Network Analysis
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Lecture – 60
Graph Theory Applied to Network Analysis - II

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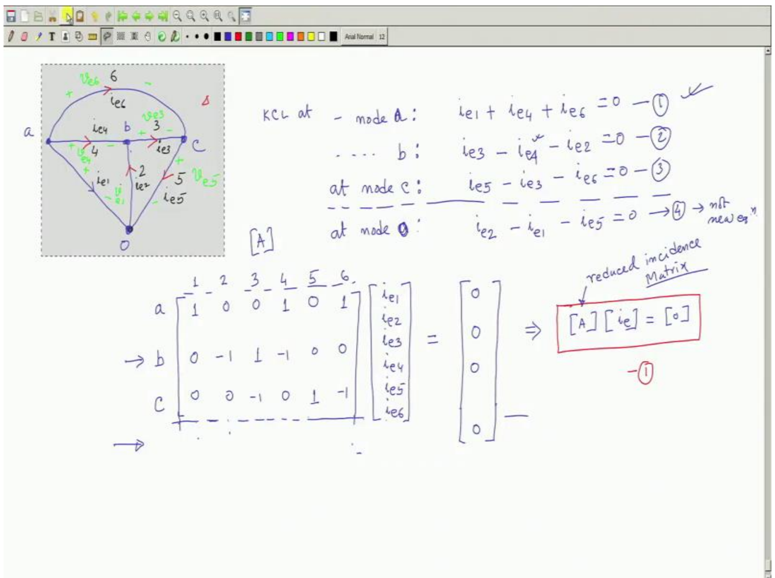

Welcome to lecture number 60 lecture and we are trying to apply the graphical method in solving a circuit.

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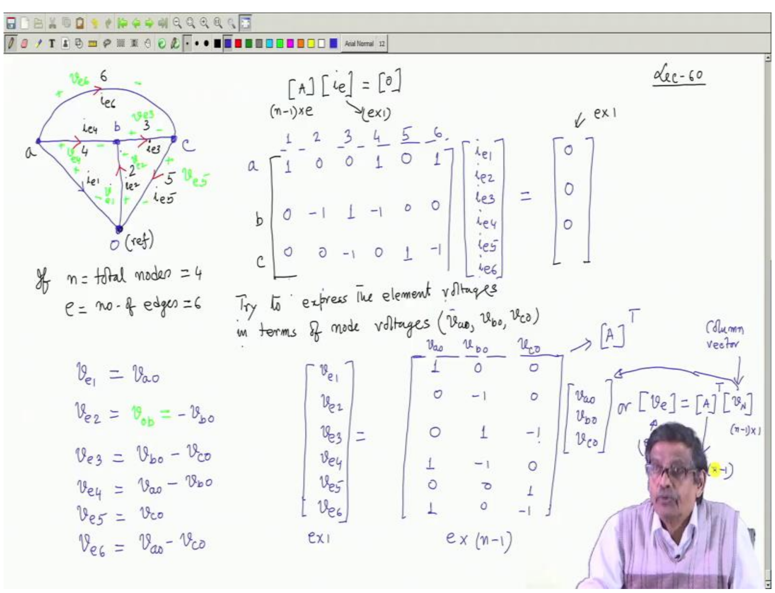

So, this was a graph of the network and we solve the circuit here for this circuit and these are the 3, 4 nodes present ABCO and we got last time we defined what the incidence matrix is. It may you should not be under the impression why we are defining the a matrix like that it turns out to be sub matrix multiplied by the currents in various branches is essentially Kirchhoff's law telling in that way. And its usefulness will be clear after some time.

So, what we got is this thing and I will say now after knowing the result that is a into that is A into ie is the column vector and this is equal to 0 and this is reduced order that is that O I am not writing the Kirchhoff’s law. How to got that result? We got it we got this result that is in this result we got this is the result we got. Where let me cleanly write it now, reduced order A matrix as this, this will be your a these are the entries these 1 2 3 4 5 6, KCl at any node there who will participate?

Expected to be participated by any element currents so there may be 6 element currents i1 i2 i3 and I fill up this it is very easily seen. So, this is the one thing now what I will be doing generally KCl we write while solving the networks by nodal analysis, we know we want to solve the network by applying nodal analysis, we write down the KCl at different nodes. Now let us see what is the size of this a matrix.

This a matrix what will be the size? You recall that e is the number of elements here how many rows are there here this is a, b, c. so, how many rows it will have? If there are 4 nodes then the number of equations I am writing reference node I am not writing KCL here. So, it will be one less, so if n equal to total number of nodes total nodes if that number is n in this case it is 4. Then KCl equations I will be writing only 3 nodes that is 1 2 3.

So, the number of rows of this matrix in general will be n - 1 into how many columns number of columns will be e, where e is equal to number of edges so it will be an n by n - 1 by e matrix and this matrix that is ie matrix will be of course e by 1 it is a column matrix. On the right hand side you also have a column matrix e by 1 you know this is the thing. (FL: 05:08) now what I will do is this let me try to express the element voltages in terms of node voltages.

Try to express the element voltages, that is V 1 V 2 V 3 etcetera voltages by in terms of node voltages. What are the node voltages? va0, vb0 and vc0, vc0 is the reference node. So, it is very easily it can be seen that ve1, let me write down ve1 this element voltage is how much v1 is this voltage this is Plus this is -is is equal to v0 over, ve2 the polarity you see I will denote it by depending upon this direction of the current I am denoting it it should be like that plus -arrow is like this so ve2.

So ve2 will be v0b which will be equal to - vb0 is that v0b is -vb0 so be it will be negative of -vb0. So, vb0 is another node voltage. Similarly ve3 which is straight forward this is the direction of the current. So, vb0 - vc0 then in node 4 the voltage element voltage existing across that branch v4 it will be equal to va0 - vb0. Then in node 5, e5 it will be simply vc0 you know that's all vc0 you know. And finally ve6, ve6 it will be va0 – vc0. So, we have been able to express the element voltages in terms of the node voltages v1 v2 v3 in terms of va0 vb0 vc0.

Now this whole set of equations 6 equations can also be written in a nice matrix form like this. I will write it like this ve1 ve2 ve3 ve4 ve5 and ve6 I will be able to write it like this. Here who will be participating in this multiplication only 3 voltage participate node voltages. This side I will write va0 vb0 and vc0 and here I will write also va0 vb0 and vc0 you know this is no entry just, so that I will never be making mistakes.

So b1 is equal to va0 only so I will put a 1 here and 00 there, so, ve2 is -vb0 so ve2 is -vb0 is - 1 00 no other I am sorry this can be nicely like this. Similarly voltage across element 3, ve3 which is equal to vb0 - vc0, so vb0 - vc0, ve4 it will be va0 - vb0, so va0 - vb0 - 1 and 0, 5 e5 is only vc0, so only 1 here 00 and finally v6 is va0 - vc0 - vc0. In fact one can straight away right because for the first time you are doing so I wrote these equations and just translated it so it can be written in this way also.

So, this is what I did that is node voltages in terms of node voltages the element voltages can be expressed. Now the question is what is this matrix should I give it a different name now it so happens this matrix is a if you see the this matrix is nothing but A transpose. See first row 1 0 0 or 1 0 1 of A matrix has become the first column of this matrix 1 0 0, 1 0 1. Similarly second row

0 -1 1 -1 0 0 0 -1 1 -1 0 0, third row 0 0 -1 0 0 -1 has become the third column 0 1 -1 so it is no new matrix the relationship between element voltages and node voltages like this ok.

What will be therefore what will be the size so this matrix these equations or can be written in a shorthand fashion as the element voltages is equal to A transpose that is this one into the node voltages. So, what is this vector column vector which is this va0 vb0 vc0 3 entries because there are three nodes. Now what will be the size of this va? First let us see the size of this A transpose matrix. A matrix was n -1 into A so it must be e by n -1.

Because A matrix size was these its transpose so number of elements this is the element voltage e -1. So, this size will be e into 1 column vector this size will be you know n -1 into 1 and A transpose size will be e by n -1. Therefore the incidence matrix which we got from the by writing down KCl equations also appears as A transpose and it connects the element voltages with the node voltages.

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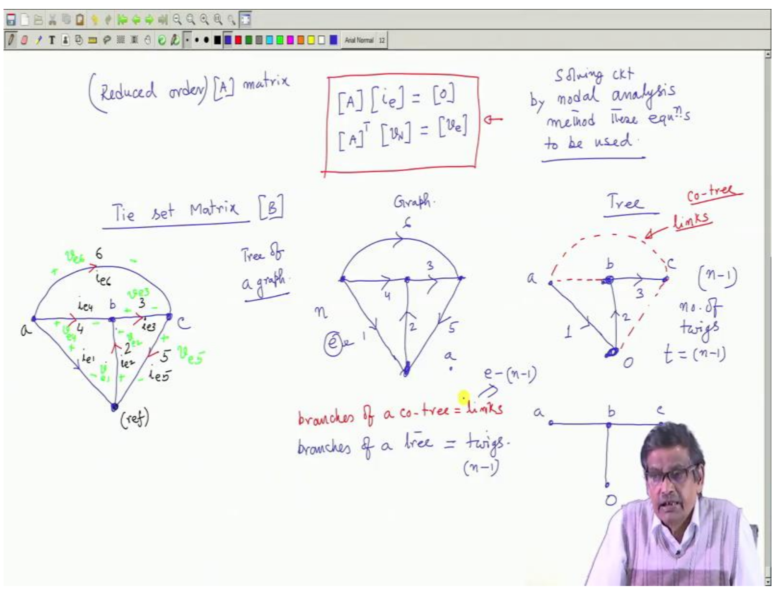

So, we say that reduced order A matrix henceforth I will not even say reduced order a matrix it is understood reduced order what is the use of writing redundant equations. So, we have got the in conclusion it is like this a into ie is equal to 0 square brackets indicates it is a matrix and also we have seen that A transpose into the node voltages a column voltages will give you the element voltages. This is the thing this is where see I have now merely stated defined A matrix and the entries of A matrix are very simple 1 -1 +1 etcetera +1 or -1.

Similarly of A transpose and the fundamental relationship of element voltages and node voltages gave me this one and simple application of KCl gave me this one. So, these are the statements if you draw the graph of a network directed graph and then these 2 are sure these 2 are correct. We will come to this further see I have not yet talked how to solve for currents and voltages. What merely I am telling is that there exists some matrix now A, whose entries are very simple numbers +1 and -1 and they relate important things clear.

So we will now go to define another matrix which is called cut set matrix so I will say this one, cut set not cut set first, let us tell tie set matrix this is also very interesting tie set matrix, what it is? I will tell you. And this will be relevant when you want to solve the circuit by nodal analysis. it is not to write down the KCl somehow I already we know that, if you know the node voltages we have solved the circuit. Therefore node voltage is involved element voltages are involved.

And then the KCl you have to write so by nodal analysis only this point I note down here by nodal analysis method solving circuit by nodal analysis method these equations to be used only make a point to be used. Now tie set matrix is B matrix first the last thing it also I know another method of solving a circuit what it is loop method write down KVL at different loops of a circuit and try to solve for the loop currents and then the element currents.

We know that there for tie set matrix B which I am going to discuss now who will also yield to equations like this. Let us see first of all what this tie set matrix means. Now you know that this always I will be needing, so I will just copy this diagram sorry this, one this I will copy and paste it next page and put it here because same network I have to talk in terms of it. So, this is the thing. So, this was the tree of the network this is how we have assigned the things.

Now given a network we will say that we must now define what do I mean by a tree of a network so these are the keywords before I really start car set matrix. So, now the this was the network graph of the network there were four nodes I do not want to put all these variable names here to understand what a tree is of a graph so this is the graph of the network and it was directed graph so you can put the arrows like this the way it has been assumed in the original circuit this was the directions and these were the name of the element simply write 1 2 3 4 and where is 5, 5 and 6.

This was the group of the network now note what a tree of this graph is tree of a graph we are discussing now tree of a graph. First draw the nodes like this then connect the nodes connect the nodes suppose like this. So, all the branches I have not joined same nodes are there abc and this is all now this will be called a tree of the graph, tree of a graph is a structure like this where all the nodes will be present and there will be no closed path that that is it.

For example this is I can call tree of the network because all the nodes I have shown something is connected to each of the nodes and no loop has been formed like this. Obviously tree of a network is not a unique configuration like this it is also a tree. If I have defined the tree like that I will say this is also a tree because all the nodes are present abc and O but no closed-form has been put. For example one should not say that this is a tree of a network.

All nodes are present this is closed this is this is not a tree no not a tree not a; there should not be any closed path present which is a subset of this elements some elements you choose such that all the nodes are present no node is left alone that is this is also not a true tree somebody says draws like this and say that he draws like this says that no closed loop is there should I call it a tree I will say no, because you have shown all the nodes but this node is left out on its own.

It cannot be also a tree so tree is a subset of the elements present in a graph in such a way that all the nodes will be there nodes will be connected and there should not be any closed loop formed that is it. I hope you have got the idea of a nodes of a tree, so this is the valid tree this is also a valid let what I will be doing is we will first take any tree you draw whatever tree is possible there are a number of trees possible independent.

For example this is also a tree let me draw this is also a tree this, this and this, this is also a tree you know no closed loop is there all the nodes are present. Choose any one of them you like so I have chosen this to be the tree for ongoing discussions now about this what should I do with this tree. And since it is a directed graph also shows its direction and the element name whatever you have given try to maintain that so this is 1 this is element 2 this is element 3.

Now the question is how many elements will be present in a tree? If total number of edges easy here how many elements will be present in a tree. Number of elements which will be present in a tree if the number of nodes are n is n -1, you see it there are 4 nodes 3 branches will be present. Now when it comes to tree it consists of also branches or edges these edges are given a separate name called twigs. Number of tweaks is equal to n -1 in this case it will be 3 got the idea.

And it is denoted by this letter t, t is not time number of two tweaks present in this one. Now the left out branches which are not shown in a tweaks I am showing them by some dotted line with red colors. These are the edges I have omitted to construct this tree of this network these are the another 3 where there, then so these are the 6 total number of branches were 6 so these 3 red lines I have shown with dotted things these are called links.

So, this is tree and this red one is called code tree. So, these are some definitions with which you will be very much familiar soon after you practice a little bit. So, graph was a full thing directed graph all the elements you are showing now I am telling you tree is a collection of these edges showing all the nodes there in such a way that they should not form a loop that is the blue one only. How many elements will be there in a tree it will be n -1 over.

The left out branches to construct the tree is this red lines this I will call co-tree I will call it ho tree. And the branches of a code tree are called links so I write it branches of a co-tree are called links then branches of a tree or edges of a tree are called tweaks. How many tweaks will be there n -1 how many links will be there o total number of phases is e therefore subtract e, n - 1 from e that will also tell you how many edges will be there.

Or so these are the statement of this in the full graph I will say branches or edges total number of edges I know what are the things I know total number of nodes I know then I constructed tree the elements or edges which will form a tree they are given a name tweaks number of tweaks will be n -1 and the left out branches which I have not eliminated discarded while constructing the tree are shown by this dotted red lines these structure is called a co-tree this red portion only.

With this blue arrest that is called a co-tree code 3 + 3 is your original graph if you make all things form line and the question is how many links will be there? Number of links those branches of a co-tree I will call links and number of links will be e -n -1 we will continue with this in the next class. Please this is the interesting topic please go through it very logical and very interesting too, thank you.