solved the network, I know, everything, ins and outs of the networks. Currents in various branches, this that, voltage across each element and so on. Now, I did draw another circuit, listen to this point carefully, and say that, this is the network. This is 30 volts and here is another network, this is 1 ohm, and here is another impedance was there, 5 ohm, and here I will just connect a battery of voltage, 24 volts.

So, this network, does not look exactly equal to this network, same as this network, because there was 4 ohm resistance here. Here is a 24-volt battery I have connected. All the impedance and other sources I have kept as it is. Now the point is, in these two networks, all the branch currents will remain the same. That is voltage drop across this element 24 volts, replace this by a 24-volts battery and redraw this network, which is nonexistent.

This is the actual network given to me, but what I am telling, in this network, if you see, what will be current in this branch, voltage 30 volts with respect to this point, this is 24 volts, so 6 volts will be there, 6 x 0.6 is 10 amperes. These two points, voltage is 24 volts, between these two points, 24 x 6, 4 amperes will be there and 6 amperes is there. Therefore, currents of these branch of this network and various node voltages, are same as in this network.

It is an observation, that means, given the network, you can replace a resistance, after solved it by the voltage drop and by a battery, say for example, no change. Similarly, I could say that, let me redraw. This is 30 volts, this is 0.6 ohm, you have suppose replaced it by a 24-volt battery, and here how much is the voltage drop, 4 volts, +-. If you wish you can write it as a battery with 4 volts. Keep it as it is. Here also everything will be same.

The current distribution in this network, this will still remain 6 amperes, this is 10 amperes, this is 4 amperes, and so on. Therefore, voltage drop across any impedance can be replaced by a battery whose strength will be I x Z, that voltage, I x R, in this case. Similarly, I could do also this one. After learning this, I will say, look here, it will be like this. You have 30-volt battery, very interesting observation, I am telling you. This is the original network.

These are my thought process, okay, this network and that network. So far branch currents are concerned, exactly they will remain the same. Similarly, here, I could say there is a current source of 6 amperes, and simultaneously you can do this one also, and this is suppose, I say, this is a 4-volt battery. That 1-ohm resistance, this is replaced by a 4-volt battery, because 4 x 1 is 4 volts.

This 4-ohm resistance through which 6 ampere flows, I have replaced it by a current source 6 amperes and other impedances I keep it as it is. Then also the current in this network, if you solve, it will remain 10 ampere here, this is 6 ampere, in any case, and this is 4 ampere, and I can easily verify. This is 24 volts + 4 volts. Node voltages will be the same. Therefore, either you can replace the impedance by a current source or a voltage source.

The currents in various branches of the network will not change and it will be there. Now, the question is okay, this is a very good observation, that way also it can be looked at a circuit. This circuit, this circuit, this circuit. They are all one in the same. So far as the current distribution are concerned, but they are not identical circuit, because 4 ohm is replaced by a source 6 ampere. That is my botheration, I must know what I am doing. If that be the case, then what is the use of this, and this I can do it, it looks like if I can solve the network, then I can do it.

Not really, this particular theorem, can be applied very nicely to solve several network theorems. Okay, for example, to find out the currents suppose I am given in a network, now I will go for general circuit, it is general statement of this. You go to any circuit.

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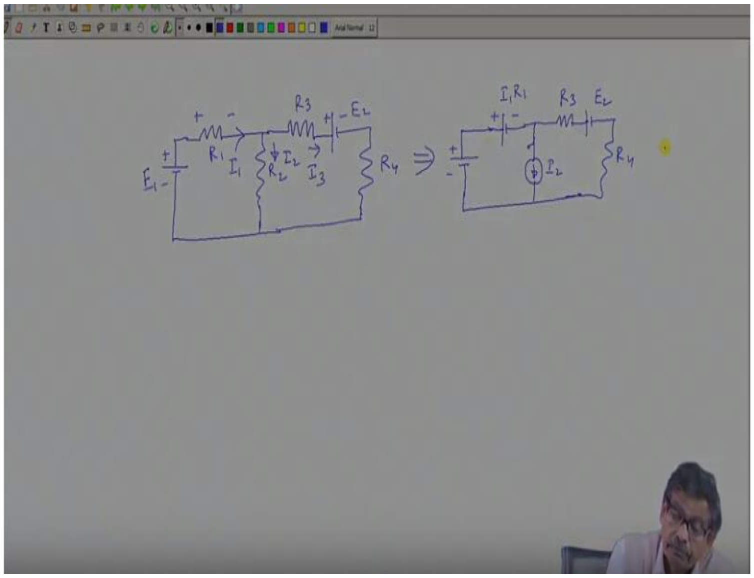

For example, here any network, you go, some network, let me write here is a voltage source, things like that, here is another resistance, another voltage source, another resistance, like this, a network is there. Now what I am telling, I am writing general things +/-. This is R1, R2, R3, and this is suppose +/- E2, and this is suppose R4. Okay, in this network, I can assume, these currents to be I1, suppose this is I2, this is suppose I3, then what I am telling, in general, this circuit will be same as E1 suppose, I have not disturbed here.

If I wish this voltage, I can simply write a battery like this, I1 R1. Currents in the various branches will not change. Got the point. Similarly, this branch you can replace it by a current source, if you wish I2 and if you wish here, you can change. I have not changed. It does not matter and here is a battery and I suppose this one. Once again, what I am telling, these are R1 is replaced by this battery, with this voltage. This magnitude of this voltage is I1 x R1, that is important.

Similarly, this way. So if you replace it, these two circuits are same as far as the various branch currents are concerned. We will use this extensibly to prove various network theorems in our next class.