Network Analysis
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Lecture – 62
Graph Theory Applied to Network Analysis – V

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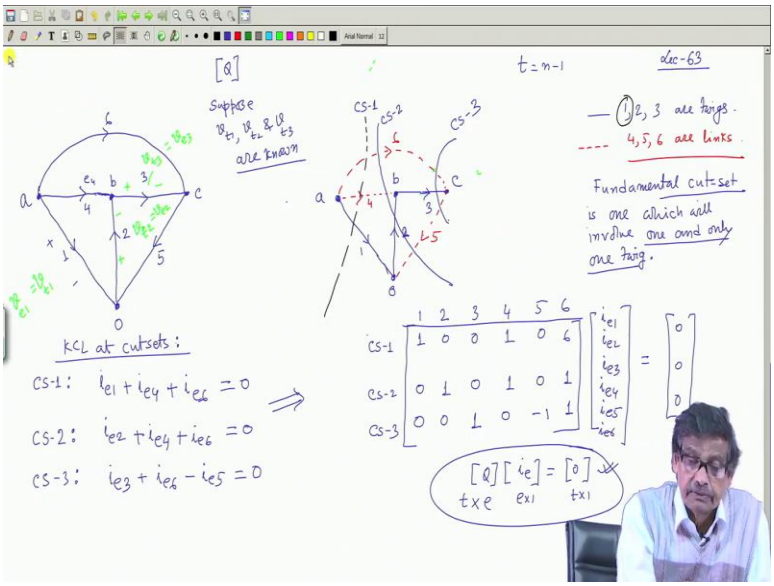

So, we are discussing about the cut set matrix Q that is what we were discussing and before that I must know what a cut set is. So, in my last lecture this was the thing we discussed predicted tree was gruff was given to me I selected 1 3 tree elements are called tweaks there are 3 tweaks which have been which have been named as 1 2 and 3. So, there will be 3 cut set equations what are essentially those equations these are KCL applied not to the node but to the cut side.

So, cut set 1 it is named as cut set 1 because only this tweak is involved 1. So, you write down KCL at this. Similarly this cut set is called cut set 2 it divides the network into 2 halves and apply KCL there but make sure that only 1 cut set is present so this is fine fundamental cut set. Then finally this cut set then do you get this relationship. Now this relationship can be written in terms of a matrix, how? This 1 from this to this in a matrix form I can write it as matrix who will participate in this matrices.

How the element current, so, there are 6 element currents 1 2 3 4 5 6 that is there. So, this column vectors will be i1 i2 that is ie3 ie4 ie5 and ie6 and then I will write here this side cut set 1 cet set 2 that is KCL at cut set 1 KCL at cut set 2 KCl at cut set 3 you see 1 4 and 6 are involved with all positive number, so 1 4 and 6 others will be this thing. Similarly at cut set 2, 2 2 4 and 6 are involved all are positive. So, 2 sorry at cut set 2 it will be 2 4 6 so 2 4 and 6 and that cut set 3 it is 3 is present ie5 is -1 and ie 6 is present and this matrix is called Q matrix.

And this is equal to 3 rows we have seen a matrix that got similar thing A into ie that was also KCL but KCL at nodes. So, here it will can be written as Q into ie is equal to 0 that is the important thing what is the dimension of Q? Dimension of Q will be this, this side number of rows will be number of 2 tweak into number of elements e what is the dimension of ie it is e by 1 so that this will be t into 1 that is what exactly we got it.

Fine and number of tweaks is how much if total number of elements is e total number of nodes is n this is the number of 2 tweaks we have seen that this is loop. So, number of tweaks will be n - 1 so number of tweaks is n - 1 because number of elements in a tree is n – 1. So, this is the equation fundamental equation. Now we are familiar with the what is called the nodal method and loop method and because of those 2 methods that A and B matrices are defined and things like that.

Now the question is here what is the implication of this Q matrix how can I use it to solve a network. Now in a network look at this diagram. In this network if you know all the loop currents then you can know all the branch currents so you have solved this circuit. Similarly if you know the node voltages va0 vb0 and vc0 you have almost solved the circuit because if node voltages are known then this branch current will be va0 - vc0 divided by impedance whatever it is I can find out.

Now the question is what about this Q matrix so that is why those 2 equations were always drawn that is this equations A into i equal to 0, A transpose node voltages in terms of element voltages. Similarly for the loop equations B into V is equal to 0 B transpose into iL equal to ie now what happens in this particular case that is what we want what should I do further the thing

is in a network if you know the node voltages you have almost solved the circuit. Al the node voltages are known with respect to a reference node fine.

Similarly I am now telling this you listen carefully in a network if you know the tweak voltages then also you know the voltages across all other elements hence almost we have solved the circuit. What do I mean come to this network? Suppose I have solved this voltages plus -81 somehow I have solved this is the tweet voltage 1 which happens to be vet1 equal to ve1 only I mean vet1 which happens to be equal to v1 only that is therefore this element because this is the tweak. Similarly this voltage plus -consistent with the direction of the current this polarity I am writing.

This is what this is voltage tweak voltage let me write it in this way be vt1 so that no doubt prevails. So, v1 is the t1 to acknowledge vt1 is this similarly this let me call vt2 and then I will say that vt2 is what this voltage vt2 is the voltage across this element ve2 consistent with this polarity which happens to be equal to -vb0 that is okay not voltage I am not considering so this is the thing where is the 3rd tweak this 1 so this is the direction of the current voltage across it is I will say vt3 which happens to be equal to ve3.

These are the things very easily I can note down. Now you see suppose I say vt1 vt2 and vt3 unknown suppose vt1 vt2 and vt3 unknown if they are known then I can find out the voltage across all other elements. For example you can see ve4 will be vt1 + vt2 will be involved apply KVL get ve4. What about ve6, ve6 can be related with this voltage vt1 vt2 + vt3. Start from this point try to reach this point you get that is vac or in other words if you know vt 1 vt2 vt3 you can find out the node voltages.

And if you know node voltages you know everything in whichever way you think. Therefore here the target is 1 is the KCL equation at the cut-set that is 1 thing. Another thing is relating the element voltage with the tweak voltages that is if I know the tweak voltages how to obtain the all other element voltages. So, let us do that now ok. So, how to do it? So, this is 1 equation better what I will do is this I will copy this portion write it down okay.

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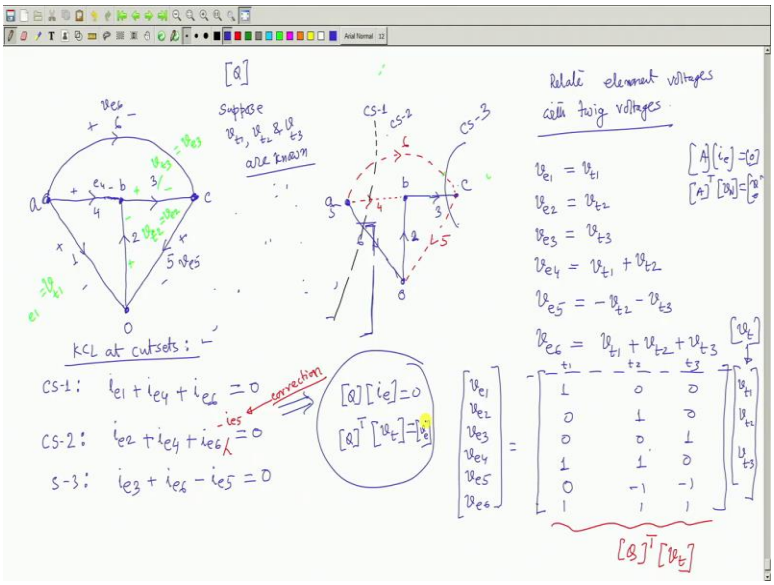

So this I have got and then I told you this I will write it these are all equations we have written it in the form of a Q matrix Q into ie is equal to 0. From the last slide now here I will now try to relate element voltages with tweak voltages this is what we will be doing okay. Let us write so for example I will say that ve1 is equal to vt1 unit ve2 is equal to vt2 then ve3 is equal to vt3, ve4 that is the voltage across this element will be this tweak voltage plus this string voltage is not vt1 + vt2 will be involved there.

Now what about this thing this will be vt1 - vt2 you know vt1 then this is opposite – vt2 or –vt1 + vt2 from this to this I come ve4 is this 1 this is plus this is minus I want to gate so it will be, this will be plus -2 plus both will be plus got the point. So, ve4 so start from this point -2 plus then -2+ this will be ve4 then similarly ve5, ve5 is the voltage across this element and what is the polarity of this ve5 is this way. So, I will start with this that is -VT - you know from this to this if you go - vt2 if I make a mistake point of -vt2 and then -vt3 and finally ve6 that is this voltage ve6 has been defined like this plus minus.

And polarities of these tweaked voltages and are known now this ve6 I will reach from see this way this voltage this voltage and this tweak voltages will give me basics. So, I start here so it will be -v this is a discreet voltage is v5 now, no so from ve6 I want to find out. So, I will go with vt1 sorry vt1 it will be vt1 in terms of tweak voltages vt1 then also +vt2 and +vt this will be

the thing. Therefore this matrix these element voltages can be expressed as let me write in longhand that is ve1 ve2 ve3 ve4 ve5 and ve6.

Here we will participate in deciding these voltages vt 1 vt2 and vt3 this will be this matrix. Here also vt1 vt2 vt3 for my own reference. So, ve1 is vt1 so it is 1 0 0 ve2 is vt2 1 0 0 ve3 is also vt3 1 0 0 this will vt what will be ve4 vt1 + vt2, so vt1 + vt2 kilo what will be ve5 is -vt2 -vt3 and finally ve6 is 1 2 3 all are +1 this will be this matrix. Now if you see the Q matrix it was like this let me copy it and put it there so that the Q matrix was this one, this was my Q matrix just.

So, Q matrix if you see if you take the transpose this is the Q matrix okay this is the Q matrix from the previous page. So, if you take Q transpose that is this first row if you see 1 0 0 1 0 1 0 0 1 1 1 0 0 this copy did not work I better write it there forget about this. Let me move to first it was here I will just go and rather read that from the previous page. What was the Q matrix Q matrix first row was 1 0 0 1 0 it was 1 actually this I thought wrongly is not.

So correct that it will be 1 so 1 0 0 1 0 1 see 1 0 0 1 1 and second one is 0 1 0 1 0 1 see this Q 2 is 0 1 0 second row 0 1 0 1 0 1 it should be 0 1 0 0 1 1, so here you note I just made 1 mistake in the mistake means I dropped 1 terms for example while writing down cut set 2 KCl there was also i5 involved and it should be –i5 this I have incorporated you take care of that. So, your Q matrix also if this entry who will get corrected a bit in the lecture 63 and in the previous lecture as well we have pointed out, so this is the thing.

Now then you note that this matrix Q ie is equal to 0 it is fine and this expression that is this matrix then will become Q transpose into vt tweaked voltages. So, I will write Q transpose in to v tweaked is equal to 0 these are the 2 equations. So, it is almost like some loop equations solving but here what is done which 1 this is ve, so you can see that this will be ve, so inconsistent with other 2 equations also there is some ACL equations a into ie was 0 earlier a into ie this was 0 incidence matrix into ie was 0 this was KCl at the node.

And then we noted that A transpose into node voltages is equal to your element voltages. In the same line if you apply KCl at the card set Qi is equal to 0 and Q transpose into vt the tweak

voltages what is vt? vt is this column vector that will give you the elemental voltages I can always express that. So, you know the element voltage or element currents you have solved the circuit so this is another way of solving the circuit where we will try to find out the tweak voltages is that clear.

So this in short is what is the 3 important matrices A, B and Q although I have not told yet how really now after knowing these how really I will solve this circuit that I will do later.

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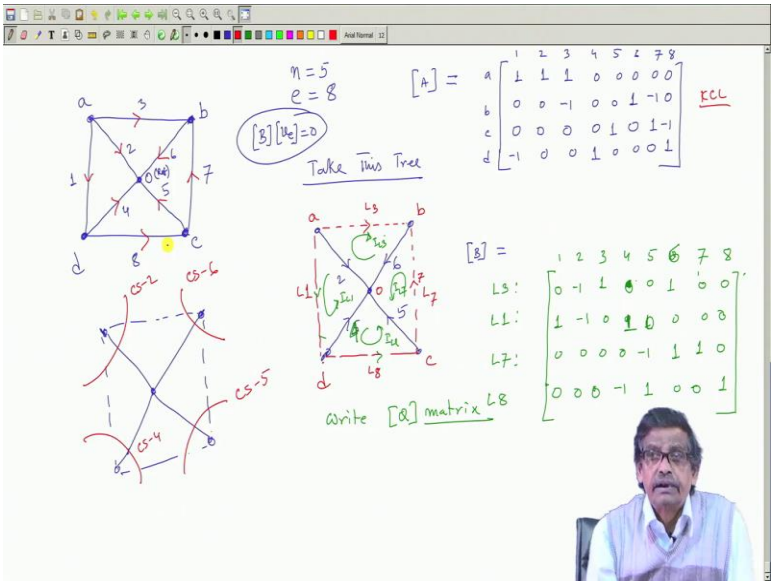

First let me now take some example so that quickly we can write down the ABC, A B and Q matrix. For example I give you now for a change some graph of a network which is like this suppose so that you also practice this. And what you how to approach the problems to find out ABC matrices suppose this is a network whose graph is this and the direction of the graph it is my choice I choose it or if it is given I will do accordingly, arbitrarily chosen how many nodes are there I will write the nodes a b c d and i choose o to be the difference.

Suppose okay so total number of nodes is 5, I come to this total number of edges or elements 448 is not because this is element and I name them quite arbitrarily 1 2 3 4 5 6 and 7 where is 8 gone 8. So, these are the eight elements present. So, first is what is their matrix reduced order incidence matrix. I have to write, so I have to write down KCL at 4 nodes so at node a b c d what

do you be the KCL and here you write all the elements 1 2 3 4 5 6 7 8. So, at node a which elements participate to decide KCLonly 1 2 3 and all are coming out.

So 1 2 3 are +1 +1 +1 others are 0 if I make a mistake point out this is the thing at a what is the KCL at node b at node b 6 is coming out away from the node so 6 is +1 and other 2 are involved are 3 and 7 and both of them should be negative 3 and 7 others I will put 0 these are the KCL at node c this is the thing 5 is coming out from the node so assign plus sign to 5 and 8 and 7, 5 and 7 is also coming out good 0 6 is not there 5, 7 and 8 is -1 other are 0 this is it.

Then finally a note d this is the reference node at note d 4 and 8 are coming out from the node so a sign plus sign there 4 and 8 and 1 is coming in so -1 0 0 0 0 0 so this is the matrix over. Now I want to find out the Q matrix of this so what I have to do I have to or let me go step by step so get the B matrix. So, to get the B matrix I have to find out the fundamental loops in the trees and loops concept should be. So, I take a graph there are so many graphs possible take this graph take this tree suppose I take this tweak like this 2 6 5 and 4 these are tweaks all nodes are there no loop is formed okay.

So, the loops are here L3 loop this loop is L7 link this link will form a loop L7 this is 7 loop 3 anyway loop 7, these this 1 is links 8 which happens to be loop 8 and this 1 is link 1 this will be the thing and here it is a b c d and o. So, what is the B matrix? B matrix is the KVL equations you know B matrix is the KVL equations where in the fundamental loops. How many fundamental loops will be there as many links are there.

So here is the fundamental loop here is a fundamental loop 4 loops are there. So, B matrix so this was KCL consideration B matrix has to be obtained and we know that v into the element voltages will give rise to 0 you recall B into ve is equal to 0 that was a starting point. So, there will be KVL equations how many equations will be there look 3. Incidentally you see this loop 3 fundamental loop should contain only 1 link.

So, this will be I am so sorry it will be arrow will be like this this is loop current iL3 this will be loop current iL7, 8 direction was already mentioned like this so this will be the loop current iL8

and this will be the loop current iL1. So, B matrix will be that is the KVL equation in loop 3 who participate in the KVL equation all the element voltages you know that is 1 2 3 4 5 6 or sorry space is less 5 then 6 7 8.

Look 3 KVL equation this way I have to go now the polarity of the voltage eL3 is the deciding factor writing +1 and -1 so this voltage 3 is 1 and 6 happens to be also 1 same direction 6 happens to be 1 but 2 is -1. Similarly the second L3 I have done then suppose loop 1 L1 the order is not that important but you must understand this is the thing. L1 it will be 1, in this loop it will be this voltage here what are the things there v1 v2 all the element voltages.

So L1 loop it will be what 1 + 5 so 105 and then -102 this, this, this are all other will be 0. And then 3 1 then L7 loop 7 in this loop it will be e7 that is this element voltage that should be assigned plus this way so 7 is 1, 6 is also in the same direction 1, 5 is in the opposite direction -1 0 0 0 0 0. Finally L8 what will be a L8 in this loop this is equal to e8 that is +1 you know this is the sign 5 is also +1. Who are the elements involved why 2 or 5 this is 4 now.

So, this is 1 this is for you just see so this is 5 is plus and 4 is -1 all these DB matrix in this something will loop 1 loop 1 is what? It is equal to that is 4 it will be 1 you know here there will be second row so it was what was written 0. So, second row it will be 1 this 1, 5 is not present other things are fully correct. So, this is B matrix I can form. Finally about the Q matrix we will discuss for this same problem how to develop.

And in the last next 2 lectures I will take up a real circuit problem and try to solve it. So, be familiar you try to your home task is write Q matrix, how to write Q matrix? You have to write down I will give you hints to form cut set, at the cut set you have to write down these KCL equations. Cut Set in this case is very simple these are the linked dotted line. So, cut set means you cut the network into 2 portions such that one tree is always present so 1 cuts it will be like this.

This cut set I will name it as cut set 2 because element to tweak this tweak is only involved there similarly this cut set I will say it is cut set 6 it is not all arrows you put there. Similarly this will

be cut set 5 and this side will be cut set 4. So, try to write down put the arrows and try to write down that, thank you.