Network Analysis
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Lecture – 61
Graph Theory Applied to Network Analysis - III

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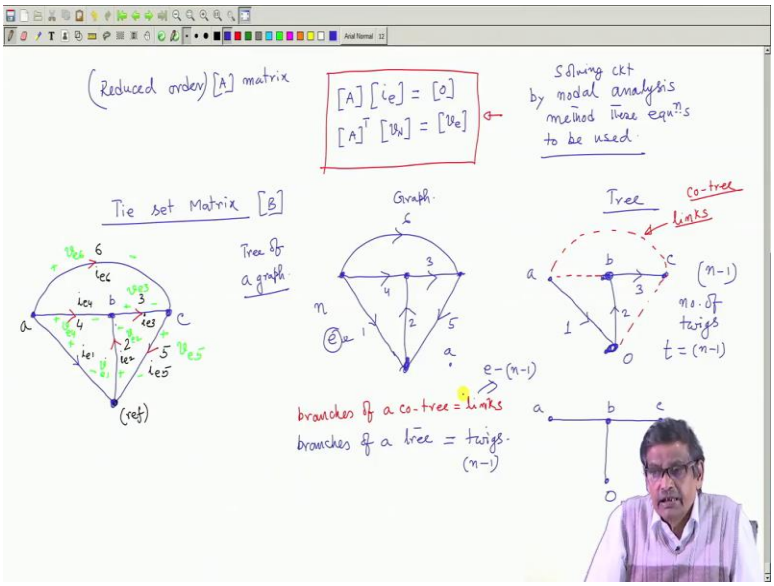

So, we were discussing about the graphs and tree of a network so this graph we have discussed directed graph it is a full graph. Then tree is a is a reduced form of this graph in which all the nodes will be present, all the nodes will be connected but there should not be do not take any elements in the to form a tree so that it forms a loop, so this is a valid tree because I omitted the element 4. If I connect element 4 then only it will be a closed path generating and so on.

So this is fine now what is done is this these are called tweaks and the red ones are called the links cleare. So, what are the links here? Element 4 element 5 and element 6 these are missing. So, what we will be doing is we will be showing those links by red lines this, this and this, these are links. So, this is the blue one is the tree and red one is dotted red is co-tree tree means tweaks. Those elements will be called tweaks associated with t co-tree elements are links.

How many links are there let me also show you show it here that is in this link this is 4 this is 5 this is 6. Now what we plan to do is this after I have drawn the links we will be writing down the

KVL equation of the loops which will be formed when you put 1 links at a time and then write down the KVL equation there. So, this method of using co-tree 3 concepts to analyze the circuit will be basically the loop analysis method.

Loop analysis method, loop analysis you in terms of suppose I did not know about graph theory I know what does it mean. I will take several different loops write down the KVL equations and try to solve the circuit. In fact Maze analysis is a form of loop analysis only. Ok so 4 5 6 are the links so there are 3 links which I will call L4 L5 and L6. Now suppose I say that I want to write down if L4 was in place here then what will be the KVL equation in this loop.

KVL equation in this loop so write KVL, KVL equations for A matrix we do not KCl equation at the nodes for in this case I will write KVL equations thinking I am putting one link at a time. For example 4 2 and 1 will be a closed loop forget about these 3 business come here in the actual circuit. This will be 1 loop, now in this loop the some of the here I am not showing it is true for all the branches. This is the direction of the current I am not writing it is ie4 we have already seen that all the numbers writing is sufficient to understand this.

So, when I say ve 4 it is with this voltage across this element you know these things I will not write henceforth it is understood. Now in this loop which is formed by this link L 4 and these 2 edges this one in this circuit it will be KVL equation will be e 4 this you follow me carefully e 4 give a positive sign when you are putting along the arrow that is the like we will be 4 so e 4 plus then -e2 then -e1 if you traverse this loop.

So the positive sign to e4 whatever direction is there e4 then e4 if you have assigned positive voltage then e2 is in opposition, so it will be -e2 -v1 and this must be equal to 0. KVL equation formed by link 4 and these 2 trees. Similarly I will write down KVL equation where L5 link 5 will be involved that is this if I put link 5 this is the loop this is the loop and what will be the KVL equation in this loop start from this first go along the link here that is e5 what about t minus this must be also plus because it is dc this direction you have a sign plus.

So, plus plus then also plus plus e3 is equal to 0 KVL equation is to be satisfied and finally in loop in third loop which I have called L6 that is this one we what is that loop now here you be careful what I mean this loop which will be formed by this link alone will be like this start from a L6 then these things this that is the; in the loop that you are writing KVL there should be only one link present, why? Because while writing down this equation KVL equation I took only one link L4 and other tools are the tweaks.

So, one link to 2 tweaks formed loop one L5 in this 1 link and 2 tweaks 2 and 3 this is the second loop and finally the loop which will be formed will have only 1 link so this 6 I will take then I cannot form this loop like this okay we will be satisfied but I will always follow this rule I will take one link at a time to form a loop. So, I go by this then this which is not a link come down here not a link this is also not a link this is tweak.

And you are correct loop formation these 3 loops will be independent groups as you can see so KVL equation in L6 I will write, how I am writing it this one going in this direction it will be e6 that I will assign positive sign. Then e3 will come do not take e5, e3 so – e3 - e2 because you opposite to that direction of 6. Then from this to this –e1 and this must be equal to 0. So, this is the KVL equation in 3 well-defined loops.

How this looks have been formed I have got the ideas of trees I know the links then I will take one link at a time form a loop like this that is the whole idea. If necessary I will repeat once again but let me proceed. Now once I get this these three equations can be written in a matrix form nicely, what is that? That is in matrix laws this equation will be written like this there will be all these 6 voltages of the elements will parties may participate in this equations.

So here it will be 1 2 3 4 5 and 6 here I will write v1 v2 v3 element voltages v4 v5 and v6. And here it is loop for 3 loops are there L4 does not L6 does not mean I am considering 6 loops I have just identified name the loops in terms of the link number that is all. So, it is 1 loop L4 formed by link 4 another link 4 5 6, so I will now put so this is outside this matrix it is for correctly writing down correctly and quickly writing down the KVL equations.

So for example in loop for it is e4 is present. So, +1 -e2 -e1 -e2 -e1 other entries are 0 and this will be equal to 0 that is all in loop in this loop number 5 it will be 2 3 and 5 involved so 2 3 and 5 all other entries are 0. And in loop number 6 this is loop number 6 mine do in loop number six it is a 6 is present so +1 -e3 -e3 -1 and that – e2 which is also -1 and -e1 hopefully I have written correctly this is the thing. And this all this row has to be 0.

So the KVL equations in these three fundamental loops they are called once you have chosen a tree then the links you know and involving links one link only write down the KVL equation in the original network. Because we have identified the links so I will call this is the fundamental loop. This I will L6 only one link should be present then I will say this is loop 5 and similarly loop 6 and so on.

So, this is the equation okay - 1 - 1 0 1 0 0 0 1 1 1 like that here also the entries are only +1 and -1, now this equation this matrix is called a matrix B and which we call it as a cut set matrix and this is the element column voltages representing voltages across each element and this is equal to 0. We got similar things in incidence matrix a into ie is equal to 0 castle and b into vb equal to 0 where from we got it from KVL, KVL in which loops I wrote the loops which is formed by at least one and only one link and links have been identified after drawing the tree of these graph that is the thing so this is b matrix understood.

Now after you get these KVL equations we will try to see how the link currents will be related with element currents. The next question I asked because it is loop analysis I will be doing essentially by drawing a tree etc at the end we will show that but in loop analysis what you need you have to write down the KVL equations. And you try to solve for the loop currents is not and if loop currents are known then all the branch currents can be calculated may be some of the loop currents difference of the loop currents and so on.

So, next thing is how to express element currents in terms of loop currents that will be the study. So, the question is so these diagrams I will be once again reading so I will copy this diagram. It is like this so I copied go to next page and paste it.

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So, this diagram is very much needed, now only thing it was not copied is a we see other things so this is the tree or 3 etcetera. Now what I am trying to do is this expressing this step is most important and you must understand this expressing element currents in terms of loop currents but the point now where is the loop currents loop currencies one loop I as I told you will be like this involving this link that is this link if you put only a loop will be formed so this is one loop current.

This is iL4 you know this will be the loop current what is the next loop current so in this diagram if I show it will be like this let me use different color so this is iL4 in this loop iL4 where will be iL5 only this link will be this one so this will be this iL5 this will be this now what is this loop current formed by this link L6 it will be like this I will use a different color so iL6 will go this way this way this way come back here iL6 this you must understand it very clearly.

So loops has been so here also if I show it is iL5 I have assumed it to be this iL5 and iL6 is this one, this is iL6 then you can easily see how to write down the element currents in terms of loop currents. So, how do I write it down element currents ie1 which one is one branch ie1 is a branch 1 what will be there are 2 loop currents flowing here okay, first let me okay i1, i1 in terms of i1 direction is this way you know.

So, there are two loop currents involved in deciding this magnitude this arrow let me put very clearly. This arrow will be dictated by the arrow of the links so this is how I have put it this arrow of iL4 was dictated by the arrow of this branch ab understood. Similarly this arrow was dictated by this link so this is how it was put. Therefore ie1 as you can see will be -iL4 -iL6 that is all ie1 will be -il 4 -iL6, ie2 where is e 2, e 2 is here ie2 in ie2 if you see what will be the loop currents involved its direction is like this.

What are the loop currents that are present in this branch 2 or H 2 or element 2 it is iL5 this is the loop current. So, iL5 it will be plus because this way it goes then also iL4 and iL6 in the opposite direction so - iL4 - iL6 this will be element currents too in, so this is the this thing then ie3 in terms of loop current ie3 is this way and here if these are the loop currents then ie3 in terms of

that loop current will be IL5 + no, iL5 this is iL5 it is a loop current iL5, ie5 I am writing which is this way so iL5 and then - iL6 this is 6 IL 6 - iL6 this will be that is all only these 2 loop currents are present in this; to decide the branch current here.

Ie4 means this branch current, in this branch current iL6 is not involved only iL4 is involved neither 6 not the other branch currents 5 is involved only iL4 is iL 4 that is all then got the point this current is only IL 4 then I if I will be equal to that is the current in the element 5 all the iL5 is involved no other loop currents are going this way. So, this will be equal to iL5 and finally ie6 will be equal to iL6. So, once again repeating very quickly so in to find out the element current in one this current i1 what I do is I see what are the loop currents that are crossing this present in this branch as well.

I find 2 loop currents are present iL4 and iL4 but they are in opposite direction of ie1 so, -iL4 iL6, ie2 is this current whose direction already assumed this way which are the loop currents that will decide ie2 all the 3 are there vertically. So, this is iL5 is flowing like this now it will complete circular. So, here all the three loop currents will be there ie2 of which iL5 will be in the same direction as ie2 therefore it will be iL5 -iL4 -iL6. Similarly ie3 you know this branch current here only 2 loops are crossing ie 3 that is iL5 this is the arrow iL5.

And -iL6 that is all and ie4 i4 i5 i6 there happens to be such that only the respective loop currents are only one loop currents are involved so this is the thing. Now this can be written in the form of a matrix what is that matrix? This matrix will be because I want to express the element currents in terms of loop currents. So, it will be if you write down it in terms of a matrix it will be a given ie1 ie2 ie3 4 5 and 6 these element currents.

It will be equal to a matrix and this will be decided by the loop currents all the 3 loop currents are there they are iLl4 iL5 and iL6 once again telling you iL4 iL5 iL3 that iL1 also exists I just named them depending upon the number of the number that has been associated with that particular links only 3 equations mind you. And this thing here I will write iL4 also iL5 iL6 this is for my own benefit quickly and correctly I will write ie1 is -iL4 so -1 here - iL6 -1 here iL5 no

contribution ie2 iL5 is +1 and 4 and 6 -1 -1 -1 -1 then ie3 L5 and L6, L5 + 1, L6 is -1 and this is 0, ie 4 is only iL4, ie5 only iL5 and finally ie6 is only this one.

So let me put it in the proper perspective. So, this is the matrix what will be the size of the matrix? The size of the matrix will be the number of elements which are present that is number of rows is number of elements so e times how many loops fundamental loops will be present it is equal to the number of links which are present and number of links. And we have seen that number of links is equal to total number of elements minus the number of tweaks this is the thing.

So, this is it will be like that and these of course will be into 6. Now I have already defined the B matrix you see here this was the B matrix okay is there any space for copying these b matrix let me copy that in the previous page. So, that this is the nature of the b matrix copy it go to next page and paste it. So, let me put the B matrix here okay to get the idea let me superimpose out the statement. Now you see this was the B matrix got earlier now look at the elements here the first row was -1 -1 0 1 0 0 the first column is -1 -1 0 1 0 0.

Second row was 0 1 1 0 1 0 0 1 1 1 0 1 0 third row -1 -1 -1 and then 0 0 1. So, this I then can write in terms of matrix it is no new matrix. The element currents ie is nothing but b transpose into the loop currents what is ie? ie is this column vector comprising of all the element current information what is iL? IL is this loop current once again a column vector and what is B transpose, b transpose is that earlier B matrix which I wrote here which here we got from the KVL equations.

Therefore in case of loop analysis we have got these 2 fundamental equation that I am rewriting here one is B into the element voltages is equal to 0 that I got in the last page that is the B into Ve is equal to 0 and this is 1 and the second one is B transpose into the loop currents is equal to the element voltages. So, these two are the equations which we will be using if we want to solve the network by loop analysis method.

Therefore please go through this 3, last 3 lecture notes carefully hopefully you will understand and then I will tell you in the next class there is another way of solving the network which is based on the cut-set method, that I will discuss in the next class. Thank you.