Network Analysis
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Lecture – 62
Graph Theory Applied to Network Analysis – IV

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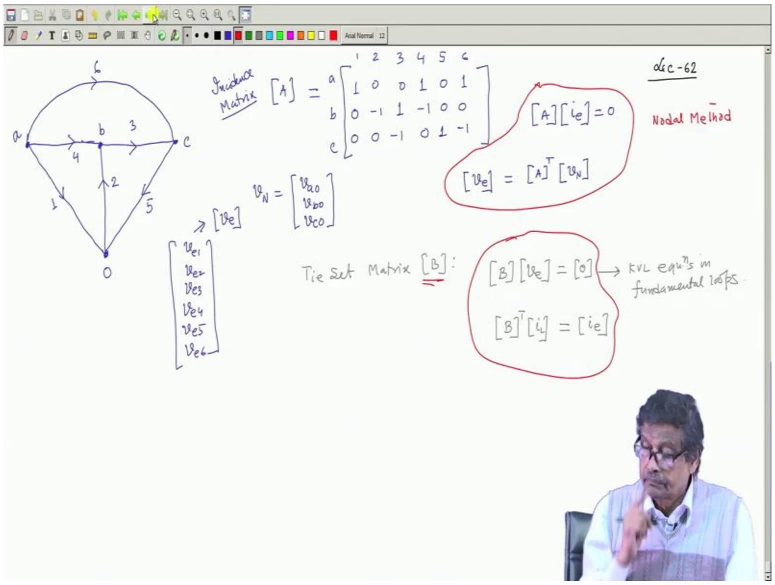

Welcome to lecture number 62 and we have been discussing the network analysis using graph theories and in the last 3 lectures we have defined several important things. One is a matrix A which is called the incidence matrix. Let me very quickly go through it incidence matrix, incidence matrix is can be easily constructed by going to each node and writing down on the +1 or -1 5 6, this. And here for example go to node a node b these are the three doors no day things going out is assigned plus sign.

So 1, 4, 6 are the participating elements while writing down KCL here and it will be 1 4 and 6 all are going out so 1 4 and 6 others are 0. Similarly go to node b things going out will be assigned plus so 3 is plus 2 and 4 coming into Junction B and they will be minus, minus 2 and 4, 4 node 2 be others are 0 this I am showing how quickly you can write this matrix looking at the graph of the network and similarly at node c things coming out is i5.

So if I will be plus 1 3 and 6 are coming in so 3 and 6 are coming in so this is how easily this matrix and this is the reduced order incidence matrix we are not writing down KCL at O because that is not going to give you any new equation and also we discussed about the size of the matrix. But at the end what was the implication of this if the branch currents which I am not showing the arrows means the branch currents in this direction.

And also the voltage of these branches before with this side plus this side minus inconsistent with the direction of the current assumed therefore we found that a into ie is the Kirchhoff's current law equation that is equal to 0, this was the finding with the incidence matrix. Similarly we tried to express the element voltages in terms of the node voltages so we defined a node voltages which is a column vector va0 vb0 vc0.

And we found that the element voltages that is v 1 v 2 v 3 a column vector v 4 v 5 v 6 this vector which I was calling it as matrix e and we also note that this a transpose element voltages are there this is the thing. And then we have to find out the element voltage like the this column voltage who are shown to be equal to A transpose into the node voltage that is what we have seen last time. So, these are the two equations which involves the A or the incidence matrix we found out that okay.

Similarly then we defined what is a tree of the network what is co-tree of a network what is tweaks, what are links and so on. Then we found out in our last class which I will just quickly go through this one no point in repeating. And we first draw this is the tree we do this is the graph full graph then from that we just chose one tree like this that is the form blue line. And then we with the dotted red lines are the links and then for each link if you put one link then a closed circuit will be formed.

And we without wrote down the KVL equations in these 3 fundamental loops what is the fundamental loop? Fundamental loop is one where only one link will be present. For example this is a fundamental loop, is this a fundamental loop? No two links are present although KVL will be satisfied there but we look for the fundamental loops which where there will be only one link present and it will form a closed circuit.

So, like this one it is a fundamental loop similarly this is also a fundamental loop and this one go like this, this, this, this also a fundamental loop while because there is only one link 6 present. Then we told you that the first equation is this b into v is equal to 0 which is nothing but the KVL equations if you write it here KVL equations in fundamental loops being to be is the KVL equation in the fundamental loop in fact this I did here.

For example in the fundamental loop say look for this is naming is not the fundamental loops are 6 in numbers L4 is one fundamental loop where link 4 is only present. L5 is another fundamental loop where only link 5 will be involved and so on. And then I told you that while writing down KVL equation in these fundamental loops you start your journey first write down the voltage of fundament of the link here and that whichever direction it is you call it e4, so e4 and then -e2 -e1 equal to 0 etcetera. Similarly for 5 it will be this e5 view a sine plus then this is also a plus and this is also plus.

Like that the third one is e6 - e3 - e2 - e1 so this matrix was b into v is equal to 0 that is what I was telling. So, the equation involving this one incidence matrix is over then we that is what we call the tie set matrix. So, this is incidence matrix tie set matrix. So, tie set matrix was defined as B and for that we have to form the tree and etcetera and then these two equations were obtained B into ve is equal to 0 this is the KVL equation in fundamental loops.

And also the link currents was B transpose into you know to relate the link currents and the element currents. So, B transpose into iL B transpose into iL is equal to ie these are the another two important equations these two which involves tie set matrix B. What is this e, you should be only careful once we have decided about the fundamental loops assign the loop currents for example this is the fundamental loop this is iL5 okay.

What is the direction of iL5 chosen along the direction of the link current because it is a directed graph I know this direction it will be like this. Similarly in this loop this link current is iL6 loop current and for this loop current is like this. So, we applied Kirchhoff’s law and found out those relations from this we got this particular relation. Generally we will show you that this particular A matrix will be utilized these two equations while solving circuit by nodal method.

Why because in node methods we find out the right down the KCL at different nodes that is this one and then we try to find out the node voltages is not which is vn this vector va0, so this equation will be necessary. Similarly this method will be utilized this tie set matrix to solve the network problems by loop equations so I have to write down the KVL equation VB equal to 0 and then I have to find out the loop currents using this one I must relate loop currents with the element currents. And about the dimensions I have already told you.

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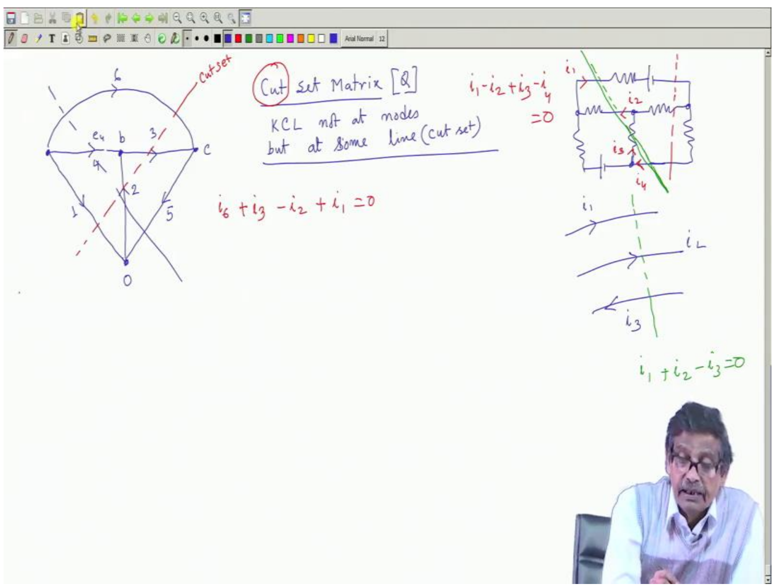

Now today what I am going to tell you is another matrix that is called a cut set matrix these also very interesting cut set matrix which will be denoted by Q we defined Awe defined B now another thing is called cut set matrix Q. Now these equations perhaps will be totally new to you other earlier to things where only we involved the KVL KCL etcetera. From that here also similar things I will do but the idea is slightly different.

It is once again writing down KCL only but KCL not at nodes, where the KCL saturated to get the idea clear we know that suppose here I do some report which I will it is suppose you have a network like this, there are resistances may be voltage sources try to get a feel of what I am trying to do here is a resistor said is some voltage what not and there is also some it is a network okay with these are the nodes.

Now in this network we always apply KCL at the nodes that is fine no problem but it is once again KCL in a larger perspective what I mean by this for example if I say there were some wire going like this which is carrying a current i1 there is another wire carrying a current i2 and there is another wire carrying a current i3 these are part of some Network element currents in some 3 branches it is like that. Now this point is important suppose you imagine there is a line here there is a line here.

There is no junctions i2, i1 whether there many junctions or not I am not sure but if you can imagine a line here, then the current crossing this line will sum up to 0 once again what do I mean by this that is current crossing this imaginary line from left to right is i1. This branch only these 3 branches are involved why this line suppose and then I will say this is -i2 no +i2 current growing from left to right I have a sign positive sign.

So i1 + i2 and then -i3 this will be also 0 although so so current crossing a line will also those current summed up with due rigger to their sign left to right or right to left whatever it is that will also add up to 0, is there a junction here no, not a junction but still this is called KCL in a broader perspective what I what is the implication of this implication is very clear. Suppose you say that in this network you draw a line like.

This divide these networks into 2 halves by this green line suppose you have set up then I will say that if you have solved this point of listen carefully suppose after solving the network you find this is i1 this is suppose i2 and this is suppose i3 and this is suppose i4, so, I have divided the networks into 2 halves as if by drawing a line you can cut the network into 2 halves cut, then I will say that in this line several branches intersect this branch this branch this branch and also this branch.

Suppose we have solved the networks and the currents were found out to be this is i1 this is i2 this is the i3 this is i4 or then what I am telling that suppose left to right I am telling positive. So, i1 this line from left to right i1 but this is i2 right to left i2 then this is also crossing this surface from top to bottom so in the same sense as that of i1, so +i3 and this one is crossing this from this side to this side because this is the arrow output.

So that will be -i4 and that will be equal to i2 has to be 0 that is the interesting part of it. Therefore if you imagine an imaginary line dividing the whole network into two parts you should not draw a line like this and say here also KCL will be satisfied suppose I have not extended this line only I stopped here that is this current and this current will sum up to 0 know. We have to take the line so that it divides the full network into 2 halves. Partly you should not take it ok it is expected to be because any line whatever current goes in must somehow go in from left to right, right or from this side to that side same amount of current must cross the other way around ok.

Otherwise there will be accumulation of charges and so on. So, this is the basic idea of this cut set matrix writing equation. It is also basically involves KCL not at nodes not at nodes but at alum but as line which we call cuts cut set or sort of thing like this. For example in this network this is the network its graph is like this if somebody draws divides the network in this way, draw a cut set this is called a cut set that line, that is it.

Then I am sure about one thing this you try to follow me i6 from left to right it is crossing this cut set suppose this see you assign plus sign then i3 these are the 3 1 2 3 4 elements are involved here it will divide the network into 2 halves so, i6 then plus i3 this way then should I write + i2 or -i2, -i2 why? Because it is crossing this line in the opposite signs as 6 and 3 did. So, - i2 and what about i1 this is the current so +i1 this must be also 0.

See I have not applied KCL at the Junction's because i6 i3 i2 i1 they are connected to different, different nodes but the fact is this has to happen at the point this is the basic idea of what we are going to discuss about the cut set matrix ok. So, any cut set the if this is the another cut set one can draw a cut set here. Then the currents involved will be i6 i6 i4 i1 i2 and i3 I can write down the KCL equation. Suppose you draw a cut-set like this then I will say that i6 suppose I assign plus sign there i6 + i4 + i2, i2 and –i5 is equal to 0 I think you have got the idea okay.

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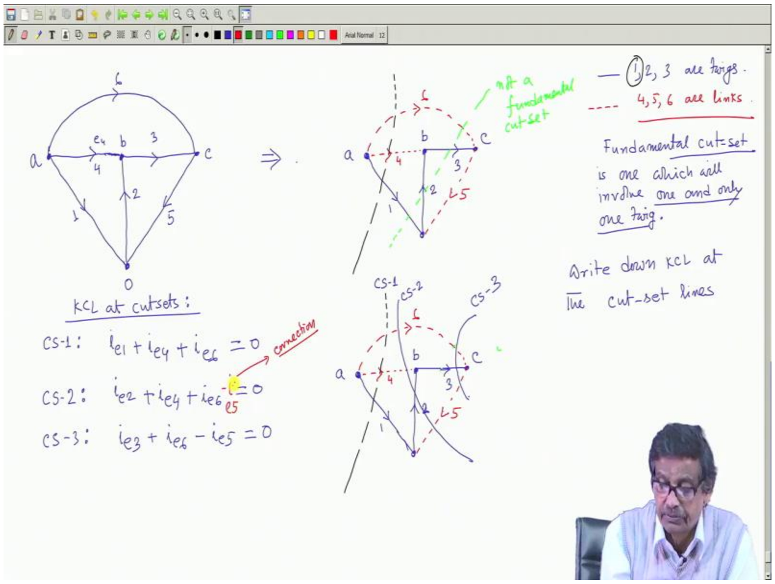

Now with this background I will now go to next page and hopefully that is the figure reading. So, suppose this point was a this was the network. Now the fundamental cut sets are those cut sets where there will be only one tree element involved. First of all what you have to do is this you have to draw a tree of this network. So, let us draw and the same tree that I took earlier I will take that it is a tree because all nodes are taken into account and there is no closed-loop existing.

And this was one this was 2 and this was 3 and where are they so these are what tweaks, so one element 1 2 3 are tweaks and the this dotted lines are dealings. So, this is blue line tweaks and the dotted lines are element 4 with this arrow this 6 this 5 so 4 5 6 are links that we already know now and these are the nodes ok. Now what is a cut-set that is you draw a line such that the network will be divided into 2 portions where on the other side there must be at least one node.

You can easily understand so you cut this network by any way this way that way we will see that but the rule is the fundamental cut set we call it fundamental cut side cut-set are those lines which will break the networks into 2 parts where there will be only one and one link present only one and one tweak will be present I am sorry so I will divide this network into 2 halves so that there will be only one tweak is present.

For example I will say that I will divide this by a line like this take a scissor think I have divided this line. Is it a fundamental cut set yes it is because it divides the network into two halves okay fine there is one node that is present on the left-hand side those things are there. And not only that this is this cut when you cut it, it will only involve one tweak, tweak number 1 will be involved. Suppose I take a cut set which will divide the network in this way it will also be a cut set no doubt.

But will it be a fundamental cut set the answer is no why because although it divides the networks into two halves but only one tweak is present is not satisfied. There are all the 3 tweaks are involved while considering these. So. this is not a fundamental cut side not a fundamental cut set. Therefore I will say fundamental cut set is one is one which will involve one and only one tweak, one and only one tweak. So, this is the fundamental cut set got the point.

Therefore so let me redraw this, this one just copy and then paste so here in this diagram I will only draw the fundamental cut set. How many fundamental cut set will be there first of all? How many tweaks are there? So, that is the thing so I will get erase is this this thing because after all this is not a fundamental cut set forget about fundamental cut set. Then you say that so this is fundamental cut set and I will call it cut set 1 I will give name cut set 1.

How to draw the other cut set this is this was 2, can I draw a cut set involving only tweak 2? Yes this is the cut set and this cut set I will call cs2 whichever tweak will be involved by that I will call that cut set so cut set 1 what means it will involve doing number one only others will be links. Similarly here is a cut set to which will be like this. And there will be a cut set here I can draw a car set car set sure what name should I give this tweak three is involved, so cut set 3 okay.

So, these are the 3 cut-sets fundamental concepts possible then what we will do is this we will write down the KCL at these 3 nodes involving the at the cut sets. So, writing KCL at cut set will be our next job. So, write down write down KCL at the cut set lines which is very easy to draw. Come to these cut set 1 KCL at cut set 1 it will be cs1 only do not think about cs2, cs3 only cs1 is there then you see only one tweak that is tweak number 1 is involved, so this current direction you assign plus sign so i1 i1 from this side to this side.

Similarly i4 is from this side to this side so +ie4 +ie4, similarly ie6 also crosses from left to right so +ie6 that is all this is the 0 go to cut set 2 it is yes cut set 2 here I will apply once again KCL at this cut set at this line not at the node. What it will be which tweak is present in this cut set only 2 that is to be 1 tweak 2. So, from bottom to top is the positive direction so ie2 that you first write ie2 that is from this side to this side i2 is flowing.

Similarly ie4 is in same way it is crossing the cut side 2 and similarly ie6 this must be equal to 0. Finally KCl I am writing at cut sets, cs3 cut set 3 what will be the KCL so here is the line, so 3 is crossing 3 is the tweak which is involved here so that will decide the plus minus thing. So from left to right it is crossing I will say ie3 what about ie6 it is also from this side to this side as ie3 did. So, that will be also +ie6 +ie6 and this one is the other way it is crossing ie5.

So, -ie5 and that will be equal to 0 got the point so this is the KCL at the cut set. So, there was a correction here because I missed this term okay ie5 while writing down the cut set to KCL equations see cut set 2 is this one which currents are involved i6 i4 i2 as well as i5 so that I missed, so it will be –i5. Please note that which I have put it with red ink here, okay. Thank you