Network Analysis

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Lecture – 59
Graph Theory Applied to Network Analysis - I

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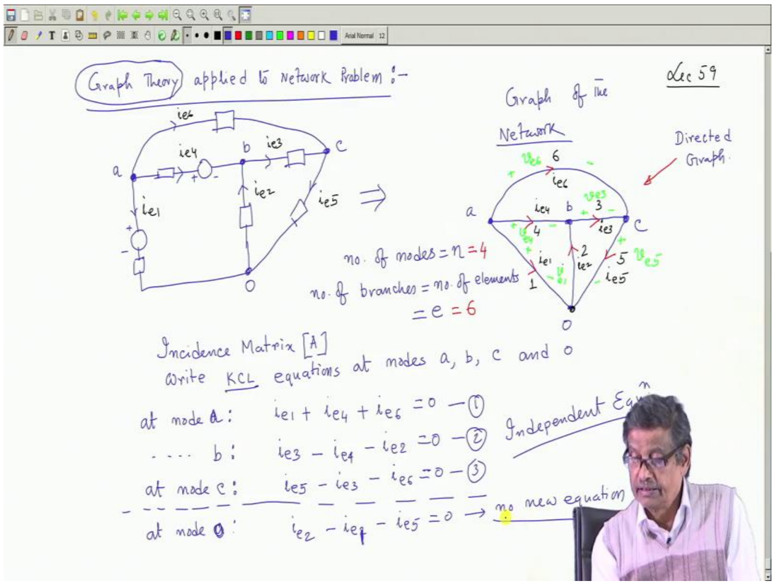

Welcome to todays 59th Lecture and today we are going to start a new topic which is application of graph theory to the analysis of network problem. This topic is very interesting in the sense mm that network problems we have now learn several techniques to solve. But then, maybe very large networks and the current power etc to be calculated that is what we meant by solving a network problem?

Then, in such cases, the whole steps could be very tedious, really very large network, large number of loops, large number of nodes, whether the analysis of any given network can be made more systematic and can be easily handled by following certain logic and can be that problem solving of network can be delegated to a computer course. You will write course to solve a network problem. Generally that is the motivation of this particular way of looking at network solving problems.

That is the graph theory is the keyword. Graph theory in itself is a separate subject and we just restrict ourselves to those ideas of graph theory which is necessary to solve a network problem. And it is very easy because network problems are such that will soon see that straight away graph theory can be applied, it looks like that. So we begin with this new topic today and suppose you have a network, let us first draw a network.

Then I will talk about graph of a network. So a network suppose you have a network like this impedances there, may be sources present plus minus and their impedances there and there is another network ok plus minus like that and other impedance. Let us consider a network problem in this way and here is another elements suppose, the network given. This network has got two sources voltage sources and several other elements, impedance elements.

And how many nodes this network has got? 1, 2, 3 and 4 nodes are there. This nodes we will mark as a b c and o, I will take as a reference node and all the voltages note voltages will be denoted by a0,b0 these are all the, are the circuit elements between a and O, the circuit element consists of a source in series with and impedance. Similarly, here like that. Now, this is the given network. Ok Now from this network, what is the graph of this network?

Idea is a very simple. Graph of the network, it will be show all the nodes. a b and c these are the nodes and there is the reference node now is o. Now, while drawing the graph, you do not show all the details what exists between a and o but simply join with the lines like this. So, between a o, whatever element is there will just show it in this simplified diagram as like this. One should not understand these are short circuit and no, this is the; we say this is the graph of this network.

I think you got the idea to draw the graph of the network, what you have to do. This is the actual network. Replace all the branches by straight lines and you call this to be the graph of this network. The nature of this graph will be different obviously from different, for different networks. So, this is one graph and in this graph total number of branches, we will call some people call number of Ages, number of cards, will call them number of branches, as number of elements. How many nodes are there?

The numbers of nodes, number of nodes have very good idea of what node is suppose n and number of branches of this network; we will call it, which is same as number of elements, number of elements? Or number of cards no. In writing so many things so number of elements and this I will denote number of edges, I will denote by e, this letter. Then, this is the graph of this network ok. Now, this network is called, this graph is called an undirected graph, because no directions, no add user put in the branches or ages ok.

In fact graph theory is straight away one can go if one has got some ideas of network. And I hope we have that background for example, in this network, when, if you solve, I can show the direction of the currents. Direction of the current, direction of the current which is arbitrary And suppose I assumed the direction of the current in this branches. So, this is an undirected graph when you also show the direction of the assumed direction of the currents.

In this graph of this network, when you say it is a directed graph, so will be talking about directed graph. So what I will do is this I will show directions also the way I have assumed. I will go rather slowly to explain to you the main aspects of what I am doing. Therefore there was a graph at network from that I will draw the graph of the network and a vacuum the direction of the current that I told you many time.

It is our liberty to assume that direction of the current, assume that in the original network. And show these Directions, send directions via by this network then it is called a directed graph. So I have a graph of this network which shows also directions of current. Then, the number of nodes in this particular problem, let me also write this is for, for this problem 1 2 3 and 4 notes are there a, b, c how many elements are there, number of branches 1, 2.

These 2 do not consider it to be separate elements and add. This exists in a specific plan. So this should be considered as one element ok. So, the number of elements will be as you can see 1 2 3 4 5 6. So in this case, it will be 6. Therefore, number of nodes are 4, which I will denote by n and number of branches as the number of elements this is the thing I will assure. Then what will do is this that I will name the branches so that we can keep track of which current which voltage, I am talking about.

So, the numbering of the branches we will be also doing. So, I will say this branch ao I will call one that is element one. This is suppose element 2 this branch 2 I will call element 3, branch 3 then this is say 4 and this is 5 and this is 6. So there are six branches and their now named arbitrarily 1 2 3 4 5 6. So, I will say that the current flowing here. I will write it like this i1 in this branch. This branch this current is ie1, I will say in this branch the current I will say if i4 because I am named this current as 4.

And this current is ie4. And this current is ie2, and this current is ie6 like that. And this is ie3. So in this graph, the details what exists in a branch is not shown just shown that joint connected by straight lines. And these are the things I can easily show here also the direction of the current in the original Network and here so this is ie3. So, that you have one to one correspondence. So this is ie5. And this is ie6. So these are the branch not only that, across each branch, there exist a voltage.

For example b, a is the voltage across this branch. So this voltages also I will say that it is like that. Let me use different colour to write that voltage in this branch. I will write it as plus minus why everything plus because current is flowing from higher to lower potential so and that voltage I will say ve1, Voltage across element 1 across this branch, branch number one. Similarly, voltage across this element, I will write ve4 voltage across this element this side plus this side minus. I will write ve3.

And similarly polarity, you also note down this voltage will be ve5. And voltage will be across this branch plus minus this side be6. That is this voltage ve6 across this element. So I have specified the direction of the current similarly based on that current direction. I have specified the voltages voltage will also agree exist across each of the branches. Now, after I have drawing the graph I will tell you about 3 matrices.

One matrix is called incidence matrix which first right then I will explain. Three matrices I will define which will be denoted by A ok. First let us do with A matrix. Let us not bring other matrices, now you see that had these nodes a b c and where we will write the KCL expressions ok. Write KCL equations at nodes a b c and o. There are 4 nodes, so I will write down the four KCL equations. So at node A, while writing down the KCL equations I will come to know that node A and Disk addition of this current i1, i4 and i6 will be summed up, will be equal to 0.

Now, whenever current will be away from the particular node, I will assign Plus sign. So I will write very simple, i1, this is going away from node A plus i4, plus ie6 this must be equal to 0 this is the KCL at node A. At node A and so sorry A family at node b, KCL come to node b, KCL current which are leaving this junctions are assigned positive always maintain that that is very important. So I will write ie3, because ie3 is coming out another 2 currents are coming on to the junctions. So –ie1 and -ie2 that will be also 0. I am applying KCL rules at a b and c nodes.

And at node c, at node c, I will be writing the KCL and I note that ie5 is coming out from the node because its direction is this, so I will say this is ie5 -ie3 and minus ie6, that is equal to 0. Then at node A, at node o, so if you want to write down the case, ie2 is coming out from the node. So ie2 coming out from the node and i1 and i5 are going into the node so minus ie2 and -i5 and this is equal to 0. Now the last equations at KCL at node at O, is known. Equation, no new equation this must be noted.

That is this equation can be generated can be obtained by suitably adding subtracting this three equations. For example, if you hear only what are the things present at node, ie1 this all are ie2, it is away from the node. That is i1, what I am telling in, from this three equation, if you eliminate 2, 3 and 4 and written only 2, 1 and 5, you will get this equation. This new equation can be obtained from the other three equations. So these 3 equations are the basic equation, independent equations. This 3 are independent equations, Independent equations.

The 4th equation can be derived from this 3. How? There are equations 2 1 and 5 is there? So, do something in these three equations, eliminate 4, eliminate 6 and eliminate 3 from these three equations and you will get back this. Anyway, so this is KCL at these 3 nodes. Understood? What I will be doing now is this let me copy this because this will be open necessary. So this diagram is important. Always I will refer to this diagram copy. So, let me go to new page and paste it there. Ok,

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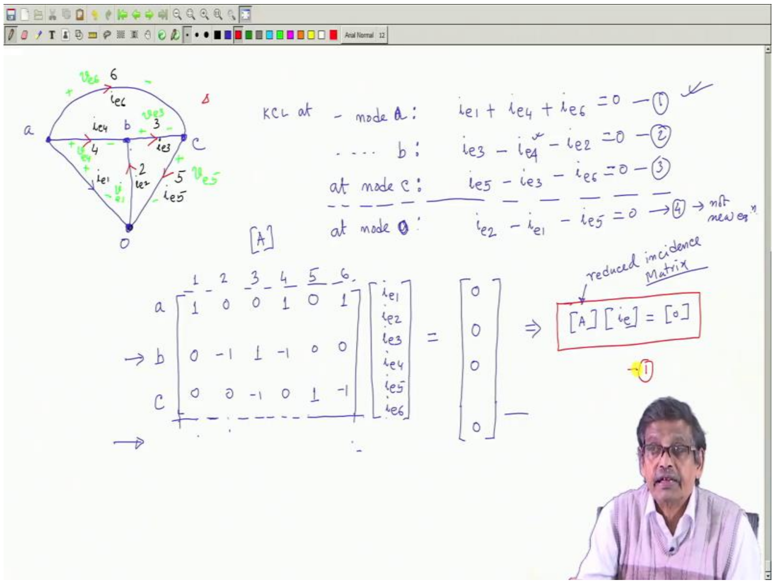

So this is the network with respect to this let us stop, ok. So, what I am telling is this. So, I node KCL at node A so at node A, KCL at this can be also copied. This 3 let me copy that why did it this three equations? So these we already got, reproducing it. So, these equations here node a, node b, node c, now, this thing can be written in a nice pattern, in terms of matrix. I presume that you have got some basic idea of matrix operation, linear algebra, of course, simple ideas no complications transpose this and that.

So this equation let me also above 4 equations let me also call it equation 4, although new equation not new. These 3 equations can be written as in a metric form like this. In terms of the element currents ie1, ie2, there are 6 elements current ie3,ie4, ie5 and ie6. This is at node a, KCL this row will be KCL at node b, this will be KCL at node 3, and KCL at node 4 I will also write now KCL at node a, tells you that Element current 1,4 and 6 will only be present and all of them are positive.

So, element 1, 4 and 6 and this will be the KCL at this node that other things will be 0. Got the idea? 1, 4, 6 family KCL at node 2, see this First row and first column if you multiply and if you write it in this way. First row, it will lead you to 0. So, i1, i4, this writing of 1 2 3 4 5 6 helps me very quickly to fill up this elements of this Matrix because it is 1, 4 and 6 positive. So 1 4 6 + 1 then i1 into 10 then, i4 + i4 + i6 is equal to 0 is the first equation. This ie3 node b we have seen it is ie3 is plus so put +1 here at node b the same.

Now, writing at node b then one and two with negative sign: -1 -1 i3, -1 so i3 -1, -1 i2 -1 at node b, at node b, it is i3 - 1 – i2. node b was there so i3 was equal to i3 + -ie2 that is there but why - i1 it will be -ie2 4. There is node i1 was involved in that node b so it is correct. This ie3 - ie4 is node b what is the KCL i3 - i2 - i4 this will be the correct. This will be correct then this entry will be I am re-writing. It is i3 +1. that is ie3 + 1 and -ie4 and -i4.

And this is let me also correct in the previous page if I have written wrongly had not be the KCL will be at node b, KCL will be i3 is going away + - i2 –i2 this is -4, -i2, -i2 this is 4, got the point this is it. So at node c, the KCL is i5 is plus so put a plus sign here, and 3 and 6 are minus, so - 1 and -1 others are 0, clear. And then the last row, last row if you write it will be what at node o if you write the KCL what was the direction of o, at node o, if you write down the KCL, it will be equal to ie2, going away and 1 and 5 is coming in. ie - ie1 -ie5 equal to 0.

So we put that in this last row that will be equal to so, this will be equal to node a, ie2 is +1, -ie1, -ie5 others are 0. Now I put a horizontal line here. Now, this matrix Ok if you look at we are going to call it some matrix A. And these are all 0s, 3 and 4 element is also right. No look one interesting thing. These entries of this Matrix will be only + 1 and -1 no other numbers that is one thing.

And if you see each column sum of the elements will add up to this. That is 1 - 1 in this column - 1 + 1, 1 – 1 is 0, 1 – 1 is 0, 1 – 1 is 0, 10 - 10 so adding up all the entries in a particular column we will add up to 0. That is why I am telling that this row is redundant. If you consider this three rows, the 4th row is known because the entries will be 0 then, I will say there must be a – 1. In this case, you see, in this first three elements add up to 0, 4th row can be generated using the results of the first three rows.

That is why it is not an independent equation a, b, c is an independent equation. One may also tell that OK I will treat a b and o to be independent equations. Then I will say you, c can be generated so but you will follow this. That is will write down the nodes and at this node, the KCL equation has already been distinct to be with this because of earlier KCL equation. So, new information now this equation can be written as a as a matrix into all element current participate and that must be equal to 0.

However, we will now point in having 4 rows because this row is known. So, we will always think that this is this thing, this is my a matrix. Sometimes it is called reduced ordered A matrix in this reduced order A matrix I will be using because 4th row of this Matrix was already know so reduced order. Reduced incidence Matrix it is called. So, this is one fundamental equation what is essentially, it is nothing but the KCL equations at day 3 nodes.

And at the difference node have not written the KCL l exist their KCL is valid all these things but it is sufficient to know the pin for because that 4th equation is can be extrapolated whenever I buy manipulating this 3 equations. So, this is the reduced incidence Matrix and A into i, what is i, i is of the branch currents i1, i2, i3, i4 etcetera. So, A times the branch currents or element current will always give you so this is equation. So, I stop here and we will see properties of A. Thank you.