Network Analysis
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Lecture – 65
Nodal Analysis with Graph Theory

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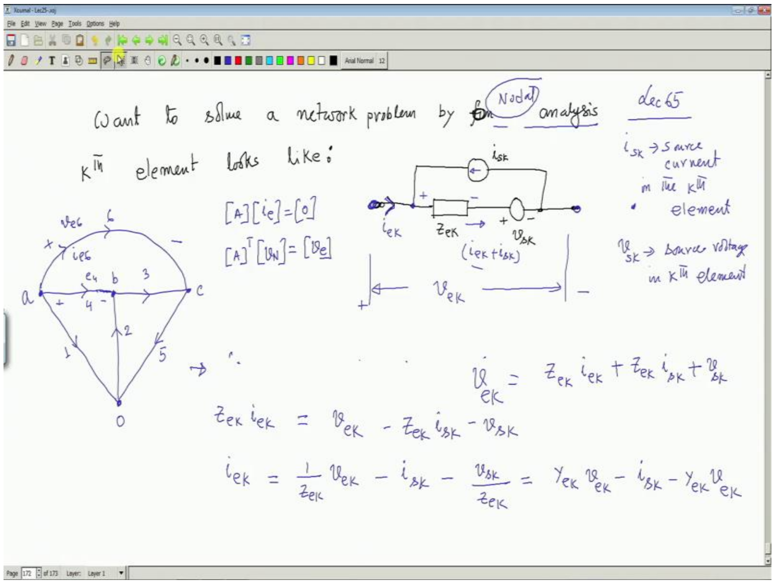

Welcome to 65th lecture on network analysis and we have seen that how graph analysis can be utilized to solve a network problem completely by using the loop analysis method. Similarly we can solve the net or same Network problem by nodal analysis. So, we started with this you recall that this is the general element and this is the thing and this portion I will just delete and straight away write that this is the basic equation which will be this one.

So, this was the kth element of the network and this time I have decided I would like to apply the nodal method that is I will invoke a matrix that is the where we write the KCL equations at the node ai is equal to 0 what are the two relations there these are the two relations terminal relations I will utilized i into ie is equal to 0 and A transpose into V nodes is equal to the element voltages. These are the two relations will utilize.

So, finally by nodal analysis your target is to find out okay what are the node voltage of the circuit. If you have got that you have solved the circuit. So, once again we come back to this equation. So, first thing is I will apply KCL so this equation this is true we have already got it and then I will say that this z ek into I ek this portion you can see this, so this is the thing zk is equal to this one this is equal to v ek. So, this is the thing I have got z ek I ek + z ek i sk + v sk so z ek i ek can be written as v ek – z ek i sk - v sk is not then what I am going to tell that this is true for all the elements k equal to 1 to 6.

Then this essentially means that ok before that what i do so i ek I want to find out what I will do i ek then will be equal to 1 over z ek divided by z ek into v ek minus only i sk z ek will cancel out –v s k by z ek divide both sides by z ek but this z ek where reciprocal of impedance is admittance. So, this can be written as y ek v ek - i sk - y ek into v ek this is the thing. So, this equation is the equation of interest when you want to find out the node voltages. So this you copy and paste next page oh sorry, so I will copy this one so from this.

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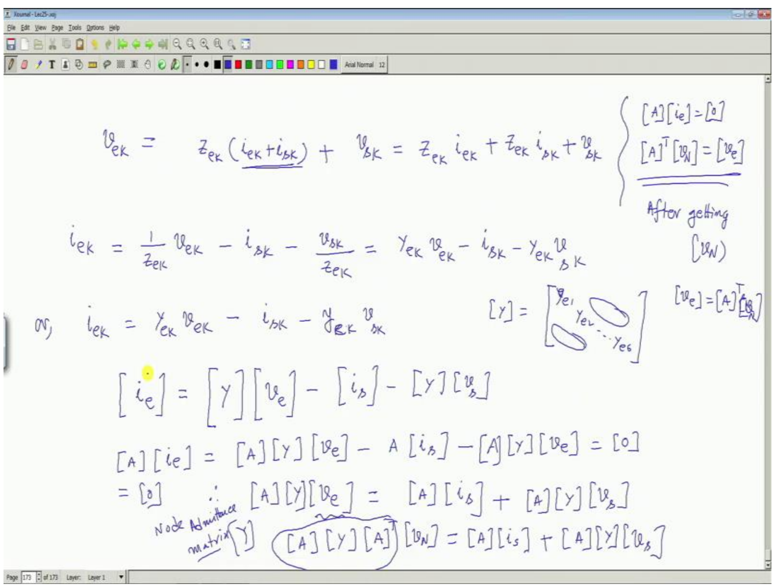

I got this okay so I ek is this one from this equation I got this equation now what we'll be doing is this thing, that is this, so from this to this I get, now this equation I will write it as i ek or i ek is equal to y ek v ek – i sk - y ek v ek and in the same way it can be written in matrix form as the a column vector comprising of all the element currents can be written as a diagonal y into a column vector comprising of element voltages minus a column vector of source current of all the elements minus why it will be another admittance matrix into ve what is y? Y is nothing but y e1 y e2 dot dot dot o y e6 for this particular problem these are all 0 elements.

What is y e1 1 by z ek1 and so on therefore. This is the basic equation we have got here. Now in nodal method analysis method you have to apply KCL, so multiply both sides by A matrix A. So, if you pre multiply both sides with A you will get A into ie is equal to A into y into ve - A into is - A into y into ve but A into ie is equal to 0, this quantity is 0 cut off current law. So, this is equal to 0 vector. Therefore we can say that this this is the A y ve - A is - A y ve is equal to 0 therefore I can say A into y into ve will be equal to a into is you take these two terms on the right hand side +A into y into ve that will be the thing.

But what is the target in nodal analysis to find out the node voltages is it not. So, we know that in nodal analysis these two we have already established A into ie equal to 0 that we have already taken care of we have used that informant and then the second one is A transpose into the node voltages is equal to your element voltages. So, this for ve here I can write A transpose into v ie in this equation. So, it will look like A into y into A transpose into the node voltages that is what I want to find out will be equal to A into is -+ A into y into vs this will be vs.

Here I wrote I sk this will be v sk sorry please I correct that otherwise v sk, v ks is v sk so this is v sk source you take on the right hand side vs here this is vs so this will be vs here this line. So, anyway so take that correction so A y A transpose into vn is equal to right side only source current and source voltage okay. What is y? y is this diagonal matrix is very easy to form give the network I will in no time make this matrix available to me.

I can also form the A matrix by drawing the graph directed graph only +1, -1, I will do that then this whole thing will be a square matrix and it will it is called the like loop impedance matrix earlier we called it will be node admittance matrix people call it, node admittance matrix may be replaced by another symbol Y or so on. And take inverse multiplied with this you get the node voltages. Then I am telling if you know the node voltages you have solved the circuit why because after you get the node voltage you can easily find out the element voltages.

After getting VN that is this one by solving this network multiplied with inverse on the both the sides you will get node voltages. If you get node voltages element voltages can be easily calculated ve is equal to A transpose into VN element voltages across each element these voltages are known and if you know element voltages for example if I know ve1 they say this voltage is known what will be I ek?

You can just subtract the appropriate source quantities in column form or individually take the equation find out I got the idea so the current flowing through the impedance z ek is ie k + i sk, so I will add up with that so this is the it nutshell you have to do now these steps after establishing these fundamental relations A ie is equal to 0 A transpose into VN equal to V then we arrived at this equation where on the right hand side on only this column vector is and vs known.

The good news is not all the vs and is will be present it is very unlikely in all the elements you have sources ok whichever elements is there I will put those numbers and in whichever branches sources not there I will put 0's there so 0 entries will be there and Y is a very simple matrix, so I can easily write down a program to find out VN node voltages of the given network. And as you know if you know the node voltages you can easily calculate the element voltages.

If you know the element voltages, you know everything in this circuit you can find out whatever? So, if you know this element voltage v ek, I want to find out this current what will be this current i ek, i flowing through z ek will be how much i ek z ek will be element current I have found out ie i ze any branch column vector it will be simply this one ie will be z ie element current vector +is is it not. So, once you get v we will be able to solve for this 2, this is in nutshell this thing you have got the element voltages.

If you know the element voltages element voltage is nothing but ve if you see element voltages this is of course in impedance term you can easily work out how this relation will be by looking at this diagram. So, this is another way of solving this circuit by loop analysis I do not know I do not have much time to complete that.

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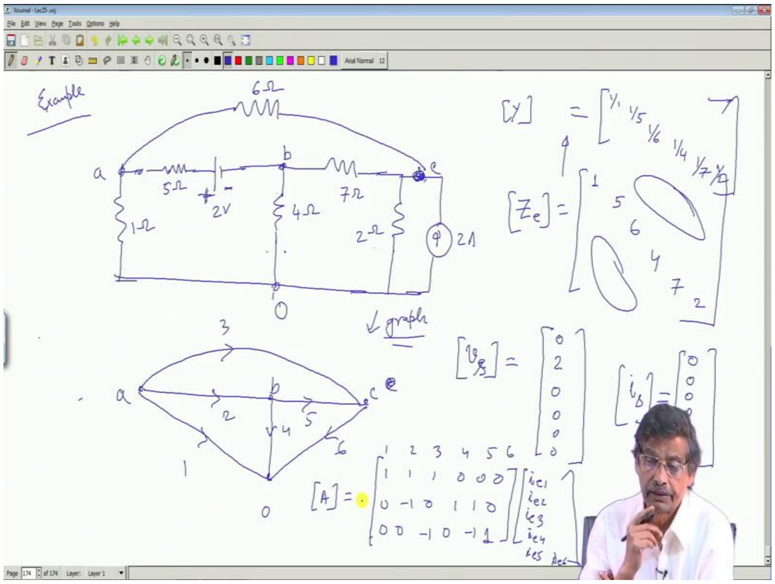

What I mean to say let us take some numbers and to fill up this matrices. Suppose you have a network like this an example suppose you have got a network like this 5 ohm this is 8 ohm, no current source there, here is an element here is another impedance this is say 4 ohm and this is this impedance I have shown it could be any other impedance in Laplace domain also it does not matter. So, this is 4 ohm it is like this and here is an element which is 7 ohm there is here I should change it here there is a source here say +- 2 volt is there.

Here there is no source and here is one, where there is no source and there is current sources say 2 ampere. And this branch is having only impedance suppose. Any arbitrary I am drawing and here is one resistance only say 6 ohm **(Fl: 19:27)** this network these are the nodes a b and this is the big node c, it has let us say this is 2 ohm. In this network while drawing the graph of this network what I am telling a b this is 0 say this is say c, you just draw a single line here also single line there are although two elements 5 ohm and 2.

Here also a single line 4 ohm here also a single line now between this c and O there is a resistance in parallel with a current source while drawing the graph I will just draw like this that is what I am telling. Here actually this 2 ohm and this thing and here is 6 ohm and here you do not show any values now what I am going to do I want to make a directed graph of it. So, this is the graph of this network and then I will say that okay call it arbitrarily this is 1 this is 2 this is 3 this is 4 this is 5 and this is suppose 6.

I have directed the curve, then let us say suppose this is 1 I want to make Ze matrix it will be a 6 by 6 matrix so 1 2 3 4 5 6 ok what will be this entry 1 1 Z e2 will be 5 Z e3 this 1 3 I called 6 what is Z e4 4 this is the impedance where is Z e5 7 and where is Z e6 that is 2 and your Z impedance matrix has been formed at the point. Suppose I want to make the column vector of the sources v is vs what should I do? Column vector will be all the source voltage is present in all the elements start with element 1 is there any source voltage? No, zero.

It will be a column vector, 2 source voltage is there yes it is there with this about sign you point note this is plus this is minus what is the arrow given this way so this is 5 + 2 what about and in oh sorry this will be a column vector. So, this will be 2 here and in no other elements voltage sources are there so 2 3 4 5 6 vs is there. Similarly what will be is another column vector in all the elements current is present your current source is present. In 1 no current, 2 3 4 5 all the current is in the 6th element so 0 0 4 5 6 element current is 2 ampere and it is +2 I will write +2 and this is your is.

And then a matrix if you remember for this particular problem the way directed graph has been drawn a will be in the meantime this is Ze what will be the admittance matrix it will be just reciprocal of these numbers 1 by Ze that is it will be 1 by 1 this will be 1 by 5, this will be 1 by 6, 1 by 4 this is 1 by 7 and this is 1 by 2 this will be your admittance matrix. Then A matrix will be needed, A matrix is very simple what at each node KCL.

How many no elements will participate in deciding KCL 6 elements? So, here I will write it nicely here so this will be A matrix for example will be this one, so that you get a filling of this so 1 2 3 4 5 6 and here this is the column matrix are you on in fact that these need not be written to write down i e4 i e5 i e6 these are the six elements and here A this is Junction a b c nodes so all currents are coming out 1 2 3 are positive 1 1 1 4 others are zero's. Similarly at node b 4, 5 are coming out so plus plus and 2 is minus 0 0 0.

And finally at node 6 is coming out 6 is plus 3 5 are minus A matrix is ready. Now the question is how to find out the B matrix ok to find out the B matrix you have to draw a tree you know then find out fundamental loops there. So that, B matrix depending upon the tree you draw identified the loops and then apply KVL in those loops fundamental loops and get B matrix. So, I stop here today and we will continue with this that is the third method which is called the cut set matrix method. And give you some example I will try to upload some solve problems. Thank you.