Network Analysis
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Lecture – 74
Two Port Network - IV

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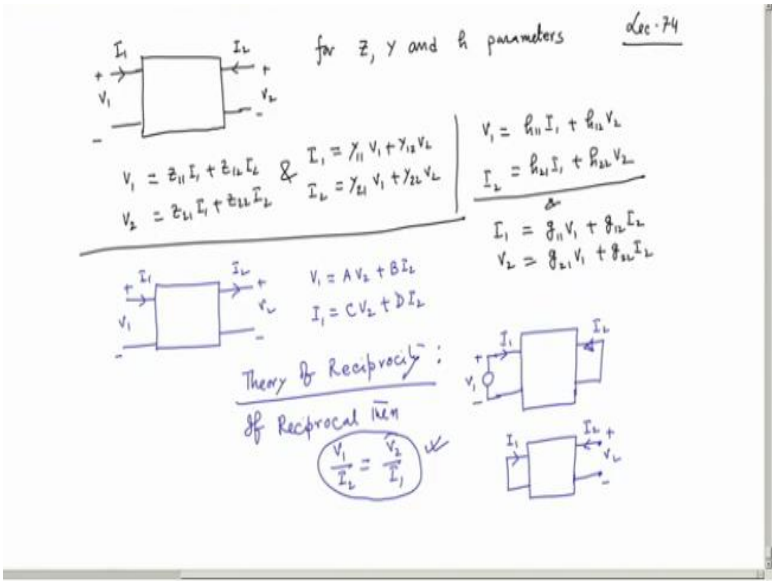

So, welcome to next lecture on 2 port networks and we have already seen that if any given 2 port network like this, we have, I think describe this network in terms of input port voltage currents and output port voltage current and this is how the convention is followed, this is the convention for z, y and h parameters. We will always assume that both the port currents are going into the network and based on that we have found out these things that is in case of z parameters, voltages are on the left hand side.

And it was like this; I1 + Z12 I2 and this is equal to Z21 I1 + Z22 I2, this is one thing and this I1 and I2 can be also represented as Y11 V1 + Y12 V2 and this is equal to Y21 V1 + Y22 V2. These 2 equations are one and the same but only thing remember that Y11 is not 1 over Z11 that we have discussed, so this is about z and y parameters and we have also discussed about h parameters.

And it is represented as V1 and I2; V1 is represented as I1 V2, so this is h1 I1 + h12 V2 and this is equal to h21 I1 + h22 V2. So, these 3 and similarly, here this I1 V2 can be expressed as g parameters like this, there is another that is and I1 V2 can be expressed as g11 V1 + g12 I2 and g21 V1 + g22 I2, if inverse exist, g will also exist of this h matrix, similarly for z and y matrix. Then, in my last class I told you that there is another parameters called ABCD parameters and here it is written like this that this given network.

But only thing is here what we will do is this, this is voltage of port 1, with this polarity, it remains same as it is and here it is voltage of the other port that polarity also remain same but only thing for I2, we will indicate it is coming out because of the nature of transmission line okay, we would like to written I2, this is how we will be written and in this case what is done is this; you can write this current entering but then you should write minus I2.

So, I will draw exactly what it means I2 and then the input voltage and input currents can be related with output voltage and output current as a V2 + VI2 and I1 is equal to CV2 + DI2, this is how and we have seen how A and B can be determined in my last class, only thing is it is 1 over A first calculate then calculate A because that will be some problem, you cannot excite a network which have voltage and still claim the current in this is 0, very difficult to keep that, okay so this is the thing.

Now, after learning and therefore, I will be able to relate A, B, C, D with z, y, h, whatever I like, we are not going to discuss. Today, first I will tell you one important property because it is a linear and bilateral networks and if it is so, then we know that theory of reciprocity will be always true; theory of reciprocity, you recall that theory of reciprocity tells us that if in a circuit, if the circuit is reciprocal, then if you apply across this port, some voltage V1.

And if you short the other terminals and the currents are like this I2, I1, you do one case like this; apply voltage, output terminal shorted and you also do this things that is you apply a voltage here V2 and this is I2 and short-circuit this, this is I1, then theory; if this network is reciprocal, then the ratio of this voltage and current and this voltage and current will be same, if reciprocal we have discussed it earlier.

Reciprocal, if reciprocal then we know that V1 by I2, this ratio should be equal to V2 by I1, it is always true, while describing intelligence theorem from that we established this, therefore this ratio will be same for a reciprocal network which demands that the network element should be linear like RLC and also they should be bilateral, RLC are bilateral things, no doubt present like that.

So, if this is V2, then this will be I1 and you see this V1 by I2, the sense of the current I1 is also same flowing this way, so this is the thing should be verified. We will now try to look at the situation that if a 2 port network is a reciprocal, then will there be some; what is the condition that the 2 port network if it is reciprocal we can see, then this must happen, that is what we are going to study.

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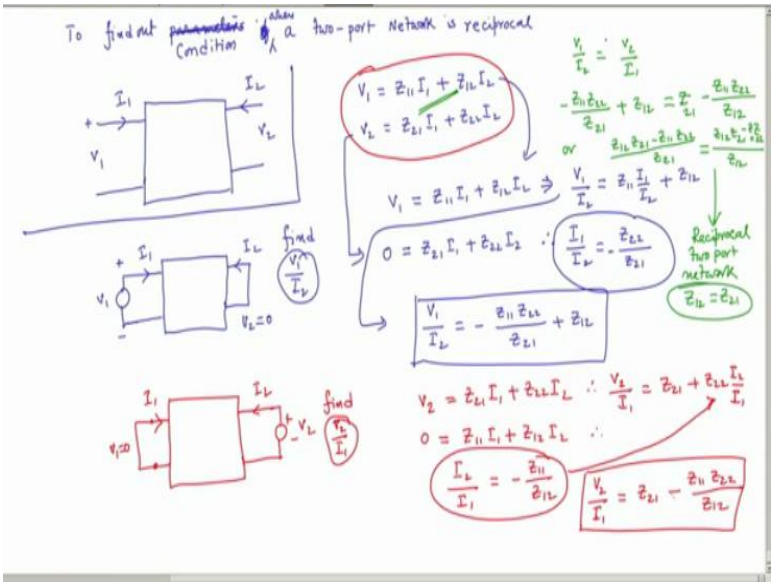

For example, that is I will write it like this; to find out parameters, if a 2 port network is reciprocal, I will better write to find out conditions, when a 2 port network is reciprocal, that way it is better to express. So, let us take with respect to z parameter, suppose this is the network, this is your V1 and this is your; first I draw the raw 2 port network this is how it is, in general these are the things.

And so what I have to do and these are the equations; V1 is equal to Z11 I1 + Z12 I2 and V2 is equal to Z21 I1 + Z22 into I2, these are 2 things. Now, to find out; if this network is to be reciprocal, then V1 by I2 with secondary shorted, I mean other port shorted will be same as the ratio of V2 by I1, when the input port is shorted, so we have to find out these 2 and equate them from that the condition will emerge.

So, what is this; so first therefore, V1 what I have to do is this; this is the general network, now if this network is to be reciprocal, we know that what you do; you will find out; you excite the input port with the voltage V1 and short circuit the output; output port and record this current I2 and this is I1 and what we have to find out is; find V1 by I2, I will find it and keep it. So, how to find out that, it is to be found out using these general expressions.

But under this condition of this experiment V2 is 0, so from the first equation you see V1 is equal to Z11 I1 + Z12 I2, I want to find out here no V2 exist, therefore I1, I2 and V1 all are present here. So, from these I can say that V1 by I2 is equal to Z11 by I1 by I2 that is divide both sides by I2 and plus Z12, is it not, this is the first equation will give you. Now, but I would expect that on the right hand side only impedances are present, why I1, I2.

So, the second equation will give you that ratio of I1 by I2 as because between 0 under this condition of this experiment and this is nothing but Z21 I1 + of Z22 into I2, therefore from these I can say that I1 by I2 will be equal to – Z21 by Z22, is it not, I1, it will be different, correct me, so I1 by I2 I will do, so Z22 by; minus, because I1 by so this is the thing, is it not, this is correct.

Therefore, from the first equation then I get V1 by I2 to be equal to this I1 by I2 ratio you put it here, it will become – Z11 Z22 by Z21 + of Z12, so this is the thing. So, V1 by I2, I have found out like this, then I have to go to other situation that is in this case, if we claim this network is reciprocal, what I am going to do is; I will excite this with a voltage V2, this current is I2 and this port I will short it and this current is I1.

And what I have to do now, find under this condition V2 by I1, this ratio to be found out and then eventually, I will equate this with this for the network to be reciprocal. Now, how to do it;

okay, these are the basic equations, once again I go to those equations and say that V2 by I1 is to be found out, so from the second equation, V2 is equal to Z21 I1 + Z22 into I2, is it not, from this equation.

Therefore, V2 by I1 will be equal to Z21 + Z22 into I2 by I1, then who will give me this ratio I2 by I1; from the first equation, V1 is equal to 0, therefore from the first equation, I will get 0 is equal to that is this equation, 0 is equal to Z11 I1 + Z12 I2, we will get that and from this I will get I2 by I1 is equal to - Z11 by Z12, this is the thing I will get. Therefore, I will put this here to get this V2 by I1; therefore, V2 by I1 will be equal to Z21.

This minus will come, so – Z11 Z22 divided by Z12, this is the thing I will get. Now, if this network is not reciprocal, then these 2 values would have been different, this is the way it is, this is the way, therefore if this 2 port network is reciprocal, then I will demand that this blue V1 by I2 should be equal to this red V2 by I1, therefore the condition is very easy to understand.

So, I will say that it should be then equating V1 by I2 is equal to V2 by I1, it will give you – Z11 Z22 divided by Z21 and plus of Z12 that is this quantity should be equal to this quantity that is equal to Z21 + - Z11 Z22 by Z12, it will be like this, is it not, therefore from these, we can conclude that this 2 will be true, if Z12 is equal to Z21 from this it can be told know or you can just multiply these because the numerator are that is Z12 Z21 – Z11 Z22 divided by Z12 etc.

And this side it is also Z12 Z21 – Z11 Z22 by Z12, these 2 will numerator same, so this is true, so for reciprocal to port network, Z12 must be equal to Z21, therefore that is this elements will be same, these 2 elements should be same, if the network is to be reciprocal. Therefore, if you have found out given the Z matrix of a network, if these 2 elements are equal, then you can be rest assured that it is a reciprocal to port network.

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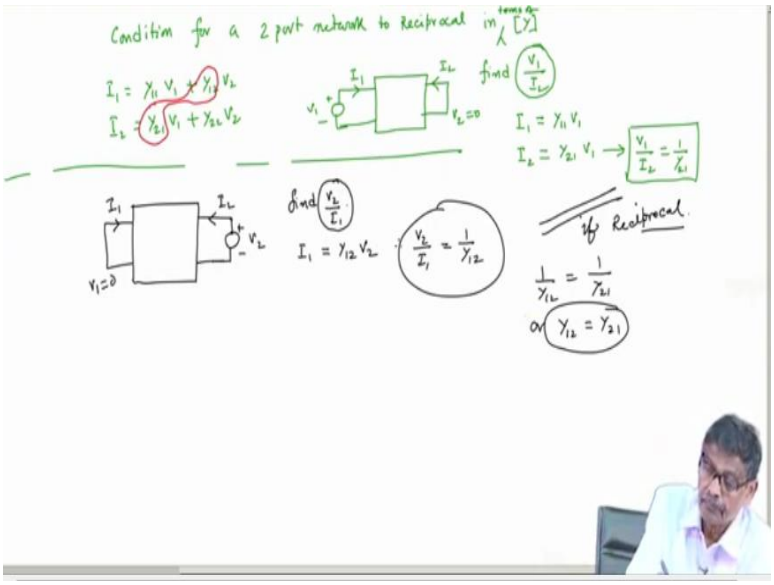

Now, similar conditions let us find out for y matrix, very quickly we will be able to proceed now. So, reciprocity condition for a 2 port network to be reciprocal in terms of y matrix, admittance; short circuit admittance parameter matrix in terms of y. So, once again same thing, in case of first I write down these 2 basic equation; one is I1 is equal to quickly let me do it; plus Y12 V2 and I2 is equal to Y21 V1 + Y22 V2, this is the general thing.

And I have to find out under these 2 conditions, I have to find out the things, I will excite in one case this suppose, input port I will excite it with same thing V1 and short circuit this one like this and find, same exactly same way, I have to find out V1 by I2, no matter whether you are represent in network at z parameters or y parameters, I will simply say this V1 by I2 should be same as this V2 by I1 that will come here.

So, find these and you short the secondary V2 equal to 0, therefore these 2 fundamental equations give you that I1 is equal to Y11 V1, is it not I1 is equal to Y11 V1 and V2 is equal to 0, this is one equation and from the second equation, you will get I2 is equal to Y21 V1 but V2 is equal to 0, is it not, absolutely okay, therefore it is I have to calculate V1 by I2, therefore this equation will not be used here.

Because these equation straightaway give you V1 by I2 is equal to Y21 and we keep this result, V1 by I2 is 1 over; it will be 1 over Y21, correct, it is admittance know, 1 over Y21, we will

leave it there. Then, the same thing I have to do is this, I have to excite this network; same network I will excite from this site with a voltage V2 and this will be I2 and these I will keep it shorted and would like to find out this ratio.

So, my goal is to find out, find V2 by I1 that is what I have to do, so to find that we come back to the fundamental equations here and write down those 2 equations under this condition that is V1 is equal to 0. So, the first equation will give you I1 is equal to Y12 into V2 because V1 is 0, is it not and no point in writing down the second equation because I have achieved what I wanted to find out from the first equation itself that is V2 by I1 is equal to 1 over Y12.

Therefore, I conclude that these 2 are to be same, these 2 should be same if reciprocal, which means that 1 over Y12 is equal to 1 over Y21 or I will say Y12 is equal to Y21, this is the condition that is off diagonal elements should be equal, so these and these; these 2 values should be equal, if it is to be reciprocal. So, once again looking at the admittance matrix, I will be in a position to conclude Y12 is equal to Y21. Now, after doing this we will then find out what will be the condition for the h parameters, let us see that.

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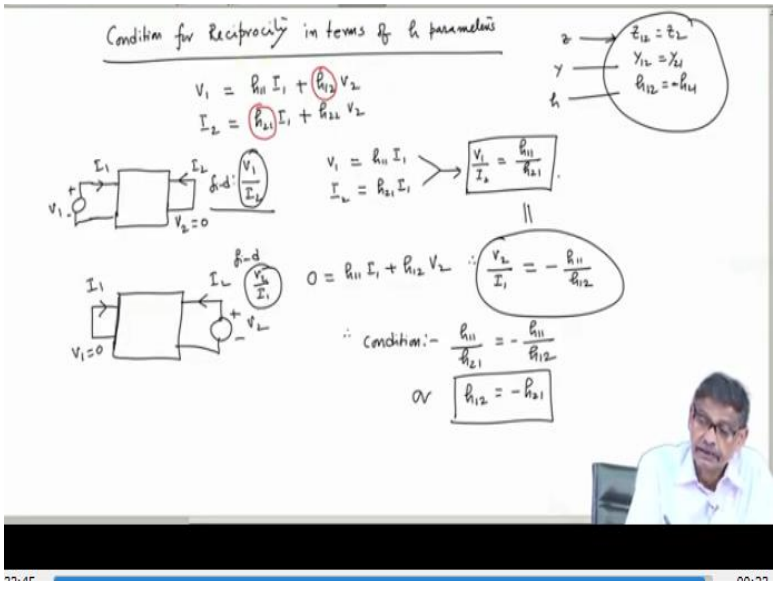

So, condition for reciprocity in terms of h parameters, so once again we will be do the same thing but let us write down, so it was V1 I2, general equation is this and it is expressed as h11 I1 + h12 V2 and h21 I1 + h22 V2, once again I will be just a rule for verifying the reciprocity is as remain

same, why it should differ, it does not matter whether you are expressing a 2 port network in terms of h, y or z or in terms of A, B, C, D that condition is to be satisfied.

So, once again you will see you apply a voltage V1 and keep this shorted and note down this current, this current like this and to prove reciprocity theorem, I need this value V1 by I2, find; so how can I find that V1 by I2, so put this condition that is V2 equal to 0, if you put, you find the first equation will give you V1, h11 into I1 + 0 into h12, so they retains and second equation will give you I2, I am sorry, this is V1 only, there is no V11.

So, V1 is equal to h11 I1 V20 and second equation will give you I2 is equal to h21 I1 and V2 is 0 that is it but my goal is to find out V1 by I2, so just divide these 2 equations and you will get that V1 by I2 is equal to h11 by h21, so we keep it this result, this should be equated with the reverse process that is in the second case what you have to do is this; you have to excite this side with a suppose, voltage source plus minus you excite V2, this current is I2.

And the other side you keep shorted, so that I1 is there but V1 equal to 0. So, once again go back to this fundamental equations and say that with V1 equal to 0, the first equation will give you 0 is equal to h11 I1 + h12 V2, is it not and what you want to find out; find V2 by I1, this ratio I have to find out, write down the first equation, V1 is 0, short at h11 I1 + h12 V2 and I have got the result because V2 and I1 this ratio I can find out.

Therefore, V2 from this itself second equation need not be written, V2 by I1 will be equal to minus of h11 by h12, this will be the thing, so this is the other thing, for this 2 port network will be reciprocal provided these 2 are equal, these 2 must be equal, which means that condition then is h11 by h21 is equal to – h11 by h12 or I will say h12 is equal to – h21, this is the condition. See, for a given 2 port network, if it is reciprocal and if you want to present it in terms of z parameters then the condition was Z12 is equal to Z21.

If the same 2 port network if you represent it in terms of admittance parameters, then also Y12 is equal to Y21 and if you represent the same networks in terms of h parameters, then the condition is this, there is a slight difference, a negative sign will appear for z parameter, this is for y

parameter representation, this is for z, y and h parameters. Therefore, looking at the parameter values, to establish whether it is reciprocal or not, look at the diagonal elements here that is these element and this element.

If it is 2 and if this is -2, then this must be reciprocal, okay so, this is the condition for reciprocity in for these 3 matrices; z, y and h parameter matrices. Naturally question comes, so what will be the condition for reciprocity if the network is represented in terms of ABCD parameters and that we are going to do in the next class, thank you.