

Network Analysis
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Lecture – 66
Cut-Set Analysis with Graph Theory

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$[Q][i_e] = [0] \rightarrow$ KCL at the fundamental cut set
 $[Q]^T [v_e] = [v_s]$
 Solving eqn by cut-set method or using $[Q]$ matrix
 $v_{ek} = z_{ek}(i_{ek} + i_{sk}) + v_{sk} = z_{ek} i_{ek} + z_{ek} i_{sk} + v_{sk}$
 or $z_{ek} i_{ek} = v_{ek} - z_{ek} i_{sk} - v_{sk}$
 or $i_{ek} = \frac{1}{z_{ek}} v_{ek} - i_{sk} - \frac{1}{z_{ek}} v_{sk}$
 or $i_{ek} = y_{ek} v_{ek} - i_{sk} - y_{ek} v_{sk}$
 $[i_e] = [Y][v_e] - [i_s] - [Y][v_s]$
 $[Q][i_e] = [Q][Y][v_e] - [Q][i_s] - [Q][Y][v_s] = [0]$
 or $[0] = [Q][Y][Q]^T [v_e] - [Q][i_s] - [Q][Y][v_s]$

Welcome to the next lecture on network analysis and today in my last two lectures I told you how to solve a network problem using graph theory. And use 2 methods one is the nodal method where the target was to calculate the node voltages and other was the loop analysis method when the target was to find out the fundamental loop current once you know, the node voltages and fundamental loop currents you have really solve this circuit.

Now today I will tell you about how to solve the network problem using graph theory but using cut set method. Ok that is give this is the network each branch in general will kth the branch has been shown put k equal to 126 here so in general all the things will be present in a branch. If no current source is present open this branch if there is no voltage source present V_{sk} will be 0 and so on. But do not put i_{sk} equal to 0 and short that you should not do that.

Ok anyway, so today will be discussing about using cut set Matrix and cut set Matrix you know that what you do is this you draw the I will just briefly review what do you know about Q

matrix, you draw the; choose any one tree this network this is what I have chosen this is the tree. These are the links and these tree elements are called the twigs. So, I told you that in this network if this twig voltages that is this voltage across 1 across 2 and across 3 if these are known you can find out all other element to voltages.

If you know the element voltages, you know the voltages and so on. So, this method; so Q matrix how to form it I am not discussing once you draw the tree you can; you know about the Q Matrix. Now, what will be the basic equations once again, you start with the kth element where this is network that element branch and here you write down the potential difference between v_k as this one this we have done several times.

Then here it is equivalent to some nodal analysis, is it not this is like nodal analysis. So you have to apply KCL equations. So first find out i_{ek} this current express it take this to this side divide by z_{ek} then i_{ek} will be admittance into $v_k - i_{sk} - y_{ek} v_{sk}$. Now if you do it for all the elements that is i_{e1} i_{e2} i_{e3} it can be a column matrix, here it will be a diagonal admittance matrix this will be one second a column matrix of different element voltages.

And source these are the source voltage and current. So, source voltage and current keep it always on the right hand side that also we have applied previously. Now after you do this you remember that Q this matrix into i_e is equal to 0 this is how do I get? By applying KCL at the cut set at the fundamental cut set, this is node. Another relationship is known that is Q transpose into twig voltages is equal to the element voltages.

That is if you know the twig voltages, you know all the element voltages. And if you know all the element voltage then node voltages can be calculated and so on. So, this is the fundamental equation to begin with here I get up to this. Then what you do is this you multiply this equation with Q both the sides therefore it will be Q into i_e is equal to q into v pre-multiply both the side by Q matrix minus Q into y matrix was already present.

Q into y into v_e this will be the first term and this will be minus minus Q into i_s , i_s and v_s are column vector v_e is column vector - Q into y into v_s this will be the thing. Now obviously Q into

ie will be equal to 0, because of this. so, Q into ie this must be equal to 0 vector column vector. So, from this I will be; so this is this thing. Now or so this is equal to 0 equal to this one is equal to Q into the nodal admittance matrix y into ve can be written as in terms of twig voltages twig voltages.

If you know the twig voltages all the other relevant voltages can be known. If you know the node voltages all the voltage of the other elements can be known. So, that was this period. So, this is Q into ve but ve for that I can write down Q transpose into B twig voltages from this substitute this and this - Q into is - Q into admittance y into vs. See here it is current therefore, right hand side also dimensionally should be current so you can easily remember this is current.

So, Q matrix has got no dimension only + 1 - 1 increase so it is in ampere and vs in volts and volt pre multiplied by admittance will give you also ampere since Q has got no dimensions. So this is the thing. So, this equation is the central equation to find out the current in the circuit. So, this you copy and paste it in the next page.

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The slide contains the following handwritten equations and notes:

$$[A][Y][Q]^T [v_t] - [A][i_s] - [A][Y][v_s] = [0]$$

or $[A][Y][Q]^T [v_t] = [A][i_s] + [A][Y][v_s] \leftarrow$

Previously got \therefore Twig voltages can be calculated.

$$[A][Y][A]^T [v_n] = [A][i_s] + [A][Y][v_s] \rightarrow \text{to calculate node voltages}$$

$$[B][Z][B]^T [i_f] = -[B][Z][i_s] - [B][v_s] \rightarrow \text{to calculate fundamental loop currents}$$

A small video inset in the bottom right corner shows a man with glasses and a white shirt speaking.

So, this is thing and this is equal to zero or I can say that my target is to compute this one so Q into y into Q transpose into vt will be then equal to Q into is + Q into y into vs that is all. So, this is once again the nodal admittance matrix here we got Ay transpose when we apply nodal

method to calculate the node voltages. Here that is should be replaced by Q and the equation is similar. So, this equation can be solved and you get the value of the twig voltages.

So twig voltages can we calculated and once you know the twig voltages practically solve the circuit that was the whole idea. Therefore this is the KCL method only but we are applying at the cut set and previously you just see whether I am writing correctly we got this one $A y A^T$ transpose previously got $A y A^T$ transpose into the node voltage equal to this it was like this A into is source voltages plus $Q A$ into y into vs. So, this was the to calculate the node voltages node voltages.

And so these two methods are similar time you are applying KCL for this you are applying KCL and the nodes and for this you are applying KCL cut sets. Ok. So this was the thing 3rd method the loop method; in the loop method we got $B z B^T$ transpose into the fundamental loop current is equal to put both sin square minus there, anyway, I will see just think that this is equal to but here you see it is z into i voltage. So voltage will remain like this, for example, it is so iL it will be equal to $-B$ into z where I am I am writing z there.

So, B into z into is $-B$ into vs, so right hand side is the source from this we will get the fundamental loop current and if you know the fundamental loop currents then. So, this is to calculate fundamental currents. If you know the fundamental loop current you have solve this circuit, it is understood. So, these are the 3 methods the final results and z it is called the loop impedance. This whole thing is called the loop impedance matrix $B z$ into beta. What is z ? It is diagonal z .

What is y ? Diagonal y in all the branches $1/z_{ek}$ is y_{ek} so like that; so this method why this assumes importance because if the network is large, you just it is solving circuits. This you can write some codes so that only thing we have to do is z matrix writing is very simple you have the network before you on pen and paper, so you can write z matrix you decide which method you will apply whether nodal method or cut set method based on that you form the Q matrix from the trees from the twigs or from the A matrix.

That is the incidence matrix or the B matrix and you know how to form them? So what are the things to be known from my side? B if you are adopting this method, then you must know z you must know B you know how to get the B elements, elements of B are very simple like that +1 or -1 or 0 similarly here and what else? This source current and source voltage this is there is a small correction it is vs, so that can be done. No matter whatever is the size of the network this can be done and then write a small codes in MATLAB in which platform you like and get the current.

Very systematic approach to solving networks therefore all the three methods I have discussed. Now at this I will of course take a very simple example just to give you a feel what exactly up to done. I am not taken a very big system otherwise I myself will make mistake and things will become very I mean too many steps will be there too many variables. Let me give you some example how to adopt this method in this case.

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Want to solve for fundamental loop currents

$$[B] = \begin{bmatrix} l_1 & 2 & 3 \\ l_2 & -1 & 1 & 0 \\ l_3 & -1 & 0 & 1 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$[i_s] = \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}; \quad [e_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore [B][Z][B]^T [i_L] = -[e_s] - [B][i_s]$$

$$\begin{bmatrix} 4.8 & .8 \\ .8 & 2.8 \end{bmatrix} [i_L] = - \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$$

$$\therefore [i_L] = \begin{bmatrix} 4.8 & .8 \\ .8 & 2.8 \end{bmatrix}^{-1} \begin{bmatrix} -12 \\ -10 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

A simple network let us say, for example you have a network like this here of course simple network which of course can be solved without resorting to graph theory because it is so simple and network but none the less it will; suppose somebody decide he wants to apply graph theory to solve this simple network even we can do so. So, suppose this is the network here it is supposed 10 volt, here it is 12 volt here 4 ohm 2 ohm and 0.8 ohm. Now what will be the graph

of this network? Graph of this network will be there are only you know this is one node this is one node.

So between these two nodes there is an element. Between this node and this no there is another element. Between this and this if you call this is node A this is node O. Then there is another element. This is the graph of the network very simple 3 elements are there ok and then I want to make a directed graph. So what I will do I will; this sum assuming the directions is totally flexible in whichever way you assuming the sum suppose I have assumed it in this way 1, 2 and 3 this is you're a and this is your O.

What will be its tree? 1 tree let me draw. These are the two nodes. All the node should be connected but there should not be any loop. So this is one tree. This is one tree. This is one. And what will be the links of this? This is only this is one link and this is one link, this is link number 3 and this is link number 2 this is a this is O got the point. Now suppose I want to solve it by; I want to calculate the fundamental loop current for this network.

So I have to form the B matrix so it is loop current first B Matrix so I will write down I want to solve, to solve for fundamental loop current. What are the fundamental loop currents? Fundamental loop currents will be i_{L2} and i_{L1} you put one i_{L2} and i_{L3} decided by the number of links present. So there will be only two links so B matrix straight away I can write it as for loop 2 and for loop 3 and how many elements are there? 1 2 3 4 write 1 2 3.

So B into v is equal to 0 that is thing. Suppose you take this one, so it will be you start from this point the direction of the link will if it is this way assign plus sign here. So 2 it is plus then only one will be there that will be minus and this is 0 that is it. Loop 3 for this loop when all this link is present. This is the plus sign so plus one. Then once again, there will be only one link present in the loop so 3 and 1 form that fundamental loop.

So, 1 will be 0 and this will be -1. This is B Matrix formation. Now, what will be ze here? So what I need? So, for this what this I need this equation that is let me copy and paste that so that you understand the steps what have to do? So, this last equation let me know I have to calculate

the I want to mention that z I have to calculate, B I have obtained. Then is and vs this 2 matrices for let us do that. Now z matrix will be simply here let me write z Matrix for this network.

It will be simply diagonal matrix and there are only 3 elements for element 1 that is 0.8 so 0.8 other side it will be diagonal. For element 2 it will be 4 ohms, so 4 0 0 and this will be 0 0 2 this is the z matrix obtain. Now, I have to write is and vs, now there is no current source in any of the branches. So vs first let me write it vs is equal to a column vector element 1, is there a voltage source? No 0. For this element, there is a voltage source present should it be + 12 or - 12, it will be +12 why this element arrow is this way and battery is connected plus to this side.

So and in the third element there is another source that is also +10 is that the thing. The matrix is that is current source present? No, so 0 0 0 that will be the thing. Now I know what I have to do? I have to calculate then keep this equation. Let me write so it will be B here B into z into B transpose into loop current fundamental loop current i_L and this is equal to $-B z e$ or z whatever you write, you write. That is a diagonal matrix z into is source current - B into vs.

Now this B is known and z is known therefore I will be able to calculate $B z$ into B transpose I will be able to calculated that. Which will be let me write down a bit about this matrix B is already there so B so this calculation means B that is -1 1 0 -1 0 1 B matrix z matrix is this one 0.8 0 0 0 4 0 0 0 2 into into B transpose so B is known its first row become first column so - 110 and - 101. If you calculate this then the z matrix hopefully it has been correctly calculated it will become equal to 4.8. into 0.8 into 0.8 I mean the second row into 2.8 this will be this calculation.

Here then I will say that i_L will be equal to then we B , z you have to calculate B into Z . And B into vs you have to calculate vs is known. And is is also known is term will be there in fact this term will become zero in this particular case. So, it will be simply this matrix that is this equation am coming it is 4.8. 2.8 0.8 0.8 into the loop currents loop currents that will be equal to B into vs - B into vs. So B was this so minus of B into vs will be -1 1 0 -1 0 1 into that vs 0 12 10 this is thing, got the point.

And this will then become equal to on the right hand side we will get 12 here and negative so -12 and -10 therefore I_{iL} the loop currents will be this matrix $\begin{bmatrix} 4.8 & 2.8 & 0.8 \\ 2.8 & 4.8 & 0.8 \\ 0.8 & 0.8 & 4.8 \end{bmatrix}$ inverse of these multiply both sides into this one - 12 - 10. And if you do that, it will come out to be -2, -3 ampere done. Therefore loop currents are known therefore I will be able to calculate the element currents. Now one my wonder it is such a simple circuit, should I do cut set? Not necessary by some our simple our traditional nodal method or any other mesh analysis you can easily solve this.

But here it is to highlight what exactly you have to done, you know now that is complicated network formation of B Matrix have to do once which we can only entries + 1 – 1 directly do that. Then get the impedance matrix is also easy to obtain see this elements source voltage and source current column vector you can easily write. Let it be very big networks. Do not worry about that. Calculate this and these calculations whatever B transpose B z B transpose. So, at the initial stage of your programming you must give biometrics, source currents and source voltage.

Then you ask it as we computed to calculate B z B transpose all the numerical values are known. Inverse you write some programming in Matlab it is readily available and then you get the loop current. After you get the look current, you know the branch current. I will stop here in the next class I will show to more examples. Thank you.