Network Analysis
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Lecture - 56
Tellegen's Theorem

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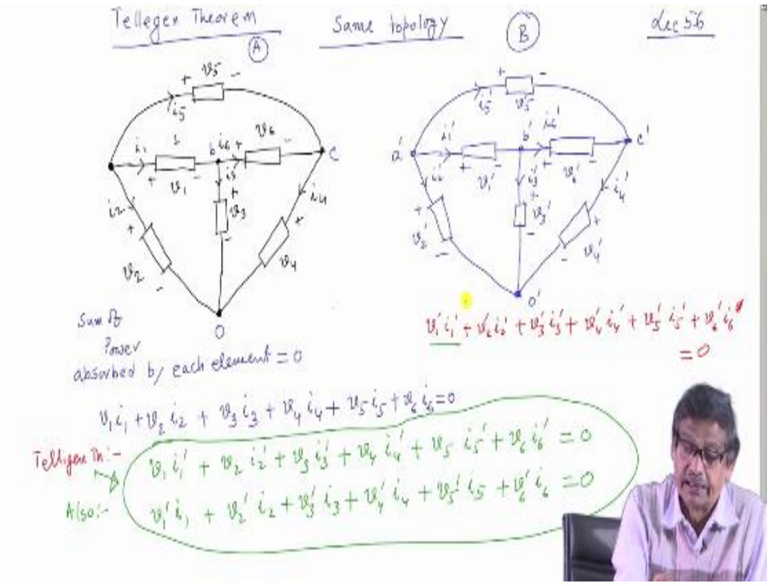

Welcome to lecture number 56 and we have been discussing with Tellegen's Theorem very interesting theorem Tellegen's Theorem. And in my last class I told it is applicable only to different networks, but having same topology, same topology and in this network 6 elements, in this network 6 elements, elements are different it is like that then about one thing I am certain.

Suppose I forget about the second network which are the this is suppose network A and this is network B where all the quantities I have market with primes. Now in the network at least this thing I know if I calculate total absorbed total power absorbed by each element and sum them up that must be = 0. Now what is the power absorbed power absorbed sum of power absorbed by each element must be 0.

What is that power absorbed by element 1 v1 i1 so the plus current is entering (()) (02:02) element to absorbing power based on the voltage polarity and the current reduction yes it absorbs so + v2 i2. What the element 3 is doing it is also absorbing power so it will be v23 into i3. What is element 4 doing absorbing + v4 i4. Element 5 absorbing + v5 i5 and element 6 okay it is also absorbing v6 i6.

And this must be = 0 that is what we have learned. Similarly, in this network separately only thing this numbers have been replaced by prime therefore if you say I will use for network B it will be similar equation will be v1 dash i1 dash absorbing power all the elements are that is how I have assigned the introduction + v2 i2 dash + v2 i3 dash + v4 i4 dash v3 dash v2 dash v4 dash + v5 dash i5 dash + v6 dash i6 dash that will be also 0 why not separate network.

So this is the power balance equation for each of the networks and nothing surprising about this, but now Tellegen’s Theorem states that if you multiply voltages of network A with corresponding currents in those elements in network B that is what I will do Tellegen’s Theorem this is not Tellegen’s Theorem it is known total power absorbed will be 0. Tellegen’s Theorem tells that you take voltage of this network let us use different color say you take this color voltage of this network across element 1.

Multiply with current i1 dash same element, element number 1 of the second network + voltage of this network across element 2 that is v2 I will not multiply with i2, but we will multiply with i2 dash and so on v3 i3 dash + v4 i4 dash + v5 i5 dash + v6 i6 dash and Tellegen’s Theorem tells that this two will be = 0. Alternatively, you take the voltage of this network and multiply with the current of network A then also it will be 0.

What does this mean. Also v1 dash into i1 that is voltage across each element of network B multiplied by the corresponding currents of network A and (()) (06:01). So v1 dash into i1 + v2 dash into i2 + v3 dash into i3 + v4 dash into i4 + v5 dash into i5 + v6 dash into i6 that will be also equal to 0 that is the new thing the above 2 equations are okay individually we know for each network it is it looks surprising voltage of one circuit multiplied with current of another circuit this 2 has got no coupling etcetera how this happens.

But that is what Tellegen’s propose that this has to happen and we will try to prove it. Proof is very simple and elementary nothing complex, but the statement of the theorem you have understood. Therefore 2 networks having same topology has to individually satisfy their own power balance equation that was known, but in addition to that this 2 lines this 2 equations are also true this is the Tellegen’s Theorem.

Now we will try to see how this can be proved, why such a thing which is not so apparent why voltage of this circuit and current in some other circuit even if their topologies are same why you multiply and sum them up we will give you 0 how do you say that. It must be told that this v1 and i1 dash for example v1 into i1.

I know specifically it has got a physical meaning. What it is amount of power absorbed by the load, but so far as v1 into i1 dash is concerned we cannot assign any physical significance to that because voltage of across element one of this network and current of network B you are multiplying what does it physically signify we do not know. It is not possible to signify anything to this product.

Although v1 i1 and v1 dash i1 dash I know pretty well what this signify, but let us come to this result why it will be like that.

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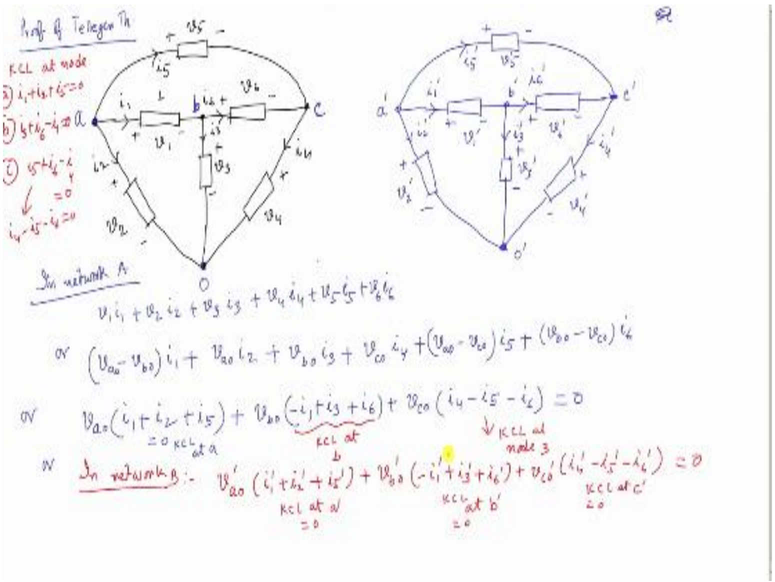

So I go to next page and paste it once again so this is the thing. So proof of this network I am trying to do. So this is so this is this network okay only thing this has not been copied this number was node a this is node b this is node c and this is node a all things have come correctly. Now from whatever network so this is Proof of Tellegen's Theorem. Now it is done like this suppose I calculate the power here that is I calculate v1 i1 of this network.

V1 i1 all are absorbed v2 i2 + v3 i3 + v4 i4 + v5 i5 + v6 i6. Now this v1 or v1 can be written as va0 – vb0 potential this v1 is va0 – vb0 va0 – vb0 into i1 this I can write + v2 voltage across this element 2 is va0 itself va0 – v00 that is 0. So va0 i2 for element 3 it will be simply vb0 = v30. So vb0 into i3 for element 4 it is + vc0 into i4 and for element 5 it will be va0 direction of current I have assumed like this.

So va0 – vc0 is the voltage which will make this side + this side – into i5 and finally the voltage across this elements 6 is vb0 – vc0 into i6 this will be the thing and this I know is certain to be 0 because power observed by all the elements in this network, network A in network A this is the thing. Now what I will do is this I will collect the terms of va0, vb0, vc0 separately.

So I can say that va0 into from this term it will come i1 + i2 this term va0 no va0 here, here also there + i5 and no other va0 this will be the thing I can write + vb0 similarly I will calculate vb0. Vb0 is – i1 this will give – i1 this will give + i3 vb0, vb0 is here once again + i6 this will be vb0 then what is vc0 all the terms vc0 I am collecting no vc0 here vc0 no vc o it is i4.

Here – i5 – i5 and oh sorry then another term is there – i6 and that must be equal to 0 that is what I am telling. You see I have calculated the power across each element and express the same quantity in terms of node voltages and some of the currents. Now you see or i1 + i2 + i5 i1 + i2 + i5 (()) (14:20) must be 0. This is KCL at a similarly i3 + i6 – i1 at node b you see i3 + i6 = -i1.

So here I write KCL this side only from this KCL at node a what it will be i1 all currents are coming out from the node i1 + i2 + i5 this must be equal to 0. KCL at node b node b what is there node b i3 + i6 going out so i3 + i6 and i1 is coming in so – i1 this must be 0 that is i3 + i6 is i1 so i3 + i6 – i1 must be 0 and at node c at node c it will be i5 is converging at c i3. So i5 + i6 and another – i4 is also coming – i4 that = 0.

Which of course mean this is if you take two other side i4 – i5 – i6 = 0 same thing. Therefore, you see this KCL at a similarly this term is KCL at node b and this term is nothing but KCL at node 3. So what I am telling is suppose you have not this is another way of proving that power balance. Suppose I am just calculating how much total power absorbed no = 0 I have put.

I know it has to be 0 this is one way of proving okay. In terms of node voltage you express and then you say va0 into 0 + vb0 into 0 + vc0 into 0 and this must be equal to 0 that is what the power balance equation is. So the total power absorbed by all the elements has to be 0 and it has come out to be 0 and this only thing I have expressed in terms of node voltages that is all. Now come to this network, in this network everything is like this.

Therefore I have no point in going on writing v1 dash i1 dash + v2 dash, i2 dash what I am telling in network B. In network B which is totally a separate network B only thing all the quantities here has been replaced by primes. So same thing you will get if you apply only thing they will become prime that is i1 dash + i2 dash + i5 dash then + vb0 dash into –i1 dash + i3 dash + i6 dash and + vc0 dash into i4 dash – i5 dash – i6 dash.

And once again i1 dash + i2 dash + i5 dash will be KCL at a dash at node a dash. This is KCL at b dash and this is KCL this must be at c dash node number c dash. So this will be once again 0 va0 into 0 so these are all 0 this term bracketed terms are 0 so this will be also = 0 that is separately the power balance will take place (()) (19:26) that is the whole idea. Now we are just the last line of proving this. What is that now I am telling that.

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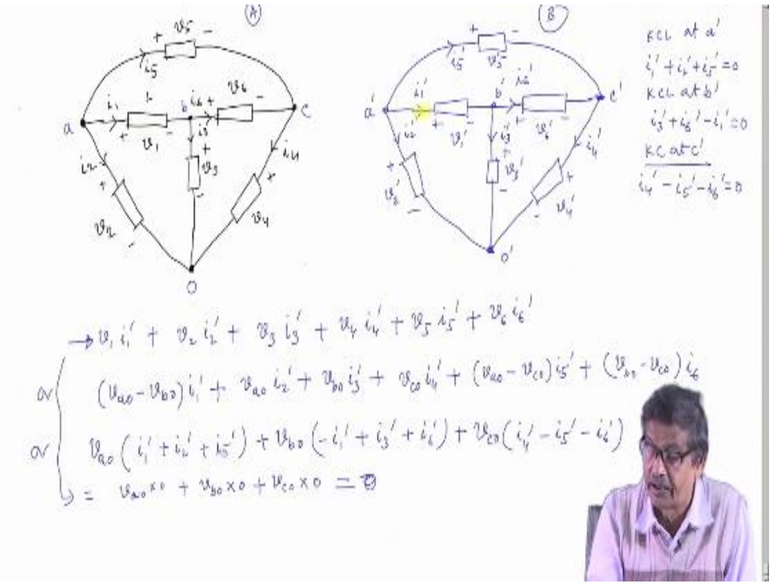

Okay let me do elaborately go to the next page and paste this one move it. Okay now come to this circuit once again. Now what I will be doing is this as suggested by Tellegen that now let us try to calculate v1 and then i1 dash voltage across this element and current in this element + power absorbed all the things + v2 voltage across this and current in second element of network B this is network A this is network B + v3 into i3 dash + v4 into i4 dash + v5 into i5 dash and + v6 into i6 dash this is the thing.

Here on once again I will do the same thing that is v1 this voltage is of course = va0 let me write that okay – vb0 into i1 dash v2, v2 is nothing but v this is a mind you va0 + va0 into i2 dash what is v3 vb0 into i3 dash + what is v4 voltage across element 4 it is nothing but vc0 into i4 dash. What is v5 va0 – vc0 i5 and finally across this element 6 v6 is vb0 – vc0 vb0 – vc0.

This will be the thing into i6 and once again I collect the terms of va0 so just out of curiosity I have multiplied without knowing really what does it physically imply I have taken the voltage across element 1 and multiplied it with i1 dash (()) (22:50) and so on just out of curiosity I am doing and then I collect the term of va0 what I will be left with i1 dash this is + i2 dash this i2 dash will come.

And va0 where else + i5 dash then for vb0 + vb0 if you collect it is – i1 dash from this then +i3 dash from this term and + i6 dash and finally vc0 into all the terms of vc0 that is i4 dash from this place – i5 dash from this and – i6 dash this one. Now what is KCL at a KCL in network B at a dash is how much i1 dash + i2 dash + i5 dash is 0. KCL at node b dash is i3 dash + i6 dash – i1 dash and that = 0.

And KCL at node c dash will be how much i4 dash – i5 dash – i6 dash = 0. Therefore with this sum a = va0 into 0 + vb0 into 0 + vc0 into 0 and this has to be 0. This is the most interesting part see this va0 a into now KCL of node see no matter this 2 circuits are independent i1, i2, i3, i4, i5, i6 will be totally different from i1 dash, i2 dash, i3 dash, i4 dash, i5 dash and i6 dash.

But i1 dash, i2 dash and i5 dash when added must be 0 because it is the compulsion of this network to satisfy KCL at all the points at all the nodes a dash, b dash and c dash. So Va0 is multiplied with a term which is KCL at node a dash, b dash, c dash and they are 0 that is why we say that and we started with this and then came to this to establish that this indeed is 0. I am not going to elaborate it.

But you can easily see similarly v1 dash if you wish to calculate you can calculate and i1 I will do okay. So understood this, this is Tellegen’s Theorem voltage across this element current in other sum them up then also it is 0.

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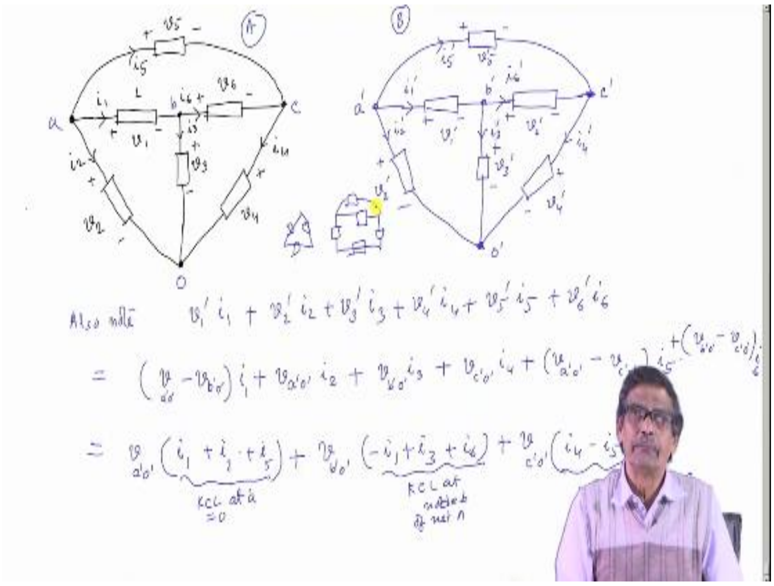

Similarly I could do which is just few lines I will write because it is necessary also this is network B this is network A also node that v1 dash into i1 (()) (27:25) + v20 dash voltages of this networks multiplied by the respective current in the first network that is network A actual current i2 + v3 dash + i3 + v4 dash i4 + v5 dash i5 + v6 dash i6 what this comes out to be. Okay same procedure v1 dash is what voltage across this element in terms of the node voltages v dash a0 – v dash b0.

I am sorry I should write it much more correctly va dash 0 dash this is this dash will come a dash 0 – b dash a dash 0 dash into i1 + v2 dash is nothing but v2 dash is this one v a dash 0 dash into i2 this is i1 + element 3 v element 1, element 2 is va dash 0 i2 element 3 is v b dash 0 into i3 b dash 0 dash + element 4 it is v c dash 0 dash into i4 element 5 v dash I mean b a dash 0 dash I am always making a mistake ba dash 0 dash – element 5 vc dash 0 dash.

And that will be into i5 current of this circuit and finally the last term that will be + that is element 6 which is vb dash 0 dash – b c dash 0 dash into i6 dash i6 and once again you collect the terms that is v a dash 0 dash it = i1 from this i1 will come then + i5 from this it will come and + i2 sorry i+ i2 also. So let me put this two comes first i2 + i5 similarly v b dash o dash will be equal to – i1 from this + i3 from this and + i6.

And finally v c dash 0 dash will come as v dash 0 dash c. C is not there here it is i4 – i5 and – i6, but what is this thing i1 + i2 + i5 is KCL of network at node a this is a b c and this must be 0. This one i1 + i3 is nothing but KCL at node b of network a not b dash but this must be 0 this network whatever it is separate, but it is satisfying KCL at around all the node i1 + i3 +

i6 hopefully it is coming correctly.

And finally this one is nothing but KCL at node c of network A these are all of network A. So this in general is your Tellegen's Theorem okay. One interesting point is that see okay it is nothing, but because KCL is satisfied that is why you get this. We will see its application next class, but another interesting thing is the voltage across each element of this network that is v1 i1 dash suppose this two circuits are having current bedding voltages and currents suppose time dependent thing.

It is also to be noted that you can take the voltages of this network I would set some time say T1 all the voltages I have taken and the current values of this network I will take at some different times say T2 still this will hold simply because KCL is satisfied of all the times no matter that it is satisfying now and then it is not satisfying not like that in fact we have pointed this out many a times while going.

KVL and KCL will be satisfied at all times in a circuit be it linear circuit, non linear circuit whatever it is KCL has to satisfy KVL has to satisfy. There are many applications in communication signal processing of this particular theorem, but we will apply here this theorem in our next class to establish another interesting circuit theorems called reciprocity theorem.

Thank you I stop here today, but go through this carefully it is very interesting otherwise two totally different circuit, but only a point is they must be of same topology then you cannot do you cannot have a circuit like this with triangular topology, with a circuit like that this circuit is different topology. I mean perhaps another element you bring in. You must see structurally there same time. Thank you.