Network Analysis
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Lecture – 58
Maximum Power Transfer Theorem

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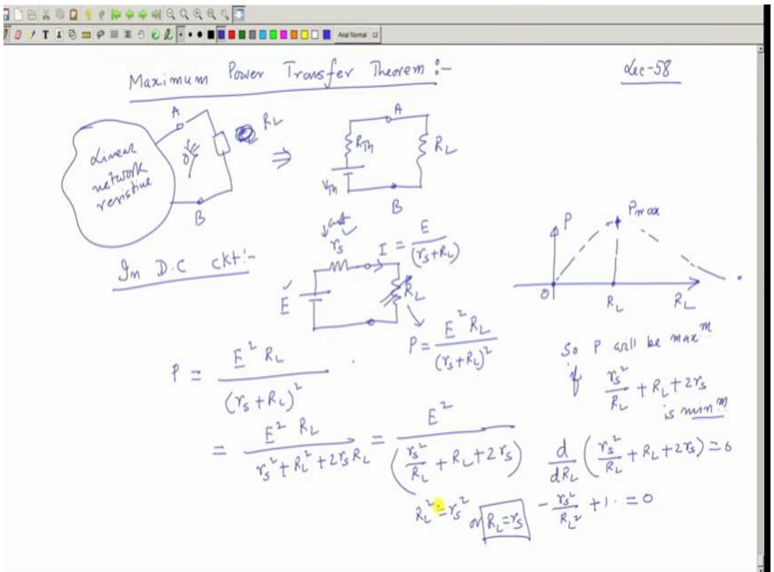

welcome to lecture number 58 here another interesting theorem is there which is not really solving a network theorems, but it is called popularly known as maximum power transfer theorem. It says that in a network between any 2 terminals what impedance should I connect? so that power consumed in that impedance are called load impedance will be maximum power delivered to the impedances maximum.

To begin with. let us tell you that you have a network here linear network suppose resistive network. this team will see also the for the AC circuit how it looks like you pick up any two points then here I am going to connect the load impedances. I want to know what should be this load impedance. ZL or if it is resistive network it is some RL I will connect what value of RL will give me maximum power to do this i behind the A and B this whole network can be a change to some R Thevenin and v Thevenin is it not.

Then these 2 points A and B and then the load resistance here. So the problem can be simplified in this fashion. Therefore, to find out first the maximum power transfer theorem with DC circuit. So in DC circuit it will be like this that you have a I will consider a battery having a voltage E open circuit tmf. These are the 2 terminals of the battery and here is your RL. This I will go very quickly and without not much maths. So and this RL am going to vary as you can see if RL =0 output power will be 0 because there will be current but RL is 0.

Similarly, if RL is very high are you getting suppose I am plotting here power versus RL in this network so for any finite value of RL the value of the current will be E/rs+RL this will be the current and power in RL this P = I square that is E square/rs+RL whole square into RL what I am telling if RL is 0 then power is 0, if RL is very large infinitely large it is RL by another this is RL square.

So then also power will be 0 and it is a smooth curve as a function of RL. Therefore, I must expect so power was 0 here at RL infinity also it was 0 but for finite values of RL it gives you a positive number. Therefore, this power must have gone up reached some maximum value and once again has started coming down is it not that must be happening? I want to find out what is this value of RL so that you get maximum power P max.

The answer to this question is very simple that P= E square mind you E is constant rs is constant. you are valuing RL only. So E square into RL divided by rs +RL whole square is the expression of the power. now therefore it looks like you differentiate this DPDRL equate it to 0 you will get the values of RL. But only thing is a it can be that calculation. can be simplified if you just do like this this you break up RL square + 2 rs into RL and there was an RL here and this RL you bring out the below this one.

So it will be rs square/ RL + RL RL square I am dividing both numerator and denominator by RL and then +2rs. So power will be maximum if the denominator is minimum. therefore, I will instead of trying to differentiate this whole thing which will also yield same results. But there the thing is our RL is present in the numerator denominator. So that computation becomes a little not tedious but compared to that this is much simpler. why? because it is there.

So I conclude so P will be maximum if this factor rs square/ RL+RL+2rs is minimum that means the DRL I will set it to there equal to 0 and if you do it will be -rs square/RL square with respect to RL I am differentiating. these will be +1 and this will be 0 this will be the thing because rs is constant I am not going to change out rs source resistance and Emf same battery. so from these I conclude that RL square = rs square or RL=rs knowing fully well that this RL will be a positive number.

So this will guarantee the minimum numerator denominator means maximum power and one can calculate the second derivative and verify that really it is minimum or not I am not going into that therefore we conclude that if you vary RL.

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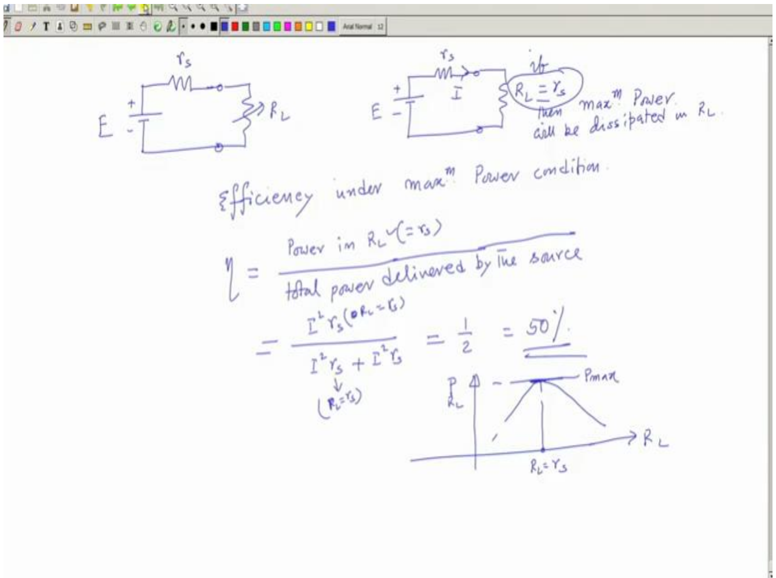

So maximum power will be delivered for a battery. The conclusion is this this is the source E here I am connecting RL which is varying and what I got is this one. this is rs and if you connect a resistance here whose value is rs, if RL=rs then maximum power in RL will take place maximum power will be dissipated in RL and only one additional information I will tell you about this maximum power transfer theorem that maximum power will be consumed by the load resistance when its value is equal to source resistance.

That is in language people say load resistance should be equal to the source resistance in case of DC circuit for maximum power to occur in RL okay that is fine now. But you know the impedance of the load is not in your hand. it depends upon the application our application requires some voltage to be applied across E it is really not in our hand. But it is worth noting that what is the efficiency under maximum power condition.

What do I mean by efficiency? Efficiency you know is the output power that is in RL power in RL divided by power delivered by the total power delivered by the battery by the source is it not? Total power delivered by the source. Now what is power in RL under maximum power condition? It is i square into rs this is the power delivered in RL because RL= rs then so I square into rs is the power delivered by the source.

What is the input power? 2 ways you can calculate it will be the output power that is i square into rds+ another i squared into rs which will be lost inside the resistance. So that mind you this rs I have got RL= rs this rs I have put RL=rs is it not output power, power in RL when RL= rs that is what i am meaning so i square rs an input. power is this output power plus the power loss in this resistance.

That is the total power delivered by the souls and this will become equal to 1/2 or 50% in percentage. Therefore, maximum power condition if you put RL equal to sources resistance okay maximum power will be delivered. But the efficiency of this overall system will be only 50% not a very good situation. So the point is this curve if you plot here some P max here if you put RL and here you put power in RL then I am telling at RL= rs this P max occurs this is the P max.

It is it may be some people may be interested after knowing this okay maximum power will be delivered. Then i will always try to mean RL =rs but you really cannot do it because load impedance is not to be decided by input power condition. Even if somebody insists that no always connect impedance load impedance to be called to source resistance. Okay you will get a maximum power delivered to the load. But the point is the efficiency of this system will become too bored. So this is not a very good proposition.

However, in case of low level of power in electronic circuits, amplifier circuits etc. Where power itself is low efficiency does not matter too much there. people will always see for impedance matching. So that maximum power is delivered is it not in your sound system. okay output impedance of the load speaker should be same as the input impedance so that maximum power loudness is important there sacrifice efficiency at low level how does it matter anyway this is how this is the maximum power transfer theorem in DC circuit.

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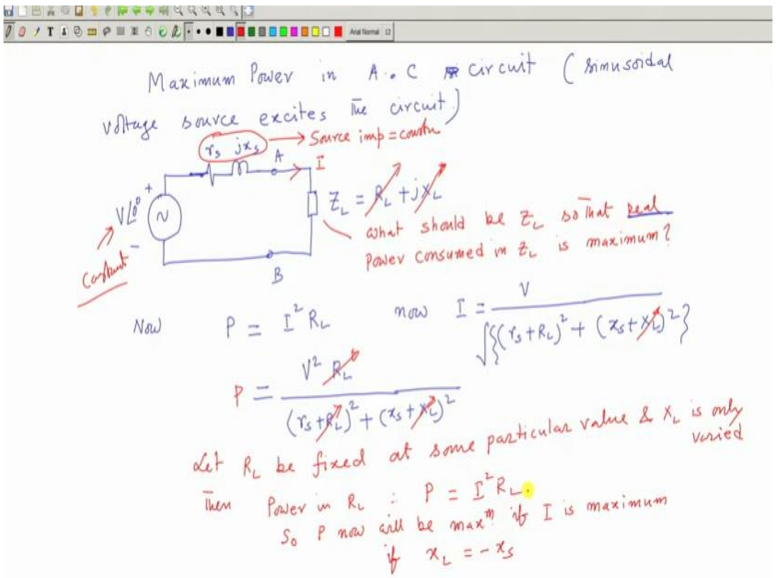

Now things will be a bit complicated if it is AC circuit maximum power in AC circuit. By AC circuit I mean sinusoidally excited sinusoidal voltage source excides the circuit. How the source can be modeled AC source it is like this this is, and the internal impedance of a source are generator will have now a resistance rs as well as a reactance jxs got the point.

Because all generators are some coils moving in a magnetic field therefore that can be modeled as a series impedance of winding resistance as well as reactance nothing like capacitance rs and -jxs is not there So these are the source terminals A and B and here there is a fixed voltage rms value is suppose v say b angle 0 degree is the voltage applied his. AC source so I will apply pressure and I am considering these circuit is operating at steady state condition. Okay now here I will connect an impedance Zn which will have once again 2 components RL+jxL

If this xL is positive, it is inductive circuit. If the value of the xL is negative it is capacitive circuit and I am telling that I will be wearing both of them. the question is what should be ZL so that real power consumed in ZL is maximum. This is how the problem should be stated okay RL, xL you arbitrarily connect you vary them this is source impedance constant this is the supply voltage constant.

Therefore what should be this valuable of RL, xL in terms of rs,xs so that power in ZL will be maximum. So power expression of power now for any arbitrary value of R and xL powered will be maximum when what is the expression of power current square into RL that is the expression of the power real power. So that real power is i square into RL which happens to be equal to this voltage into current into cos theta of the circuit we know that is it not.

So power consumed in this load. impedance will be the expression of that will be simply i square into RL is the rms value of the current. Now what if the rms vale of the current rms value of the current will be the supply voltage rms value divided by the total impedance of the circuit which happens to be series in nature so it will be rs + RL square +jyj xs+xl square is it not under root this is the expression of the current.

So put it here so it looks like it will be v square this algebraic equation mind you voltage by impedance decides the rms current we know that. So this will be then equal to p square into RL divided by rs + RL square +xs+xL square. And in this expression, you know what are the things I will be varying, I will be varying RL as well as xL other things are constant. I want to find out what should be RL and xL .

Now it looks like there are 2 variables here who decides the power because I will be going change both at RL and xL. Now to deal this problem most efficiently what I will do it try to understand this step. I will first choose any value of RL any fixed value and keep it fixed for example RL = 2 ohm and i will try to find out the condition of xL. So that power will be maximum there got the point.

Instead of varying RL and xL simultaneously now this step is very important. Then you can save lot of time in derivations differentiating things just like that. What I will do is this let RL be fixed at some particular value any particular value say RL =1 ohm i will not vary RL, xL only i will vary. So in that case what I am telling is that is this problem of getting this value of the load impedance for maximum efficiency I am doing in 2steps.

First step I will fix RL to some many arbitrary value I will fix it then I will say that if that be the case then power in RL will be P = I square into RL is it not and RL being known fixed at a particular value. who in power will be maximum? when i is maximum in the rms value of the current is maximum then only power in this RL will be maximum is it not I will vary xL the particular value and xL is only varied.

So under this assumption I know what will be the power in RL, power in RL is i square into RL, RL I have made fixed who is changing them? i is changing what for because you are varying xL which value of xL will give maximum rms current this scenario. So P will be maximum P now will be maximum if I is maximum and I is these expression where rs, RL is fixed in this case the same expression is valid now for i.

Now I am telling rs,RL I have now fixed I am not changing RL, xL only I will vary now only i then when i will be maximum does it require any derivation? I will simply say i will be i will be maximum looking at this expression. if the value of xL =-xs then only will be maximum other ways out. Got the point but then you will say look you have still not completed your solution. You have simply said that if xL value is chosen to be -xs then you will get maximum power in RL for a particular value of RL. Now to maximize RL no matter what is the value of RL xL must be saved to -xs.

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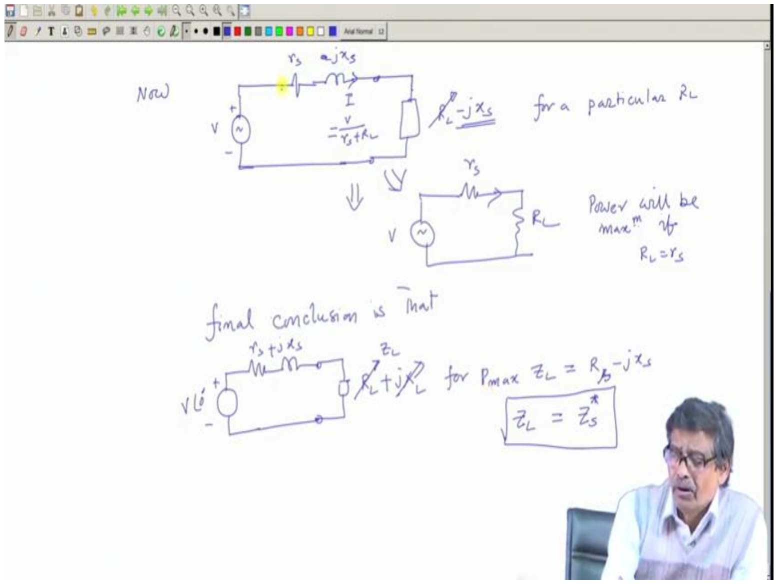

So i then conclude that here the now I then write that. Now what we have got in this circuit this is your rs jxs there are the 2 terminals I have got one thing maximum power will take place if the load impedance is set to this value for a particular RL. So for a particular RL if this is the thing current will be how much current will be this voltage divided by rs+ RL is it not under this scenario.

No matter what I suppose you choose RL = 5 ohm I will say maximum power will take place in this RL = 5 ohm provided the load impedance is 5-jxs, jxs is fixed if somebody chooses 8 ohm RL equal to then also you will say maximum power will take place when xL = xs of course the level of these two powers will be different. That is different issue therefore essentially once this is chosen, I will vary play with RL and this circuit this rs the magnitude of the current this circuit becomes a unity power factor circuit looking at this.

So this is equivalent to rs and here is RL is it not what else? This is equivalent to this and this case. I already know when the maximum power will take place when RL = rs is it not. The moment the reactance value of the load impedance is fixed it has to be -jxs then only current can be maximized rm is the value of the current. Therefore, I will say now with this choice of xL, xL choice should be -jxs then power will be maximum power.

Now I have fixed xs and vary RL this the second stage I am doing power will be maximum if RL = rs and this need not be redone because I have already it a circuit supply voltage v rs, RL resistance circuit what is this? So the final conclusion is that in a circuit better I draw it this is the AC supply v0 degree whatever it is, it is source impedance rs+jxs.

These are the two terminals where I am connecting load and now, I will say I will vary both load and source impedance. Simultaneously you imagine you are varying then for P max this is ZL for P max, ZL must be equal to RL- jxs , Rs-jxs or people say that load impedance should be complex conjugate in general although source impedance cannot be capacity but for academic interest you can also see if your source has got capacity input impedance then the load should be inductive than r + jxs complex conjugate of that maximum power transfer will take place got the point.

And the last case so this point you see how nicely this problem has been tackled in general I wrote this expression my goal is to find out both RL and xL I will arbitrarily vary one to know what should be the value of rs, RL then I say that look here instead of trying to vary both of them together imagine that okay RL will have fixed some value and all the xL you were varying then we came to the conclusion no matter what value of RL you have chosen. maximum power will be only when xL =-xs because current is to be maximized. then we allowed RL to vary and xL is said to -jxs.

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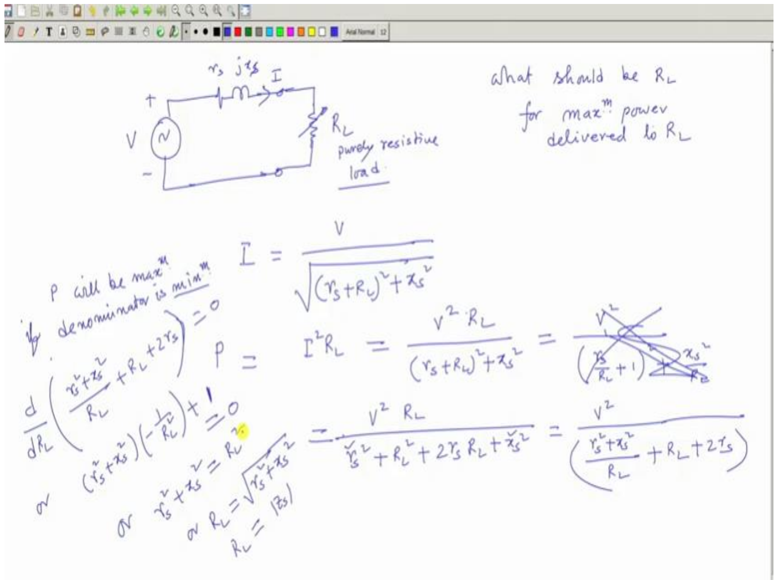

Finally, the thing is suppose you have a situation like this AC circuit this is a standard topic. this is suppose source impedance jxs and here is. the load impedances ZL I have connected Z is the load impedance. Suppose I say load impedance is such that it is purely resistive. That is only RL is connected and I will vary this purely resistive load what should be RL for maximum power delivered to RL.

Once again you will go by this current is I here of course one variable I will just give an indication what will be the magnitude of the current. That is the rms value of the voltage v divided by square root of rs+RL whole square +xs square is it not this is the thing. and what is the expression of power for any valuable RL? It is I square into RL which is equal to v square put the value of the current sorry.

So v square/rs + RL whole square + xs square into RL this will be the maximum power. Then once again for because RL appear both in numerator and denominator you can differentiate DPDRL say to 0 but I always do this bring these RL below. If you bring these RL below this can be written as rs/RL+1 whole square +x square/ RL is it not this is correct. This RL you bring and put it in say no it is not square is not there let us not do this.

Let us do it is like this v square RL let us do this step expand this xs square + R+2 rs into RL +x square of which rs and xs are constant this is constant internal impedance of the source. Then what do you do? you bring this RL below this RL So it will be rs square+ x square this one I grouped together and divide by RL this will be the thing +RL square/RL that is RL +2 rs all the terms are okay hopefully 2rs.

So power will be maximum if the denominator is minimum is it not because v square is constant in the numerator. So p will be maximum if denominator is minimum and denominator minimum will occur if your vary DDRL this whole thing rs square+ xs square divided by RL +RL+2rs this quantity differentiate it and set it to 0 what will be the differentiation? this will be rs square + xs square RL square + rs square + xs square and this will be -1/RL square and +1 this will give you 1 and differentiation r is being constant is 0 and this is equal to 0 this will be the thing.

Or you will get rs square + xs square if you manipulate this it will be equal to RL square or RL which has to be a positive number so is this one which happens to be the magnitude of the source impedance. Therefore, remember if it is only this resistive the maximum power for this value of RL has to take place that is for maximum power RL is to be said to this. For example, if I say rs+ jxL is 3+ j4 I will say connect a resistance of 5 ohm and maximum power will be delivered to. I hope you have understood this from next class I will start the application of graph theory in network analysis. Thank you very much.