Network Analysis

Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology – Kharagpur

Lecture – 64
Mesh Analysis with Graph Theory

Welcome to the next lecture on network work analysis and we have been discussing in last few lectures about the application of graph theory to networks solving.

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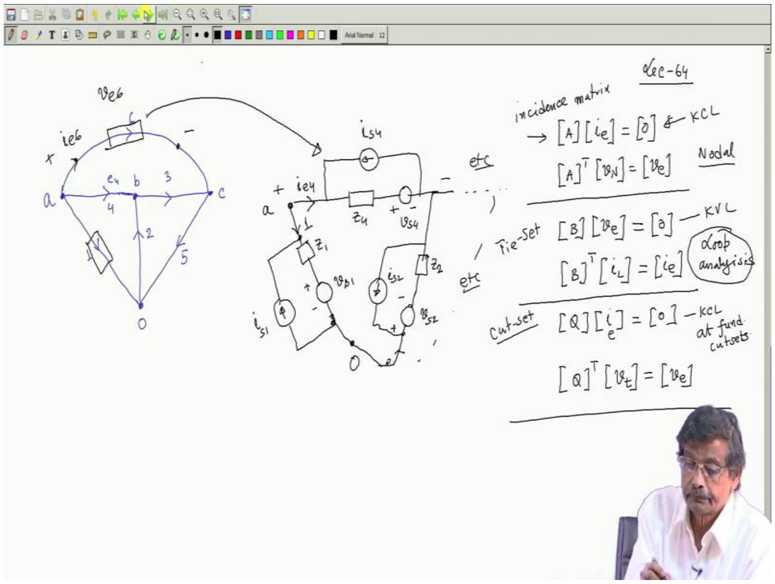

Now if you recall that given a network will first draw the graph of the network and then from graph of the network will draw a tree and then from tree I can also get the number of tweaks number of leaves and based on that different techniques different important relations were found but really we have not solve network problem a given network problem for finding out the current yet we have not done what simply we have done is we have found out this 3 basic relationships that I will rewrite it here.

A into ie is equal to 0 that is KCL A incidence Matrix and then we found out that A transpose. V nodes equal to the element voltages. This is one relation we found out this is A is called the incidence matrix. Incidence matrix and this A matrix we will get in touch with for a given graph when I want to solve the network in nodal analysis this is KCL equation. Similarly we got another 2 interesting relations that is B into the element voltage matrix. this is equal to 0 this is

KCL this is KVL vb equal to 0 and also I have seen that B transpose the loop currents is equal to the element currents.

And we have discussed at length how to get AB matrix from a given network. So iL is the number of fundamental loop currents. And finally we got a relationship like U into and this is called incidence Matrix this is called cut set, tie set Matrix this is this is tie set I am going to write tie set and this is called this Q into ie is equal to 0 this is called a cut set Q Matrix and this is also KCL but KCL at not notes KCL and fundamental cut set this we have already discussed and then finally we got this Q transpose into tweak voltages is equal to your element voltages.

In last few lectures we develop this. Now any of these methods looks like can be applied to solve the circuit. So this 6 direction is ie6 henceforth I will simplify this I am not going to write ie6 ie5 it means ie3 in this direction ie5 in this direction there are the nodes. Now, today what I am going to do is this how do I get the exactly how I am going to solve the network problem. Now in general this network this branch for example branch 6 it will contain in general some impedance some sources as well.

And it is true for all the element that is what I meant to say is this if I draw that is a this is suppose the given network. Which is actually given is like this suppose some impedance is there and some voltage sources is there. This is suppose I say it is z1 and this is vs1. This point is a, this point is O this branch and this is I have specified as 1 branch number, got the point. And in general it could also have a current source in parallel is1 so this line although I just just drawn a single line in general in this line there will be an impedance.

Their name will be impedance and there will be sources voltage source and current source is connected here. Similarly here in this branch there will be say branch number 4 you can have and impedance can have another voltage source vs4 element 4 and then maybe current sources is4 got the point. And similarly in this branch so this then this branch should also had a voltage source where is the error given this error is given. So it can have a and impedance Z2 because this branch number is 2 and there will be some vs2 and this is a class and there may be a current source in parallel with this element.

So current was this. So this is the thing is2 similarly this two branches will have impedances. I am not trying that got the point therefore this branch 6 between these two points what may exist in general is a voltage source is an impedance in series with this. So, there are something here. What is this box this box is this? There will be impedance there, but while drawing the tree of the network we have considered this to be one thing draw a horizontal line like that. Therefore to find out and these current I am telling ie4 mind you.

This current is ie4 not this impedance current got the idea and this voltage between this to this point is ve4 etc, here I am not completing this in order to 1 2 3 branches another 3 branches is similar thing. Therefore now the question is I have only told you that for a graph you do not show this all these details you just draw lines and say that this current is ie6. That is what I told and this voltage you as shown as this + - to be ve6 and similarly for the other elements of this graph.

Now today what I am going to do is this I am going state forward that is ok. These are the relations which exist between this element current and the voltage across the whole combination then how daily I am going to solve a network problem. That is the thing. I think you have got the idea. So next point is first suppose and this method mind you this B Matrix will coming to picture when you want to solve this circuit solved for the fundamental loop current cycle.

And this one is nodal essentially and this one is loop analysis. And this one is also like nodal but slightly different and it is called cut set method. So, first let us try to given a network. I want to solve it by loop analysis. So I will now go here.

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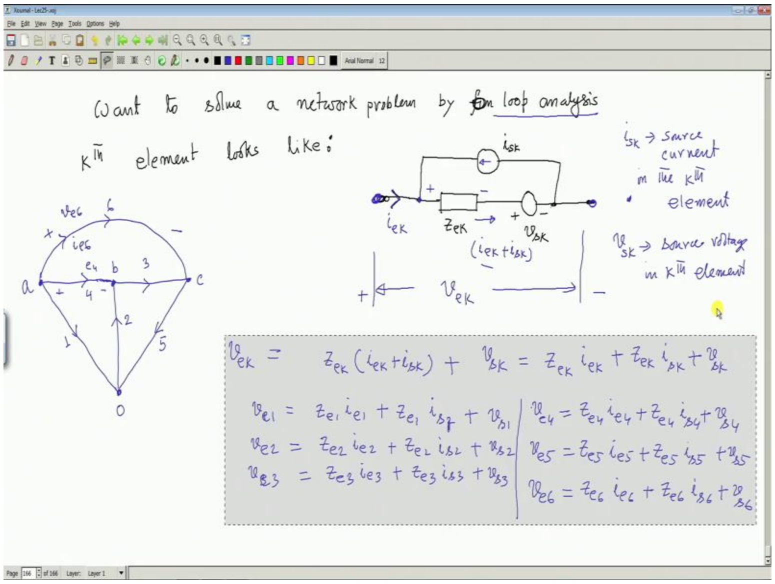

So, want to solve a network problem by fundamental loop analysis loop what I am going to do? So first thing is a General kth element looks like every arrow you have given like this between two notes and element exists, so I will say this you carefully follow me. So, here is your; I will say this is a Zek the impedence let me not put the arrow first. This is a Zek and then there may be a voltage source. This arrow is known so always put I will follow this convention the source with this polarity is connected.

So, these are the two loads and there is a current source. In parallel and these 2 are the nodes this is kth branch k is equal to 1 to 6, put all the branch and if you put k equal to 1 then this node is A and this node is B you have understood now so this is the thing. Now what I am telling while for a particular node say node 4 this is the arrow. This is the arrow suppose k equal to 4 you substitute. So the polarity of the voltage will be like this class plus minus this C for voltage and ie is this current this current is iek this you must understand.

Henceforth this picture I kept behind this I just replace this whole thing by a line to understand what are the relationship of this current with different mattresses, like KCL this that and this is inside this in this particular element this box was replaced by line. But this box in general may content like this. What is the isk? Is a source current in kth branch in kth element. What is vsk? Source voltage in kth branch in kth element, so when I asked to solve a network problem, I will be interested to know what are the current flowing in Zek.

This iek is the total current understood the point. If you just allow me, I will make one correction always. I will assume current is injected in the positive terminal of the node isk direction is like this and also polarity of the vsk is this arrow decides what is that. If this arrow coming like this class is there, so this you please follow. Then I must find out relation first thing is I want to solve this problem by loop analysis.

Loop analysis means that tie set matrix will be invoked that is this what you want to find out the loop current. So, first of all you see that so isk is the source current and this one this isk arrow I will put so that it also inject current like iek. So, iek those current which we have already found out relations with this is and this is the voltage across this element. vek is this voltage, this is voltage and assign plus here and minus here that is for this element this is this is minus and this is ve6 and this is ie6. So ie6 comes but inside this actually this I can expect.

In some branches all impedance will be present then vsk will be 0. And isk will also may not be present in most of the cases, few branches will have impedances. But in general this will be the thing. Now you see the current flowing through this branch will be isk iek apply KCL here. So this current iek +isk flowing from left to right. So there will be a voltage drop here Zek into this current so you can see that vek will be vsk this to this plus this voltage drop. This voltage drop will be Zek into iek + isk with this side plus side minus to plus or plus to minus both the terms are added.

So this will be equal to these elemental voltages. In case of Meyers analysis, you would like to apply this equations. So we would like to find out the in terms of loop current how things look like. So what I will do is, this is vek and this can be broken up into two terms that is Zek into iek + Zek into isk + Vsk mind you is and vs are source voltages there will be node it is there. Therefore, it looks like I can and I will just show you write this one like ve1 will be in general Ze1 i1 ie1 put k equal to 1. You get ve1 + Ze2 is2 present in Ze1 i1 plus Ze1 only Ze1 plus if I miss the term plus vs1 this will be the thing.

Similarly ve2 let me write one Ze2 + ie2 + Ze2 is2 + vs2 ve3 will be equal to Ze3 ie3 + Ze3 is3 + vs3 finally another for ve4 let me write at least one so that Ze4 + ie4 + ze4 is4 + vs4 v5 quickly Ze5 ie5 + Ze5 is5 + vs5 finally the last element for this particular problem. It will be Ze6 ie6 + Ze6 ie6 + vs6 now all the 6 equations can be written in matrix form as this. So, this one I copy to next page.

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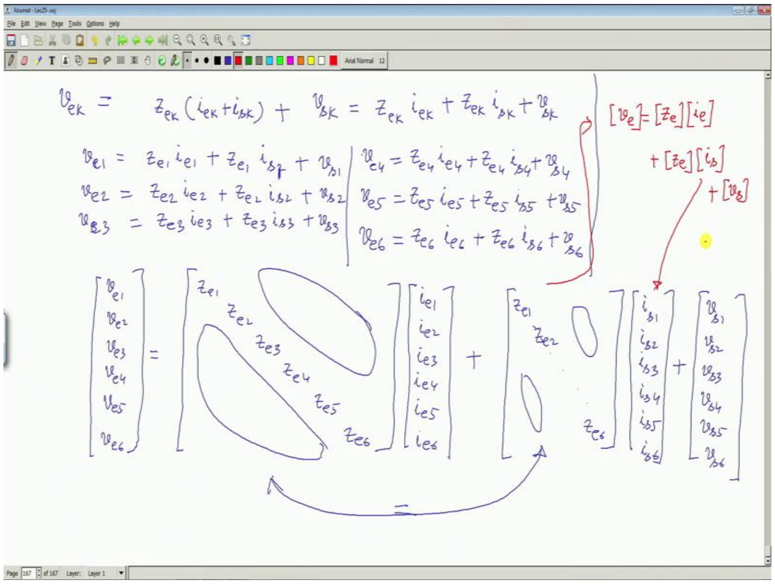

So, the all the six equations can be witness in matrix form as difference equations ve1 ve2 ve3 ve4 ve5 ve6 this one can be written as Ze1 Ze2 Ze3 Ze4 it will be diagonal Matrix Ze5 and Ze6. all other elements are 0 and here you will have ie1 ie2 ie3 ie4 ie5 ie6 other elements are all 0 it is a diagonal matrix which the diagonal elements are nothing but the impedances of the several branches and this is the elemental voltage. So, this into ie this is ie6 plus then you have the source things. Source thing will be also this matrix the same metrics comes in here I am not liking that is this Ze1 to Ze6.

You see Ze1 into is1 + Ze2 into is2 and this column will be is1 assuming all the branches that having current sources is4 is5 and is6 and this plus column vector only it has got no Z which will have all the source voltages present got the point. This will be the equation. So, all the elemental voltages can be written as this free current I mean element currents and some diagonal elements this two are same this 2 matrices are same this is the thing.

So this whole thing therefore can be written as here let me right as a column vector of the element voltages. This equation I am writing v is equal to I will say it is Ze, where Ze is the a diagonal Matrix into the all the element current ie + Ze into is where is is a column vector having is1 is2 is3 is4 plus a column vector of the source voltages got the idea. This is the in short and this is the nice way of representing the things.

Therefore coming back to this previous slide each element in general can have this form. So I wrote down and vek is this vek the trees and here there was a box each element has a box that box is this one inside. So, actual impedance current Zek will be some of isk + iek. So if we solve iek ok. So, now we will discuss about that. So, we get up to this point.

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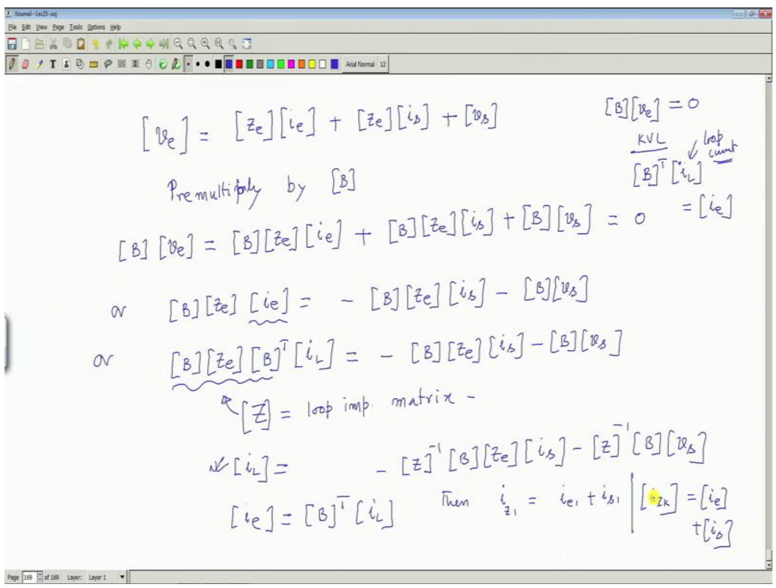

So, we write that this equation as; so we have got so we have got ve is equal to Ze diagonal matrix into ie is it ie + Ze into is and plus vs that is this equation ve equal to ze into ie + Ze into is + vs where we should only remember that that this is a column vector. This is a diagonal Ze. This is Ze into is and this Ze into is, vs a column vector ok. Now you see loop analysis what you do what element voltages ve1 ve2 ve3 ve4 like that.

Now, we multiply both the sides by B transpose multiply pre multiply pre multiply by pre transpose by B pre multiply both sides by B then it will be equal to this will be equal to let me write in steps B into ve this will be equal to B into Ze into ie + B into Ze into is + B into vs this is the thing. But what is B into Ve is 0, KVL equation, this is KVL terminal voltage in any loop sum up to 0 so this is equal to 0 then.

What I want to find out my goal is to find out the element currents. So, I will write or B into Ze into ie then will become equal to take this two terms on the right-hand side because this involves sources and this are known, so this will be -B Ze into is - B into vs this the thing. But in loop analysis, which I am trying to do I must find out the fundamental loop current and I say if I know the fundamental look current I solve this circuit.

So what is the next relation? The next relation that I will be using is B transpose the element currents can be expressed in terms of the loop current this is loop current fundamental loop current. So, what we are going to do here is B into Ze and for ie I will write B transpose into iL about this fundamental loop current I know what it means. So this is equal to -B into Ze into is - B into this. Now this will be a square matrix ok and this is sometimes called loop impedance matrix.

So, this one if you call Z is called the loop impedance matrix. So, this will be then equal to Z into iL so iL the loop current will be equal to minus pre multiply with this Z inverse you multiply both sides with and then -Z inverse B Ze into is minus this Z inverse into B into vs this will be the loop currents. This is excited. Correct. So minus so I will be able to solve this circuit now, I was always telling you that this graph technique is applied to circuit analysis just to solve a big circuit.

Because you see now I will be doing mechanical given a network the formation of this Ze is very simple. Only put the in the diagonal this elements. Finding out B matrix? B Matrix elements are + 1 - 1 that also I know how to do it? Cut set matrix and then I have to calculate this loop impedance matrix Z which is this computation have to do B Ze B transpose and then take its inverse and multiply. So, this problem can be dedicated to a computer and loop current will be obtained. Once the loop current is obtained then what I am going to do?

I will calculate the then calculate after you get this then I will say element currents is equal to B transpose into iL I will be able to calculate this element currents ie1 ie2 ie3 but ie1 ie2 ie3 and not the impedance current in a particular branch you know iek you will get ie1 ie2 ie3 ok add with this source current so ie + is will give you actual impedance current iZe1 for example if somebody writes. So, after you that is what I am going to tell if you are solve the loop current then we solve the circuit.

What else you calculate ie after you calculator ie add to this is to get the actual impedance current. Current source is already known, so then finally if you say what is iZe1 we will say it is equal to ie1 + is1. In other words, if you say the current flowing through that impedance if you say that is Z k It will be simply ie +is which is already known. So everything is no systematic and you get all the impedance value. So please go through the steps very simple with simplified networks. It will not be a tough thing to construct B matrix. And to construct impedance matrix that I will do in some later class some numerical problems. I will solve.

But today I am going to just tell you I have got this to summarize, this is what I am trying to tell is this; these are the basic relationships that we got Ai0 but this ie is the actual current here this current not this part of the current. So, if you want to adopt loop analysis, then B will be involved that is what we have done. Next time we will solve by applying nodal method, thank you.