

**Network Analysis**  
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**Lecture – 68**  
**Circuit Analysis with Dependent Sources - I**

Welcome to lecture number 68 and today we will start some new topics and if you recall that some previous few lectures we have discussed about the Graph theory applied to network analysis. I have only considered I have not really discussed about graph theory in grid detail. It is a different subject in its own right? But just the relevant portions which are necessary to solve network problems that I have discussed you must and there are various properties of A matrix, B matrix this that we are not going to discuss in the network analysis problems.

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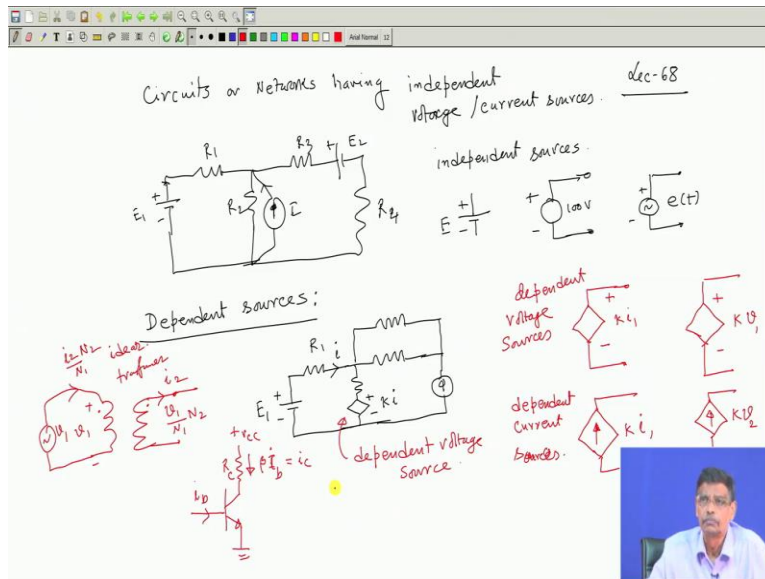
- Circuit Diagram:** A circuit with nodes a, b, c, and 0. A 5Ω resistor is between a and b. A 2Ω resistor is between b and c. A 0.5Ω resistor is between a and c. A 2Ω resistor is between a and 0. A 4A current source is between b and 0. A 4Ω resistor is between b and 0. A 10V voltage source is between c and 0. A 1Ω resistor is between c and 0.
- Graph:** A graph with nodes a, b, c, and 0. Edges are labeled 1 through 5. Edge 1 is between a and b, edge 2 between b and c, edge 3 between a and c, edge 4 between a and 0, and edge 5 between b and 0.
- Equations:**
  - $[Q][Y][Q]^T[V_n] = [Q][i_s] + [Q][y][V_s]$
  - $[Q] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} cs-1 \\ cs-2 \\ cs-3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \end{matrix}$
  - $[i_s] = \begin{bmatrix} 0 \\ -4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
  - $[Q]^T[V_n] = [V_e]$
  - $[V_s] = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ +8 \\ 0 \end{bmatrix}$
  - $[Y] = \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/5 \end{bmatrix}$

But 1 good thing is that in graph theory what happens this group theory is looking at the graph of a network and the deduced graph you can easily identify this can be associated with currents and these are the node voltages things like that. And there may be awkward situations, for example in this case y matrix is like this 1 by 2 things like that. It may so happened between 2 nodes there is no impedance connected. Suppose an ideal battery is connected across a and b Then the value of zy will be infinitely large.

In such cases ok that y matrix of the Nodal analysis, I can avoid and I can apply the Z matrix. Z Matrix will be finite Z is equal to 0. If there is an ideal battery across ab. Similarly there may be a current source only connected between 2 nodes in that case Z matrix will have infinite value. And you cannot proceed further in such cases nodal approach will be appropriate. There are techniques you can; there are voltage source ideal voltage present between 2 nodes that can be transferred to other nodes and try to solve the circuit.

Those are the things if time permits I will discuss one day but that is not the big issue. Big issue is you one of the approaches you can adopt after all for the larger network you are going to do it. You can write your codes the formation of Z Q or A matrix is very simple. That is what you have to do and enter those data then solves this equation or another case equations involving loop impedance matrix are nodal impedance matrix. Anyway you solve problems small problems on that.

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Now today, I will be telling you ok there may be circuit still now we have considered circuits or networks having independent voltage current sources that is any source present there independent what do I mean by this term independent it means that whatever parameters are connected in the network RLC this that the terminal voltage if it is an ideal voltage source is independent. What you are connecting across it that will remain independent.

Similarly whatever current sources are connected in the circuit if it is an ideal current source the current in that branch is independent of what you are connecting in other branches. It does not depend upon that, parameters which we have connected to form the network apart from the sources. So that is why do sources are called independent voltage or current sources. This is in fact, we have done that but there may be circuits.

Where the for example if I draw it you will understand better what I mean to say, I have a network like this, there is a current source, there is another voltage source like that. Ok, so this, this these are the symbols for independent sources say  $E_1$   $E_2$  or this is some current source. This parameter values are known. Now what I am telling is no matter what is the value of  $R_1$   $R_2$   $R_3$  mind you I am writing resistances in S domain you can write it  $C_1S$   $C_2S$   $R_1S$   $Z_1S$   $Z_2S$   $3SZ_2s$  I think you know this now.

Now also this source  $E_1$  is an independent source because no matter what is the value of  $R_1$   $R_2$   $R_3$  this voltage between these 2 points is going to be  $E_1$  always. Similarly this current in this path will remain  $i$ , no matter what is the value of  $R_1$   $R_2$   $R_3$  of  $Z_1$   $Z_2$   $Z_3$  that is why they are called independent. So, independent sources are generally drawn in the circuit with this independent sources can be AC DC what not. There can be if it is a small battery it will be shown like this  $E$  independent source or big DC supply that is generally shown by a circle and this I say 100 volt DC is it not.

These are the representation of independent sources and then there maybe AC voltages. These are the representation of independent sources. Now the question is what are than dependent sources? To tell a verbally a dependent sources in a network which by tab several branches several nodes etcetera. If suppose I say that in one of the branch there is a voltage source whose magnitude depends upon current in some other branch. Ok similarly current in one branch may depend upon the voltage in some other branch then those sources will be dependent sources. Let us take a simple example to explain the idea.

Suppose, you have a network like this, this is the symbol for independent source. This is suppose  $R_1$  and this is suppose  $R_2$  this is another source here there is maybe a current source there. These

are independent sources, but I may say that current in this branch, this one what I will do is this. This current whatever is the magnitude whenever it flows there will appear a voltage in this branch with this polarity the value of which is  $K$  into  $i$ .

$K$  is a non constant maybe  $2i$  got the point. Therefore this; what is a dependent voltage source, the unit of  $K$  will be volt per ampere so that this becomes volt. So, a dependent voltage source may will be represented all dependent sources will be represented by a diamond like this and the voltage across this one will depend may depend upon current in some other branch of the circuit.

For example  $Ki_1$  what is  $i_1$ ,  $i_1$  is current in some other branch a definite other branch of the circuit when  $i_1$  want flows between these 2 points a voltage  $ki_1$  will appear or another dependent voltage source may be plus minus the voltage between these 2 points will depend upon the voltage across some other branch  $V_1$  may be, so in that case  $K$  will be unit less and what is  $V_1$ ?  $V_1$  is the voltage in some other branch number one whatever it is that will be very clearly specified.

So this dependent voltage will be marked with polarity with the diamond shaped thing like this and the relationship that is what is the magnitude of the voltage will be also clearly written so that you know what it is. On which so naturally the voltage between these 2 points will depend upon voltage in some branch which is  $V_1$ . It simply tells that. So, similarly you can have a current source, current dependent source which will be also represented by Diamond shaped like this, but this time the polarity instead of that the direction of the current will be mentioned.

So these are dependent voltage source. Representation I am talking about dependent voltage source, similarly dependent current sources will be represented like this. So magnitude of this current may depend upon some current in other branch in that case  $K$  will be dimensionless or dependent current source here, the magnitude of this current with the deduction specified as we have specified field is the voltage polarity.

It may also depend upon some voltage in some other circuit and other branch understood. So, some there may be some sources which is not a; neither battery or a current source as we have

dealt with earlier and those are called independent sources. There now in some branch this voltage or current sources if they are drawn. It may depend upon some value of current in some other branch or value of voltage in some other branch, but clearly this polarities will be mentioned in things that.

For example now where why do they come into? Why we are interested to know how to solve circuit which are having both independent and dependent sources, what is the reason for that. The reason is there are situations for example in a mutually coupled coil circuit, if at anytime this voltage is  $V_1$ , if it is transformer the voltage between these 2 point will be  $V_1$  by  $N_1$  into  $N_2$ ,  $N_1$  by  $N_2$  is a constant. Therefore here suppose connected voltage source which is  $V_1$  that is why this voltages become  $V_1$  then the potential between these 2 points gets fixed because of this thing say transformer ideal transformer.

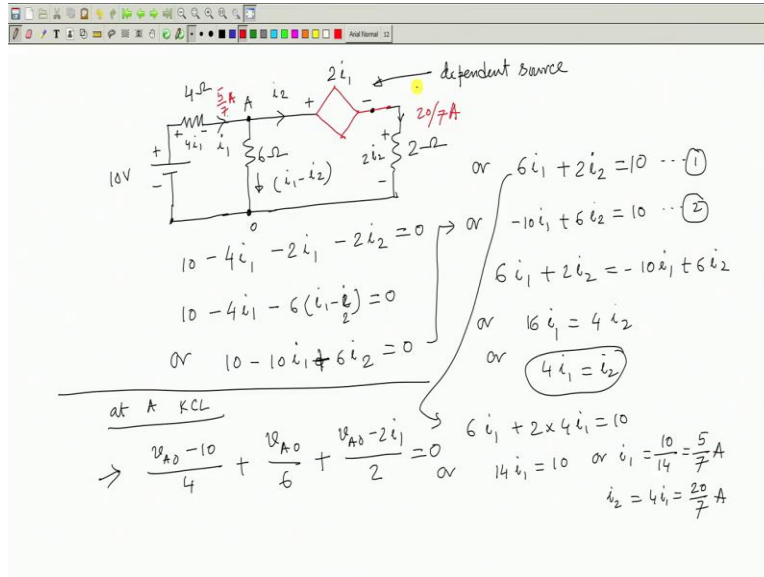
Similarly when a current is delivered here  $i_2$  the current in this branch drawn from the supply gets fixed by this current  $i_2$  which will be equal to  $i_2$  by  $N_1$  into  $N_2$  got the point. So here we can assume that there is a current source in series with this. So this is for mutually coupled coils. Similarly there may be situation for example a transistor common emitter transistor operating in active region. The collector current is beta times the base current is it not like this, for example, you have a circuit like this.

This base current beta and this collector current here plus BCCA etcetera this current is beta into  $i_V$ . If the transistor is in the active region and magnitude of the current is not dependent upon what is the value of  $RC$ . So, current in this branch is fixed by base current beta times that beta is the transistor constant, that is amplification factor of the between collector current and base current. So, collector current  $i_c$  will be beta time that there may be situations like that.

We are not discussing the circuits but in general what will do you tell you this is this that okay in a circuit if a circuit is given with both dependent sources and independent sources present then how to analyse the circuits that will that way I will introduce to you.

And nothing is better than I take an example for because we know already how to analyse a circuit problem. In the same way we will go write down KVL, KCL or whatever method may be Thevenin's theorem to find out current in various branches on nodal method.

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So here it will be; so I will explain this whole thing by considering a network problem where both dependent and independent sources at present and would like to find out the current in the circuit? So let us take one example, for example I take this simple network, which is suppose if I give you a network like this here is a resistance. Here is a voltage source 10 volt. Is this a dependent or independent source? It is independent source, is the how it is specified, this is opposite 6 ohm resistance connected 6 ohm.

And here I show it with the different colour. Here it is shown like this ok and then you have a resistance connected here say 2 ohm. So, 2 ohm 6 ohm and this is 4 ohm given and it is given to be a voltage source. So this is dependent source this one who tells me that this symbol dependent source and on what thing the magnitude of the it is a voltage or current source? It is a voltage dependent source. It will be a voltage source dependent depends on voltage in other parts of the circuit.

Suppose it is given that whenever this current is i<sub>1</sub> the magnitude of the voltage is 2i<sub>1</sub> here. So this is well defined circuit. Now suppose I say that I want to find out the current in various

branches of the circuit so there is a dependent voltage source that is an independent voltage source like that. Now how to find out the currents these calculations will be pretty simple in various ways you can do because I know how to solve a network.

I will treat this while writing down the equations as a voltage source of magnitude,  $2y_1$  and so on. Now the point is how to solve it. There are various ways of solving this network and first letter C that in this network in this outer loop there is only one unknown  $i_1$  I have to form one equation involving  $i_1$  if I can do that my problem is solved. So, considered this loop and write down the try to write down the KVL in that equation got the point.

So, how to do it? Let us assume this current to be  $i_2$  if this is  $i_2$  the current in this branch will be  $i_1 - i_2$ . So what I will do I have to actually 2 unknowns are there  $i_1$  and  $i_2$ . So, I take this outer loop or any loop you take outer loop suppose I take. Then I start from this point from this to this what will be the voltage drop here  $+4i_1$  so 10 volt from this to this 10 volt then minus if any mistake point out  $-4i_1$  then I reach this point and then from this to this it is a voltage source with this polarity  $-2i_1$  I reached this point and the voltage between these 2 point is  $2i_2$  is it not.

With this polarity so  $-2i_2$  And I have come back to this point no drop here. So that is equal to 0. So one equation is therefore  $6i_1$  take everything on the right side  $+2i_2 + 2i_2$  and it is equal to 0, is equal to 10. This is equation 1. My goal is to solve for all the current in various branches of the network. So that is the first equation, second equation is I would like to develop another equation which I can do that any loop this loop will also do.

So I write down the KVL equation in this loop. What will be the KVL equation in this loop so this will be once again 10 volt  $-4i_1$  then I come here this will be  $-6i_1 - 2i_2$  and you reach this point and this is equal to 0 is this is equal to 0,  $i_1 - i_2$  so this will 0, which is equal to or this one is  $10 - 10i_1 + 6i_2 - + 6i_2$  is equal to 0 or you get  $-10i_1$ . This equation I am writing here  $-10i_1 + 6i_2$  is also 10 this is equation 2 is it not.

Therefore I will be able to solve these 2 equations so able to solve this equation 2 you will get the relationship  $6i_1 + 2i_2$  is equal to  $-10i_1 + 6i_2$  or  $16i_1$  is equal to  $4i_2$  to you take to this side,

so therefore  $4 i_1$  is equal to  $i_2$ . Then use any of these equations, for example,  $i_2$  you substitute so you say  $6 i_1 + 2 i_2$  into  $i_2$  means  $4 i_1$  is equal to 10 or you will be getting  $6 + 8$  is 14  $i_1$  is equal to or  $i_1$  is equal to 10 by 14 is equal to 5 by ampere.

If  $i_1$  is this then  $i_2$  is equal to  $4 i_1$  and that will be 20 by 7 ampere. So, I have been able to solve the circuit all the branch current etc. Did I apply any new theory? No. I applied simple KVL, KCL and so on. If I wish I could also apply Nodal analysis, for example. I said that this point is O this point is A then also you can write at A KCL will be  $V_{A0}$  potential of A with respect to all who is reference point  $- 10$  divided by  $4 + V_{A0}$  divided by 6 is this one and plus current in this branch.

This current will be potential of this point will be  $V_{A0}$  then to potential of this point with respect to O will be  $V_{A0} - 2 i_1$  and then that voltage appears across 2 that is 2 this is equal to 0. This is one equation and the other equation is  $V_{A0}$  and  $i_1$  you write some loop questions for example  $V_{A0}$  and  $i_1$  can be related here. And similarly the other equation you write down KCL and you see that  $V_{A0}$  by 6 is nothing but  $i_1 - i_2$  and so on.

So, same result will get at the end but what I am telling you can apply Nodal analysis, you complete this problem with this method got the point. And you will get the same result. So what is the current in this branch? Suppose this is the load resistance. This current is  $i_2$  and  $i_2$  is 20 by 7 ampere you note it down 20 by 7 ampere will be flowing here. Then you have solved for  $i_1$  also that is 5 by 7 ampere. And then  $i_1 - i_2$  I can calculate and get this current. Is it clear.

5 by 7 ampere 20 by 7 ampere is there. So, you can solve this state by any method you like. Now, so here is a voltage source which is independent and there is another voltage source which is dependent and it is telling that the magnitude of the voltage with this polarity across this whenever I want flow this magnitude get decided with this polarity plus minus, that is all. Otherwise nothing very complicated situation like that.

So it is better you only I will give you some circuits in my next lecture and tell you how to solve those network problems having both dependent as well as independent source. Thank you.