Network Analysis
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Lecture – 57
Reciprocity Theorem

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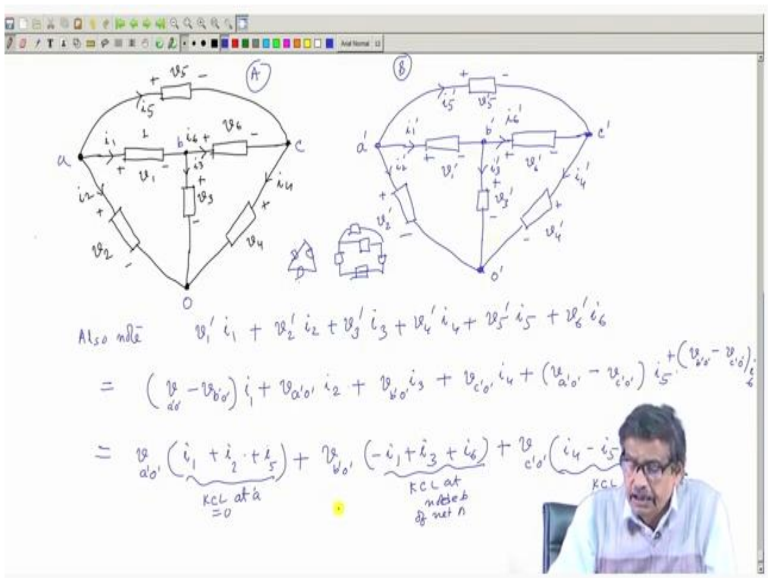

Welcome to lecture number 57 and in this class, I will tell about reciprocity theorem and the proof of this as an application of Tellegen's theorem I will utilize that one you recall that in our last class I told you that if you have 2 networks A and B which are structurally same that is their topologies are identical.

These elements of course could be different in these 2 circuits totally different circuits and if you assign voltage and current polarities and their directions same in both these networks then we found out that voltage across any element of network A multiplied by the corresponding current in that element in network B which are denoted by these prime letters and sum them up they will be same and not only same that will be equal to 0. Now this Tellegen's theorem has got several applications in the field of signal processing and so on.

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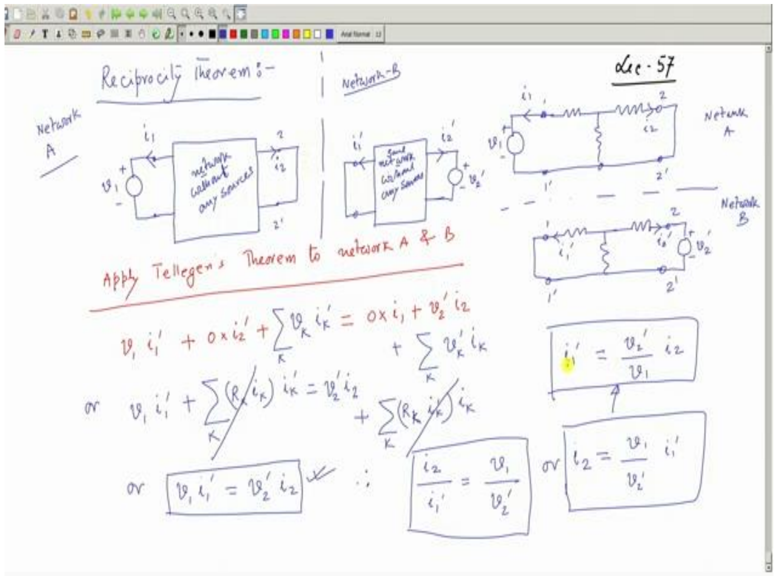

But so far as network analysis is concerned as and when required I will apply that, but it is interesting to note to prove reciprocity theorem this particular theorem that is Tellegen's theorem can be very nicely applied to establish reciprocity theorem. Now to do this let us consider you have a network for example any network like this inside the box these networks are there inside the box what could be the things it could be a networks for example like this without any sources.

So network without any sources inside and for example this could be inside, and they could be also impedances with no initial conditions and things like that. So if that is the thing then these 2 terminals are these 2 terminals and this networks inside could not be as simple as this they could be several other elements connected but somehow 2 terminals come out here and 2 terminals come out here like that presumably I will call it as my input terminal or this could be my output terminal and so on now in this network if you apply a voltage source with this polarity.

Suppose v1 this is network A and these 2 terminals that is I am applying as if a voltage here please be with me v1 and then if you short these 2 terminals which has an example this is suppose 1,1 dash this is suppose 2,2 dash so thigs is 2,2 dash this terminal so this if I short it then there will be currents in this circuit I can find out this current solve this networks and so on. So let this current direction which I can arbitrarily assume as i1 in this direction as if this all the elements will absorb power that way, we will try to apply the Tellegen's theorem.

In this case that is this current I am assuming i1 corresponding thing and the current here is suppose i2 and this I will call network A. Now the question is if I apply instead in this network which for an example, I am just showing it by a simple network like this to drive the key things to derive the key things here. So this is the thing now suppose in this network if I apply some voltage say v2 dash I call it and this 2 I short the opposite thing I do I apply the voltage across 2,2 dash and short the 1,1 dash terminal then can I predict this current.

So these 2 networks so far as their topologies are concerned, they are identical therefore for both of them and this is suppose network A and network B is it not are not the topology same topologies are same. These parameter values are of course same as this one that is, I am going to apply a Tellegen's theorem to the same networks when one of the elements have been changed 2 of the elements have been changed here there was some voltage here nothing was there shorted and things like that got the point.

So the network topologies are same this current was i2 here also there will be some current i2 dash like that. Now therefore network B in general will look like this one same network that is this is the same network as this box same network without any sources inside there is no sources without any sources. So what I have done I will apply a voltage here like v2 dash and I will short these 2 points here and in this network this 2 whole network including this 2 input and output terminals things are connected and their topologies are same.

Therefore I say that this current is i2 dash and this current is i1 dash. So all quantities in this network I will denote by prime letters like this therefore I will be able to apply so what is our goal to find out i2 dash and v2 dash is applied here and these 2 are shorted that is the goal. Now to find out that quickly what we'll do is this we will apply Tellegen's theorem so apply Tellegen's theorem to network A and B.

Can I apply this yes because their topologies are same so I will say that v1 into i1 dash so v1 in this branch v1 into i1 dash this current plus then I will complete this branch it will be equal to what is the voltage here this voltage and the current of the other network I am multiplying when these 2 are shorted the voltage between these 2 is 0. So 0 into i2 dash is it not plus voltage of this into this current and so in all the elements I have to do this. So this is considered one element here between these 2 points.

So that one I am going to write it like this it will be vk into ik dash and summed over k whatever total elements are there. So therefore total number of elements I have considered is to be k+2 summed over k how many elements are there across each element I will go voltage into current, current of the corresponding element in the other circuit like that I will go on doing. So and here how many terms will be there 2 terms there then another k.

So total number of terms will be k+2 this I am telling is to be equal to the voltage of the corresponding elements for example voltage in this branch in this one is 0 because this is shorted therefore 0 into i1 and all are absorbing power is it not this is how I assumed this direction of the current. So 0 into i1 always be consistent with that 0 into i1 plus then this term I will first write that will be v2 dash into i2+vk dash, vk dash into ik summed over all the elements here in the network present okay summed over k.

So first term second term then the k values here we will start from k = 3 and so on. So this is how this is what Tellegen's theorem tells us that this will be true then you see that or I will write that v1 i1 dash this one is of course 0 plus this one summed over k vk into ik dash what will be vk if it is purely resistive network I can write it as Rk into ik this is my vk because no source is involved.

So across each branch there will be voltage drop so into ik dash and on the right hand side you will get this is 0+v2 dash into i2 plus this term this will be vk dash which will be equal to because this network has not changed exactly same as this with same parameters. So parameters have elements have changed only in this are in these 2 branches you got the point so inside it is same so Rk same Rk into a ik dash and into ik can summed over same k.

These 2 terms if you see there they will they are same Rk into ik into ik dash this is also Rk into ik into ik dash therefore they will cancel out these 2 terms will cancel out and you will be left with v1 into i1 dash = v2 dash into i2 it is needless to say that if this impedance only condition is that this this internal network here there should not be any sources present. However it could be impedances Zs you know it could be Zs also because vk can be written as Zk into ik.

So whatever the distances I have put this could be replace by Zs that is the energy storing elements may be there but with the condition that they are not having any initial currents that is the thing anyway we have got this very important result which says me that if you apply v1 here this is i1 will be the current here and i2 will be the current. So what was my problem to determine all things I have written correctly.

So v1 I will say that i2/i1 dash =v1/v2 dash this way also I can write it or I can also write that i2,i1 to find out it will be v2 dash/i2=v1/v2 dash into i1 dash is it not if I have applied a voltage v1 current was i2 if I apply a voltage v2 dash here what should be i1 dash, also can be written in this way it will be v2 dash I think this is the final thing I should concentrate upon into i2.

What was my problem my problem was if I apply v1 I noted this current to be i2 that is known what will be the current i1 dash when you apply v2 dash here it is this value i1 dash will be v2 dash/v1 into i2 if these 2 voltages happens to be same.

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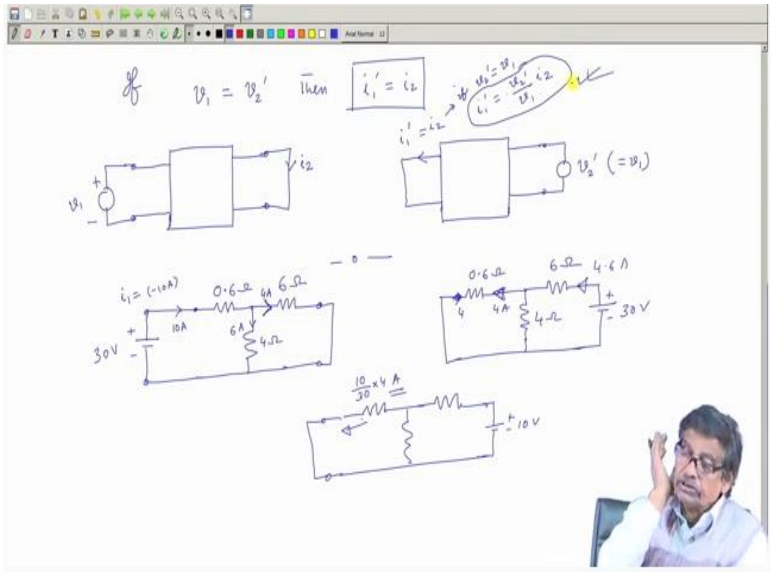

If v1 is equal to that is this voltage v1 same voltage if you apply as a special case v2 dash, then then we note that we note that i1 dash this current will be same as i2 these 2 are same you can cancel it out so i1 dash will be equal to i2 as a special case that was more general and expression. So what is the conclusion, conclusion is that if you have a network linear bilateral in linear networks like that and here is 2 terminals here you apply a voltage with this polarity v1.

And here suppose these 2 are shorted and this current is i2 is it not v1 applied i2 current flows then the same network to this pair of terminals if you apply a voltage v2 dash which happens to be equal to same as v1 I am telling if you apply same voltage. Then if you short it then this current which is equal to i1 dash because this is the element i1 dash this will be same as this current i2.

This is the reciprocity theorem in general if this bit it is v2 dash this is true if v2 dash = v1 otherwise also I will be able to calculate this current i1 dash is equal to it will be i1 dash will be equal to v2 dash /v1 into i2 in general. So if you apply to this terminal voltage 2 times then this current will be i2 and this is called reciprocity theorem. It will be clearer if I do a very simple but useful example to highlight the concept.

Suppose you have a network what do I mean by this you have a network like this. This suppose a 30 volt source you have applied, and the networks are this is about 0.6 ohm this is 4 ohm and here it is supposed 6 ohm suppose here between these 2 terminals I have applied 30 volt and I will short this. So this is network A you know that example then I am asking you what will happen 0.6 ohm this is 4 ohm this is 6 ohm.

And here I will apply this voltage 30 volt and how much will be the current in this branch these called reciprocity theorem. If you short these 2 there will be some current here for a given applied voltage between these 2 terminals, then to these 2 terminals if you apply that same voltage then what will be the current. The calculation of currents are very simple I have taken the figure if you solve it, it will be 4 ampere here and 6 ampere there.

And here it will be 10 ampere is it not it will be 10 ampere. So I am predicting that if you apply same voltage to these 2 terminals where 6 ohm is connected 30 volt then this current in this direction, I am predicting it will be 4 ampere. Now this I have shown 10 ampere like this it gets reversed. Now this current in this circuit if I solve how much will be this current 10 amperes in this direction forget about your Tellegen’s theorem conventions.

This will be the exact current 10 amperes, 6 ampere, 4 ampere. Now to be consistent with that you can only say this is -10 ampere flowing into this this is your the value of i1 okay that way if you want to think this is this one this, I note it down then I will say that this is -10 ampere, and this is 4 ampere that is fine. Now if you apply 30 volt here what will be this current top to bottom 4 so -4 ampere here. So from bottom to top it will be 4 that is let us solve this circuit individually okay so what will be this current independent of this Tellegen’s theorem.

This current will be first you calculate this current that is how much it will be this current will be total current and 4.6 ampere that is do see this parallel this thing. Then what will be this current 4.6 will get divided into 2 parts. So 4.6 into 4 divided by 4 so this will be 4 ampere is not this will be 4 amperes from bottom to top therefore this was 4 ampere this way flowing and if you apply 30 volt here 4 amperes from right to left it was flowing.

Now if you maintain the polarity of the supply voltage 30 volt here this current will be because that was derivation this will be 4 ampere is it clear. So what I am telling in the second case I need not find out series parallel combination if I know this particular theorem I will even say that if supposed to these terminals 2 and 2 I will short this to the this information is known to me this is 30 volt this is 4 ampere this way if this information is not known to me suppose this to I short.

And here I apply a 10 volt source got the point what will be the current I need not solve this circuit once again it will be how much 4/3 ampere why? Because that current will be 4 and voltage ratio is one third instead of applying 30 volt I have applied 10 volt v2 dash/v1. S this current this current will be v2 dash 10/30 into this current 4 ampere in this branch. So 4/3 ampere you see the usefulness of this problem this particular theorem.

If you apply across 2 terminals a voltage then current in some other branch you calculate it then you need not once again resolve this circuit when you apply across these 2 terminals a voltage and other 2 terminals where you connected voltage earlier put a short circuit there these magnitudes of current remains same or if you apply any fraction of that voltage here by virtue of this relation I will be able to calculate that current. So this is called reciprocity theorem this is of course one can apply it for any complicated or even AC circuit.

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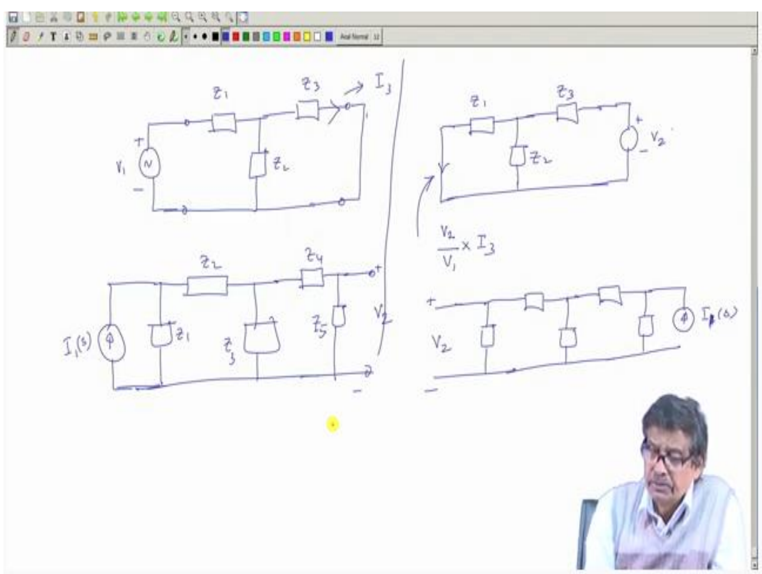

For example I say that I connect a voltage here where these are impedances not just resistances Z1,Z2,Z3 and these 2 are the terminals I have identified and these 2 I have shorted. So I will solve this network for this current I will solve that is i3 I will be able to solve this complex impedances are there suppose sinusoidal applied voltage is there v1 I will get this current i3 here. Then I am telling the same network here you apply a voltage v2 AC voltage because complex Z3 and Z3 then this current this current in this branch you need not resolve the circuit it will be equal to this v2/v1 into i3 where i3 was this defined is it not.

This is the crucial thing v2 dash/v1 , so v2 dash v2 here so be careful about whether I call it v2 dash or v2 it does not now mean. Now after learning all these things I will say okay here is a circuit you apply a voltage between these 2 terminals v1 note down the current in this branch its direction then if you apply instead this voltage v1 there some other voltage v2 then you can find out the current in this branch efficiently.

So it can be applied there sometimes it is very useful. Similarly in a network if you apply a current this, I leave it to you suppose we have a network like this I mean extension of this suppose you have a network like this. Suppose you have a network like this suppose you connect a current source here instead of voltage source i1s.

Then calculate the voltage between these 2 points here there will be some voltage available then whatever is this voltage say v2 you calculate solve this network these are impedances suppose Z1, Z2, Z3, Z4, Z5 calculate the voltage with this plus this minus then if you open circuit this one just opposite to that of this here if you apply the current source i2s i2 then same current if you apply i1 then I am telling the voltage you will get between these 2 will be same as this v2 that is the whole idea.

Therefore solve problems we will solve some problems and upload it therefore reciprocity theorem I hope you have understood simple ideas that in a network linear network like this with no initial conditions of course if you know if you have applied a voltage v1 between 2 points and you are examining currents flowing through another 2 points when they are shorted whatever is the value of the current instead of applying voltage here if you short it and connect the voltage source here in this branch the same current will flow.

With this I conclude the what is called the reciprocity theorem and Tellegen's theorem as such there are several interesting problems you try to apply wherever possible nonetheless, we will solve some problems and upload it as an time as and when it is required. Okay now another small theorems which I must tell to complete this one that is called maximum power transfer theorem.

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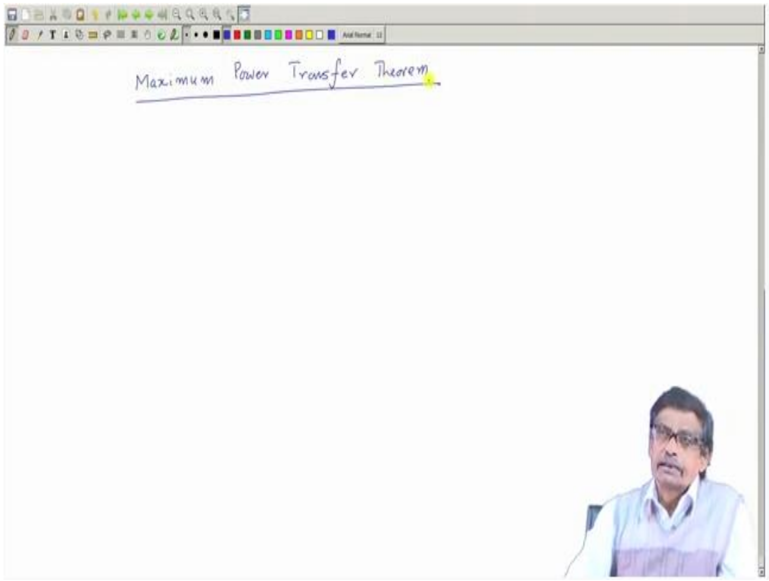

Okay this maximum power transfer theorem we will take up in the next class.