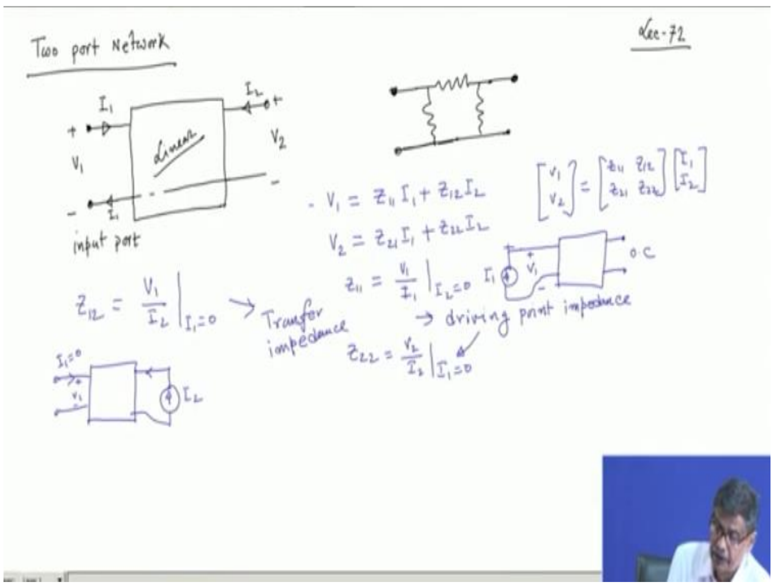
Network Analysis
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 72
Two Port Network - II

(Refer Slide Time: 00:27)



Welcome to the lecture on two port networks, which we have started in lecture number 71 and last time what I told you is this that a network, which is inside this box; details we are not drawing what is there inside, but it is a linear network and then you can identify terminals where you will give input signal and the current drawn by the network is shown like this and then these two terminals together is called the input port. Very quickly, I will recall input port.

So a pair of terminal is needed to define a port and it will qualify from being called a port is that the current supplied and current is written same. Similarly, here it is output port where the output voltage will be V2 and the input current is I2. This is the conventions, the way direction of currents are mentioned. So this is a linear network, okay. Now the kind of network maybe it is like this.

Suppose we have a network and this can be also Z impedances in terms of S, as I was telling you later. So perhaps these two are output port and these two are input port and if you see this type of configuration, you can easily see that there are actually three terminals, 1, 2 and all these points are together. So this is one terminal. So 3 terminal 2 port network some people may call it. Of course there may exist a 2 port network where the port terminals are not connected.

We will discuss about that a bit later. For example, a transformer, primary winding has two terminals, secondary winding has two terminals and so on, but anyway for our purpose, so it is a three terminal. Why 3 terminal? This is one, this is one and this one is running through inside like this and with respect to this, I will calculate all the voltages potential of this point V1 and V2 and then we can write down input voltage and output voltage as a function of the input and output port currents.

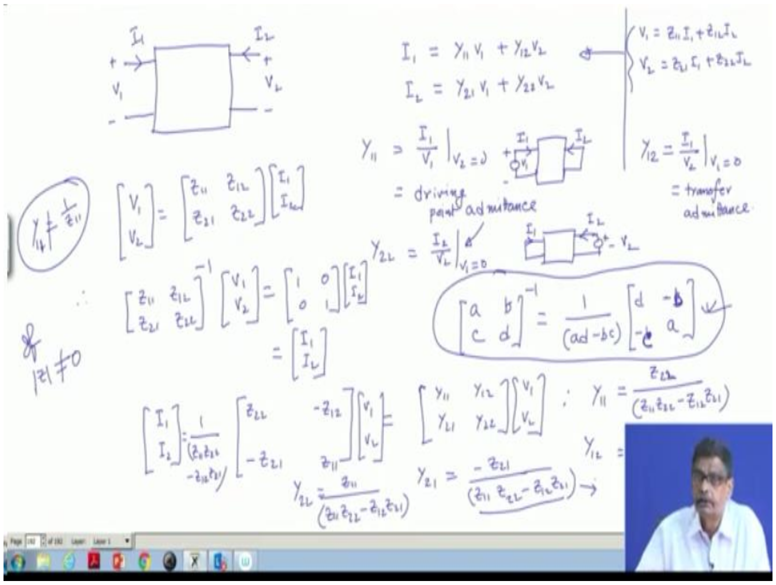
And we told you that this is Z12I2 and Z21I1 plus Z22I2, is it not? Then it is called Z matrix of the ABCD and it can be written in matrix form V1 V2 is equal to this one Z12, Z21, Z22 and this is I1 I2 and in my last class, I told you how to calculate Z11. For example, Z11 is only one, I will repeat, that is equal to V1 by I1 with I2 equal to 0. What this means? This means that, to this network if you apply a voltage here, if you apply a current source here, below it is current.

So I1 you apply to port 1 and port 2, I2 is 0 means, this is open circuit and measure what is the voltage now comes here. So excite it with a current source I1 measure V1 and take the ratio V1 by I1, which will be giving you Z11 and this impedance the dimension of this quantity is ohm. And this impedance is called driving point impedance. Similarly, Z22 will be V2 by I2 with I1 one equal to 0. That is excite the other port with a current source I2.

Find out what is the voltage between these two points with this open and this is also another driving point impedance and other impedances, that is Z12 is V1 by I2, is not, with from this Z12 is V1 by I2 with I1 equal to 0. That means open circuit, this input site I1 should be 0, I1 is 0 that will be obtained if these two are open circuited and on this side, you excite it with a current source I2 and measure this voltage.

Some voltage will appear here, although current is 0 between these two terminals. So Z12 is V1 by I2 and this can be termed as a transfer impedance. Its dimension will be ohm, transfer impedance and so on. Similarly, Z21 will be also a transfer impedance. So you can understand this.

(Refer Slide Time: 07:21)



Then I told you in the last class, that this relationship, another way of writing this one code. Express the input port voltage current and output, one can relate rather, the input port voltage and current in various combinations. For example, I say that this is the thing. Suppose I wish that I will write this equation that currents I1 and I2 is equal to some Y11V1 plus Y12V2 and I2 is equal to Y21V1 plus Y22V2.

And recall that in impedance matrix we wrote these equations like this V1 equal to Z11I1 plus Z12I2 and V2 is equal to Z21I1 plus Z22I2. Now this is another way of relating the input port current voltage with output port current voltages. But what I am trying to tell this two sets; these equations are same. That is one can easily obtain from this what should be the values of Y11 Y12, is that clear?

Because of the fact that from this, we know that V1 V2 is equal to and these are called Y parameters and let me first tell you at least that admittances. These are admittance matrix and Y11 as you can see, it will be equal to I1 by V1 provided V2 equal to 0 from the first equation. What does this mean? This means that excite the network. This is the network. Excite the network with a voltage source V1, connect a voltage source V1.

And see how much is the current it draws, with secondary terminal shorted, is not? V2 is equal to 0 you have to make. Then take this ratio I1 by V1. The dimension of this quantity is reciprocal of Ohm. Therefore, it is admittance. So you call it Y11 then admittance and it should be called driving point admittance. Similarly, you can see Y22 is equal to, from the second equation it is equal to I2 by V2 with V1 equal to 0, is not?

And I2 by V2 means, you excite the second port, output port with a voltage source. Whatever it is below, with that you have to excite. So excite it with V2. This is I2 and with V1 equal to 0 means this is shorted and this is I1. Then in this experiment, you take the ratio of I2 by V2. So this is also driving point admittance of the output port. So Y11 and Y22 are the driving point admittances respectively with respect to port 1 and with respect to port 2.

And obviously, these are transfer admittances Y12, for example from this equation it will be equal to I1 by V2 with V1 equal to 0. That is from this network, you can get it. So you excite the input port with V1 voltage record this current I2. So you are taking ratio of current in some other branch with voltage applied in some other port. Therefore, this is called transfer admittance for obvious reason, transfer admittance okay.

Now the question is, what will be the relation? If I know Z11, Z12, Z21, Z22, can I also find out the admittance matrix parameters and these admittance matrix parameters can be obtained by doing this sort of experiment and under short circuit condition. So they are called short circuit admittance parameters. Similarly, impedance parameters, we have seen can be determined from some open circuit short of test, where either the output port or input port are kept open and various values of Z11, Z12, etc, are obtained accordingly.

But in any case, since V1 V2 is equal to this. In a matrix form I can write this easiest way to do is this, I1 I2. Therefore, I was telling last time that you multiply with Z inverse both sides. Z is this matrix. So you multiply with Z inverse, Z12, Z21, Z22 premultiply with Z inverse both sides of this equation. Then this Z inverse and this Z will become an identity matrix diagonal 2 by 2 it will become 1 0 0 1 and this will become I1 I2 and this is nothing but I1 and I2.

Therefore, the coefficients of this Z inverse is nothing but this Y11, etc. You know that inverse of a 2 by 2 matrix is very easy to calculate. For example, ABCD the inverse of this is divided by the determinant AD minus BC and this matrix will be DA and this will be minus C and minus B, very easy to remember, interchange these two and also interchange this two and do not interchange B and C. This will be the inverse, because cofactor of B is minus C and transpose.

So minus C will come here. So this is the thing AD minus BC it will become and this is the inverse of this matrix, very easy to calculate. Therefore, it looks like that I1 I2 will be equal to the Z inverse. What will be Z inverse? Determinant, so you divide by determinant that is 1 over Z11 Z22 minus Z12 Z21. This will be below, 1 by the determinant AD minus BC and this one will be simply interchanged, that is this will become Z22 and this will become Z11.

And about these things, it will remain only thing their sign will reverse, that is all. Therefore, I will say and I am telling that this into V1 V2 and I am telling that this is equal to Y11, Y12, Y21, Y22 and V1 V2 okay. Therefore, we can easily say the relationship between Y11 and if you know the Z matrix parameter, you can find out Y11 as Z22 divided by the determinant Z11 Z22 minus Z12 Z21, is not. This first element Y11 is Z22 by this determinant.

Similarly, Y12 will be minus Z12, this is 2, Z12 divided by this determinant Z11 Z22 minus Z12 Z21. Let me write at least for one matrix to tell you the idea. So if you know Z matrix, take its inverse. You get the Y parameter, so Y12. What will be Y21? Y21 is this one, which is equal to minus Z21 by the same determinant Z11 Z22 minus Z12 Z21 and finally Y22 will be equal to Z11 divided by Z11 Z22 minus Z12 into Z21, easily it can be related.

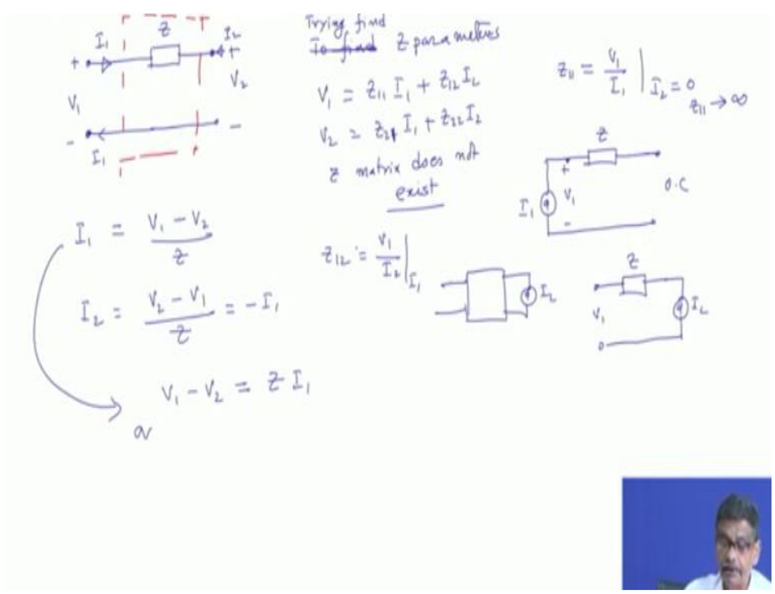
But the only thing one should remember that Y11 is not equal to 1 over Z11, no, that is the mistake. Similarly, no, this is not true Y11 is not Z11. It is not like that. So you have to take the inverse or manually you can always play with this equation to express I1 and I2, keep on the other side, bring other things there, algebraic manipulation you do, you will get the same result.

So if you know the inverse of a 2 by 2 matrix, which is very easy to calculate and one should remember that 2 by 2 matrix, the inverse is this elements gets interchanged without any sign change and these elements remain where they were, but a negative sign creasing preceding them and divided by the determinants, which is AD minus BC. So this should be at your fingertips and then you can express it.

Now the question is, is it always possible to if Z matrix exists, Y matrix should also exist? Not really, see all these things we then find is this that you are dividing with this product to calculate the different Y parameters from Z parameters, is not? If this determinant of Z matrix happens to be 0, then you are gone. I mean no Y parameters can be found out, although Z parameter exists. So provided this determinant, if the determinant of Z matrix is not equal to 0, then only we can find out the equivalent Y parameters of this two port network otherwise not.

So this is the thing, so we have done this. We will see several examples where it will not be possible and where it will be possible. Let us take a simple example, before I talk about the, what is called H parameters, same two port network can be represented by another combination of voltage and current and ABCD parameters.

(Refer Slide Time: 21:19)



Before I do that let us take a simple exercise. For example, somebody says that this two port network is as simple as this. There is a single Z in series with these two and these two are my input port and these two are my output ports. What will be the Z matrix? So as per our convention, that is this is the two port network okay and suppose I know inside it is like this and I want to find out what will be the Z parameters of this two port network and Y parameters of this two port network.

So let us try to do it plus, minus. This is V1, is not? This is the convention we had up there I. Here in the output port, the assumption is this. This is the polarity of the voltage and current enters I2 like this. Now to find out the Z parameters, we know what I have to do. One way of looking at thing, I will tell you, so that in simple logic also it can be done. For example, I want to find out, trying to find rather Z parameters of this two port network.

Okay that means V1 is equal to Z11I1 plus Z12I2 and V2 is this. This is the basic equation Z21I1 plus Z22I2. This is what I want to do. Now suppose I want to find out Z11, then what should I do? Z11 is V1 by I1 from the first equation with I2 equal to 0, is not. That is what I have to do. So what I told you, you have to excite this two port network which is like this now with a current source I1. You excite it with a current source I1 and this is Z and try to and measure this voltage.

Is not, that is what I told in the previous lecture okay. I2 0 so second port is open circuited, nothing is connected and to the input I have supplied the input port with a current source and then I will record this voltage and divide it by this current to get Z11. Now here is the trouble, you can easily see, a current source and here is an open circuit, so what will be this voltage? A current shorts as you know always try to force current impedance between these two point is infinity, because open circuited.

So this voltage will be infinity and also this voltage V1 because this voltage plus this voltage, this will be finite, I get no doubt but this is infinite. This will make it V1 also infinite and therefore, Z11 if you want to do, which you should not do, you know that a current source should not be kept open circuited, high voltage appears. So this is what he will be facing on pen and paper at least, we can easily argue that, oh, in such case Z11 we will try to go to infinity like this.

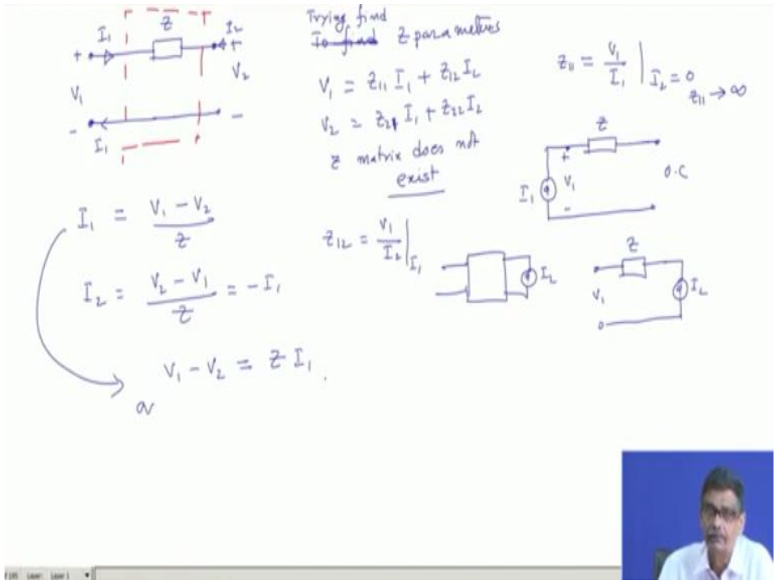
Is it not? So the elements of Z matrix becomes infinitely large means, what is the point of writing a equation with infinity as coefficient and we will say for this simple two port network Z matrix does not exist, not possible z11 is infinity. Similarly, Z12, if you want to see, what is Z12? Z12 is V1 by I2 with I1 equal to 0. So some open circuit is to be made and other port is to be fed with a current source. So in this case similar thing will happen.

If you apply a current source where I2 and open circuit, this one. That means for this network, it will be this network is known network as I told you Z inside I know what it is. So if you pass I2 current and here it is open circuit. So infinite voltage here, therefore V1 will be infinitely large. So you can easily see that impedance matrix does not exist. This is the thing. In fact, behind all these arguments in this way, you can easily see that for this network what is I1?

It is such a simple network. I1 is nothing but V1 minus V2 by Z. This is the value of I1 is not. V1 minus V2 potential of this is known as V1, potential of this with respect to this point is V2. Therefore, what is the current that I1 is from left to right? So I1 is this one and also I2 is equal to V2 minus V1 by Z, that is all this is how V and I are related. Can I express this V1 and V2? let us write. So from this we see that V1 minus V2 is equal to Z into I1. That is good.

This is the impetus Z, that is all V1 minus V2 is ZI, but I cannot separate this V2 out to write this. Therefore, it is not possible to express V1 in terms of I1 and I2 and which happens to be, this I2 happens to be equal to minus I1 for obvious reason. Therefore, here the impedance matrix does not exist. Therefore, you understand this point? Does the Y matrix, can I find out the admittance matrix of this? Let us try to do it.

(Refer Slide Time: 29:19)



So what I am telling for this network, let us try to find out, does Y parameters exist for this network, which has got a single impedance in series and these two, I am calling it as input port V1I1 and this I am calling as output port V2 and I2. This is by convention; I am telling this okay. Recall that if you want to find out the Y parameters, these are the concerned equation that is I1 is equal to Y11V1 that is the input port current and output port currents are written like this.

Y21V1 plus Y22V2, is not? This is the thing. Now the question is what is Y11? We have seen earlier and as you can see from this equation Y11 is nothing but I1 by V1 with V2 equal to 0. This is the thing you have to do, I1 driving point admittance with respect to input port. Now with V2 equal to 0, then the circuit is like this, V2 0 means you short it and here you apply a voltage V1 and the current drawn is I1 and obviously V1 is equal to I1 into this Z. Therefore, I1 by V1 is equal to Z and that is equal to Y11. So Y11 exists, no problem.

Similarly, you can find out the other Y parameters, but in this case I will do it in this fashion also I can do it, because it is such a simple circuit. For example, V1 this input voltage is nothing but V2 plus I1 into Z, this voltage drop is I1 into Z, is not. If you consider this current to exist I1 into Z V2 plus this okay. Therefore, I can manipulate this equation and write that I1 Z is equal to V1 minus V2, I can say or I will say that I1 is equal to 1 by Z into V1 minus 1 by Z into V2.

What is this Z? This Z is the original two port network. I told it is a simple network like this. So this is one equation. I have been able to express I1 as a function of I2. What do I do with I2? I can write straight away in a different way that I2 is nothing but V2 minus V1 by Z, is not? I2 is this current, so potential of this point with respect to this applying that V2 minus V1 is the voltage applied. This is plus, this is minus. So current from right to left is this one I2.

And I will say I2 is equal to then minus 1 over Z1 into V1 plus, not Z1, Z plus 1 over Z into V2. So what is the value of Y11? Y11 is nothing but 1 by Z. What is Y12? Minus 1 by Z, is not and so the Y matrix from this looks likes 1 by Z minus 1 by Z Y12 and this is minus 1 by Z once again and this is 1 by Z. So this equation can be written as I1 I2 is equal to this into V1 V2. So this must be the Y circuit parameters. So Y matrix exists.

From this also, we can conclude there cannot be a Z matrix, because of the fact the determinant of this matrix is equal to 0 1 by Z square minus 1 by Z square and that 0 is to be then divided below, this quantity will become 0 and that is why the Z parameters do not exist, but Y parameters exist. So it is not true that for any two port network, there exist all the matrices that is Z matrix will exist, Y matrix will exist and so on.

Because 2 by 2 that parameter value impedance, if its determinant is zero, it is inverse will become infinite. It will give you infinitely large elements and that is we cannot handle infinity in equations. Anyway, I will continue with this in the next class. Thank you.