Network Analysis
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Lecture 71
Two Port Network - I

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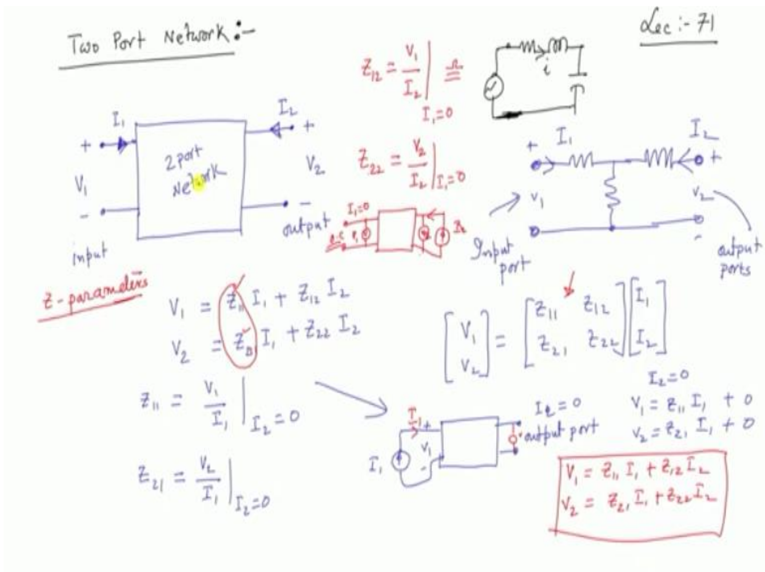

So in this lecture, we will start a sort of new topic, which is called two port network. Now any network, for example, RLC network, any network. It could have other combination of impedances, just to give you the idea. You have to excite this network with some input signal and suppose, I am interested to know what is the current. Then, this network, these are the two input terminals and you excite it with current or voltage sources.

And find out the desired output, either current or voltage across the inductance, what not. Now this sort of network is one port network. You excite it and maybe your current is the output. But what is a two port network? A two port network will, as a black box, first I draw, will be a network where two terminals will be available for input terminals. Across these two terminals, you connect the input, voltage source or current source and across these two terminals, you get the output.

And often this input and output, this thing will be shorted. For example, a network like this. I declared that these two are my input port and these are the output port and there are impedances in between. No sources should be present in this. Often this is the case. For example, in a transmission line, you excite the transmission line from one end, there will be network impedances, line impedances combing in between.

And at the output, you receive the voltage, with respect to same common point here. So first, let us try to analyse a two port network from this perspective. That is, there will be a distinct input port and output port and the convention is very important. What they say is this, that in this input port, the voltage across this is, I will always write with this polarity, plus minus and current drawn by the circuit network, so two port network, is I1.

This is the input port and these two are output ports, where the voltage of the output, I will always mention with V2 and each current direction, I will specify it like this, because I told you several times that detection of current in which direction it flows, I will assume it. So it is the convention always, in a two port network, except ABCD parameter network, that will come later to assume the direction of voltage, polarity of the voltage and the current in the input and output port in this way.

This is how, we will specify, because we are now matured enough to understand the direction of the current. Otherwise one can assume in anyway one likes and also remember, although I am writing V1, I1, V2, I2, there may be V2S, V1S, I1S, I2S. This S, I will not continue to write and the impedances here can be resistances or inside these two port network, several impedances will be connected, there may be Z1S, Z2S, etc.

So that, we will not go on writing S. In general, that is true, because in s domain, if you write, you will get the response of the network for a given input both for transient and steady state things like that. But anyway, first thing is, this is how the input terminals, what is the thing, inside there should not be any source. All elements are linear elements inside these two port network, okay. Now what people do is this.

They try to relate this V1I1 and V2I2 in some algebraic form of equations. For example, I will say that how the voltages V1 can it be expressed in terms of the currents. That is Z11I1 plus Z12I2. Similarly, V2, I will say it is equal to Z21I1 plus Z22I2. See as a matter of fact, this two port network, only thing I am told, they consist of linear elements and have no source inside and there may be a common point. This is V1 and this is V2 like that.

With respect to this point, this voltages are measured and this is I1 and somebody is telling also assume the current to be I2 in this way and we try to relate the input voltage and current with output voltage and current. So as a black box, if it comes, what I am telling is this, it is possible to relate the input and output voltages V1 and V2. I can express them in terms of currents of input port current and output port current.

And this can be written also in metrics form like this, is not? So this is one way of connecting the voltages and currents of input and output ports. There will be several other ways. Why I have written Z here, because it looks like left hand side dimension will be in volts, I is ampere, so it must be known. These coefficients of I1, I2 must be known. So that is why I have written Z, impedance, so that you know the same unit it must leave of the left hand side and right hand side.

So if a network is given in the form of a black box, I do not know Z11, Z12, Z21 and Z22, but I know perhaps they can be expressed, these voltages V1 and V2 in this form, then the question is how to get these values of these impedances. How much is Z11, how to model these two port network in this fashion. That is in that case, I must tell that Z11, Z12, Z21, and Z22 must be known, then only useful utilizations of these two equations can be carried on, otherwise how.

So the question is how to find out Z11. By doing simple test of a given two-port network, the inside of which you are not allowed to see can be done in this way. Let us see. For example, Z11, I will go slowly, so that you really understand this. I can write it as V1 by I1 with I2 equal to 0, is not. What does this mean? Take this network, excite the input with a current source, keep this, I2 equal to 0 means what? Output port open circuit, let me write, output port.

Do not connect any impedance, so this current I2 will be 0. You will be maintaining that, such that I2 is 0 and excite this with a current source I1. This is the input port and measure the voltage existing between these two points, that is V1 as per this. Then ratio of V1 by I1 with I2 equal to 0 is Z11 that is the first equation I have utilized with I2 equal to 0. What other parameters can be found out from this, with I2 equal to 0, Z21 can be found out.

Z21, this first column elements Z11 and Z21 can be found out with making I2 equal to 0 and Z11 will be equal to V2 by I1 with I2 equal to 0. That is, if you excite the input port with a current source I1 and these two output ports are kept open circuited, which will ensure I2 equal to 0. Therefore, the thing will be V11 with I2 equal to 0, what are these equations; V1 is equal to Z11I1 and V2 is equal to Z21 into I1. This is the thing. This is 0.

Therefore, I say that Z11 you know, it is V1 by I1 with I2 equal to 0 and Z21 is V2 by I1 with I2 equal to 0, got the point? Therefore, although a two port given network, these parameters can be found out by doing simple tests, excite this with a current source, note down this voltage. Now you can easily see that how to find out Z12 and Z22. Is that clear? These two elements we have found out. How we have found out?

I have told you, excite this with a current source, get this, measure this. So to calculate Z21 you have to measure these voltages with a voltmeter V2, open circuit voltage V2 by input current I1. I1 is known. So this is how, Z11 and Z21 can be measured. It is very easy to see, how to calculate this, I will write with red ink. Z12 should be what? Come to this equation, Z12, I can write V1 by I2 with I1 equal to 0, is not?

And this element also Z22 should be V2 by I2 with I1 equal to 0. So which experiment I have to perform these two parameter values in ohms. So this experiment I did to find out Z11 and Z21. Now I say that I have to also calculate Z12 and Z22, then these two ports network description will be complete and I find that Z12 will be V1 by I2, provided I1 is 0. Similarly, Z22 will be V2 by I2 with I1 equal to 0. So next I do this experiment, what is that?

This below quantity is I2, so you excite this network with a current source from the output port I2 in this direction and keep the primary open circuit, that is input port, I am sorry, input port open circuit, so that which will ensure I1 equal to 0, open circuit and measure this voltage, that is V1. There will be voltage existing, although it is open circuit and I say that, okay then you Z12 will be this voltmeter reading V1 divided by the current with which you have excited the output port with I1 equal to 0.

That is this side is open circuited, excite this with a current source and what will be Z22? Z22 will be with primary open circuited, I1 equal to 0, you must specify. This Z22 is V2 by I2 with I1 equal to 0. So that is input port is open circuited. I have excited with I2. Then, I will measure this voltage as well. This voltage will give you what? V2. So this voltmeter reading divided by this I2 will give you Z22 with I1 equal to 0.

So, all the four parameters can be obtained, provided this network is a good candidate, so that it can be represented as a two port network. What should be values of Z11, Z12, Z21, and Z22 by doing simple experiments, open circuit, keeping this port open, exciting this port with a current source, you get two parameters. Similarly, excite from this site with a current source I2, measure the voltages of input and output port, you can get all the parameters and then you will be happy.

Then you say that V1 in general is Z11I1 plus Z12I2, Z21I1 plus Z22I2, all the parameters are known and as you can see the dimension of these quantities are known. So this matrix, the parameter value, it is called the Z parameter. So this two port network, this popular method of representing network like this are called Z parameters of the network. Z parameters are nothing but open circuit condition parameters.

While finding out this parameter, you have to keep the other port open, then only you can get these values, as I have elaborated here. Is that clear? So in a two port network, when the voltages are expressed in terms of currents, you will be getting what is known as open circuit Z parameters. Why is that open circuit business comes in? Because to measure those parameters, you have to keep other port open and excite the other port with current sources.

That is all, and measure the voltage and current. So this is how a two port network can be described. That is I rewrite here V11. Now after knowing those things, I will use it. Now I will say that okay, I have found out these parameters. Then you know, your input voltage, input currents are related by these algebraic equations. Mind you, these are algebraic equations. If it is in S domain, V1S, V2S and so on, everything will be in Laplace domain then.

Then also that is true always. So this is the Z parameters of a two port network. Okay, now there is another way. There is no restriction. Somebody says that, I want to express for this two port network.

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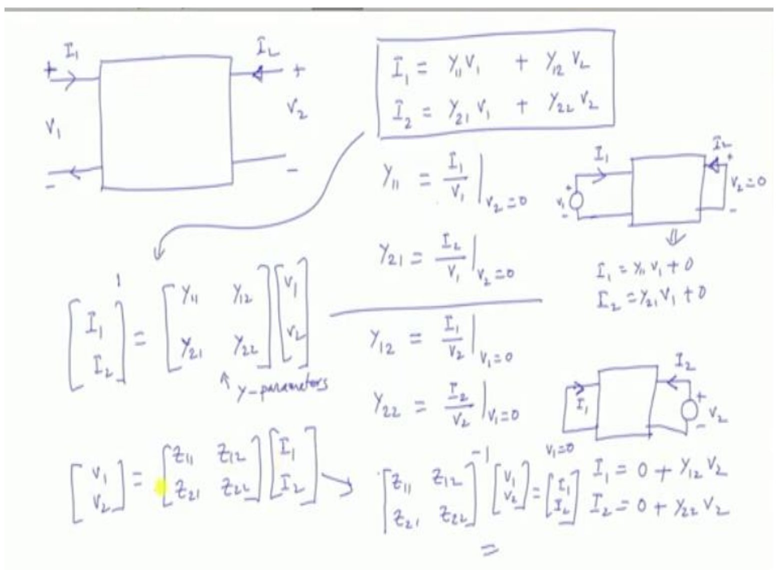

So two port network remains same, same network, two port network and as I told you, I will always express the terminal voltage and current with this polarity. This is the convention. With this I am doing. Same two port network is it possible to express current in terms of voltages. For example, I will say that I would like to express I1 with some constant into V1 plus another constant into V2. That is the input current is linearly related with input and output voltages.

And obviously, if you do that the dimension on this should be admittance. What is the dimension of Y11? This side it is ampere, this side it is volt. Therefore, these coefficients must have a dimension of mho, that is 1 over ohm and this quantity I will write it as this. Similarly, I2 perhaps can be expressed in this form, Y21V1 plus Y22V2. Therefore, in these expressions, same two port network I am talking, one could also say that relate the input and output currents with the input port voltage and output port voltage.

Now once again, what are the values of Y11, Y12, Y21 and Y22. You can easily see. Y11 will be equal to I1 by V1 provided V2 is equal to 0, is it not? So suppose, I will draw the network, now like this. This two port network, what it says is that V2 is 0 means, you short circuit this. Then only, V2 will be equal to 0, is not? With V2 is equal to 0, short circuit, you excite the input port with a voltage, voltage is below.

So excite this input port with a voltage V1 and see how much current it is drawing I1 and you know Y11, then will be the ratio of this current. So connect an ammeter or whatever it is, measuring the current. So I1 by V1 is to be the admittance Y11, which other parameter can be found out by doing this experiment. V2 is equal to 0, that means, these two terms will be absent. So I can find out the first column terms Y21, which will be equal to I2 by V1 with, do not forget to write V2 is equal to 0.

That is you short circuit the output port, excite these with a voltage source. Whether I should excite it with a voltage or current source depends upon which quantity comes below. In the previous example, it came that Z12 for example V1 by I2. So I say, okay I will excite the other network with a current I2, like that. So I excite this with V1 volt, keep this V2 is equal to 0, then what will be the relations.

Now I1 is equal to Y11, V1 plus 0 under this condition for this network. Same two port network and I2 will be equal to Y21 and V1 plus 0, because V2 is 0. Therefore, ratio of I1 by V1 with V2 is equal to 0 and Y21 will be I2 by V1 with V2 is equal to 0. These two parameters you will get. These are called admittance parameters, okay. Similarly, you now know better than me, perhaps if you are following me truly, then I will also say this is okay. What about Y12?

How to find out this parameter? Y12 can be written as I1 by V2. It will be indeed I1 by V2, provided V1 is equal to 0 and Y22 will be I2 by V2 with V1 equal to 0. So which experiments will give you these parameters. You have to then think that okay, this is two port network. V2 is below, so I will excite this output port with some known voltage. So V2 I will connect and V1 equal to 0. V1 I will short, V1 equal to 0, I will short.

And then, this current, please forgive me, I forgot to mention this direction of the current, but it remains same. This Y21 is this current you must find out I2 by this V1. I think I just missed this I2, because the direction of the current is what people all over the world has decided to be like that to describe a two port network. I mean no point in fighting with the fact that why not I2 you assume like this and we know in a network detection of currents or polarity of the voltage, it is up to you to choose.

So anyway, we will follow this one where worldwide people follow it. So I2 is this way. So you measure this current, so I1 by V1, etc. So in this case also, you show these currents whenever you show it is like this. So what is Y12? So in this way, if you connect the circuit, what this equations gives. This equation will give I1. Since V1 equal to 0, this will be 0, 0 into Y11 plus Y12 into V2 and I2. Since V1 is equal to 0, 0 plus Y22 into V2.

So with this input port shorted, apply a known voltage, measure this current and your Y12 will be the ratio of this current with this voltage, applied voltage. Y22 will be this current I2 divided by this voltage and therefore all the four admittance parameters can be found out. Is that clear? So this is the thing and this obviously can be written in this form, in a matrix form, in a compact matrix form like this.

That is Y11V1 plus Y12V2 divided by, oh sorry, Y plus Y11, Y12, Y21, Y22 and V1 V2. That is the same network, which we found out Z parameters can be also expressed in terms of admittance parameters like this. Earlier, these are called Y parameters and how to know the Y parameters? You have to do some short circuit. So sometimes, they are called short circuit Y parameters and for impedance, earlier we have got voltage, you try to express in terms of impedances and this will be I1 I2, is it not, 2 by 2 matrix.

And it looks like there will be a relationship between these two, why not? Because after all, you see from this equation, this is the Z matrix. This is true, know? Multiply with Z inverse, Z12, Z21 and Z22 inverse, multiply with the inverse of the matrix, premultiply both the sides. This into V1V2, this will then become an identity matrix, is it not? On both sides, you multiply with the inverse, this one into this. So inverse into the matrix itself is an identity matrix.

And this will be equal to simply I1 I2. So it looks like this Y parameters and Z parameters, if you have found out the impedance parameters, Z parameters, Y parameters. For that not extra test needs to be done, but its inverse has to be found out, but we will see later all the time, this inverse may not exist. Those things will come later, but today therefore, what I have told is that any given two port network, the idea is first this convention.

There will be input port, where voltage and current, voltage polarity and current deductions should be specified like this V1, I1 and there will be a definite output port terminals where V2 and I2 will be also specified and the directions of the currents and polarity of the voltage, I will always stick to this and then I told that okay this two port network, whatever may be inside, maybe inside means, there should not be any source, that is for certain.

Only impedances are present, then it is possible to express input voltage in terms of, I mean, input and output port voltages as a function of input port and output port currents or vice versa, input and output port currents can also be expressed in terms of voltages. There are other ways, for example, H parameter, hybrid parameters also is possible, okay and those things we will discuss in the next class. Thank you.