Network Analysis

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Lecture – 76
Two Port Network – VI

So in this lecture we are going to tell you about the symmetry property of a Two port network under what condition we call this 2 port network to be symmetric and will be followed by the series parallel and cascade connection of 2 port network. So let us see what does that symmetry means.

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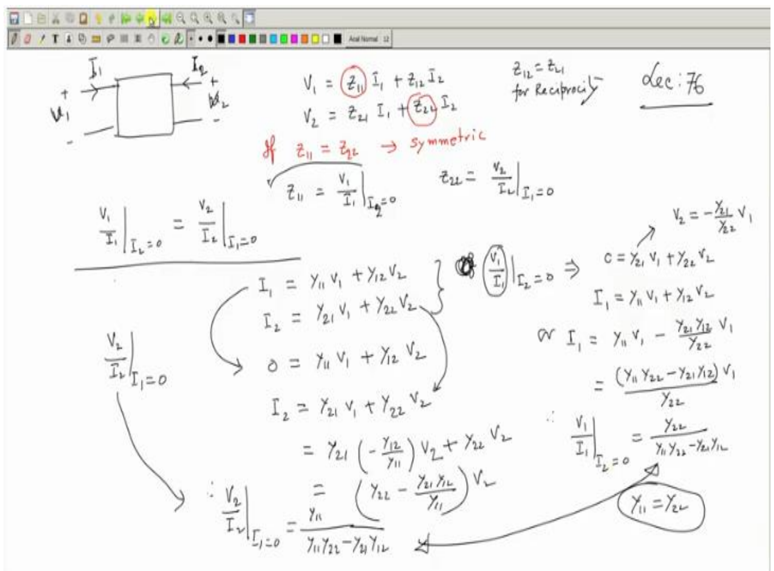

You recall that in a 2 port network we have drawn these figures several times but this is V1 this is i1 and this is v2 this is i2 and we are writing sometimes capital sometimes these one because they may be in time domain they may in Laplace domain they maybe in frequency domain it does not matter but the point is we have seen that the if these 2 port networks is expressed in terms of z matrix it will be and that is the input voltage and output voltage can be written as we did not ask they the Z11 I1 okay we are writing capital letters let me write that.

So these are I2 and this is I1 and we have seen that V1 is so V1 = Z11 I1 + Z12 I2 is it not and V2 = Z21 I1 and Z22 I2 we have seen that if this a network 2 port network is when this 2 port network will be which property that is reciprocal if these 2 port network is reciprocal you have seen that Z12 should be equal to Z21 that we have established earlier for reciprocity.

Similarly, whether there exists any condition under which these metrics will be symmetrical. These 2 port network will be symmetrical okay. A 2 port network is said to be symmetrical if these 2 elements are same for an impedance case if Z11 is it not is equal to Z12 then it is a; to be symmetrical Z22 symmetric now. What is Z11, Z11 = V1 / I1 with I2 equal to 0 that is the driving point impedance of the network looking for port 1 and driving point importance of these 2 port network looking from a port 2 that is V2 / I2 with I1 = 0.

If these two are same if these two ratios are same under the condition that I2 = 0 here and I1 is equal to that is secondary is opens circuit driving point impedance are same, then it is this one. So naturally this is the condition for reciprocity that is no matter in which form you describe the Two port network that is in short circuit admittance matrix or A, B, C, D parameters or h parameters whatever you do this is the thing driving V1 / I1 with I2 = 0 you have find out and V2 / I2 with I1 = 0 you find out and for impedance metrics this one it happens to be Z11 = Z22.

So the thing is this thing I have to calculate that is V / I1 with I2 = 0 should be equated to V2 / I2 with I1 = 0. So let us see what happens if it is admittance parameters if we describe the function so for y matrix so it will be called I1 and I2 it will be equal to Y11 V1 +Y12 V2 and this will be = Y21 V1 + Y22 V2 is it not therefore to find out V1/I1 with I2 = 0 I have to calculate this under this condition that is if we want to find out V1/I1 with I2 = 0 first I will find out this.

So this equation then will give me for this condition I2 being 0 the second equation will give you 0 = Y 21 V1 + Y22 V2 this is the second equation will give me this because I2 = 0 and the first equation will give you is I1 = Y11 V1 + Y12 V2 my objective is to calculate V1 / I1 so V1 is there I1 is there but here is a V2 and this V2 I will calculate from this first equation as Y2 = -Y21 / Y22 into V1 and this I will substitute here or I will say I1 = Y11 V1 and then for V2 you substitute this and this will become – Y21 Y12 / Y22 into V1 and this will be = Y11 Y22 – Y21 Y12 / Y22 into V1.

Therefore, you can easily say that V1/ I1 with the condition I2= 0 will be =Y22 divided by Y11 Y22 – Y21 Y12 this will give then I calculate V2/I2 this of course is obvious hopefully I will get Y22 = Y11 why not driving point admittance should be same. So anyway I will do this very quickly, so V2/I2 I want to find out with the condition I1 = 0.

So from the first equation I will get 0 = Y11 V1 + Y12 V2 this is the first equation and second equation is I2 = Y21 V1 + Y22 V2 this is the second equation and I want to find out V2 / I2 so V1 I will substitute from the first equation so it will be V1 will be equal to minus Y12/Y11 is it not into V1 into V2 and plus Y22 V2. So this will be the once again Y22- Y21 Y12 / Y11 into V2. So V2 / I2 this is what I have to find out with I1 = 0 will be simply Y11 divided by the same thing Y11 Y22 – Y21 Y12 this and these.

If you equate these two, the conclusion is Y11 should also be equal to Y22 that is driving point admittance should be also same. So this is the condition for a 2 port network which represented in the form of admittance matrix this condition should be satisfied.

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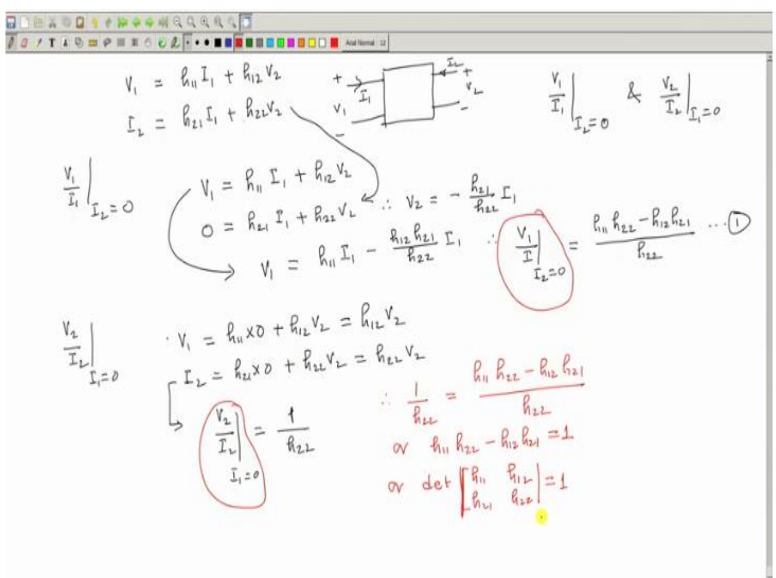

Now let us do it for h metrics what condition it will satisfy in order that it becomes a symmetric matrix. Therefore, you recall that this h matrix equation is V1 I2 is equal to is it not h11 I1 + h12 V2 and this is h21 I1 + h22 V2. So this is the 2 port network plus minus these are the conventions which we should not break to find out anything. So this is V2, this is I2 suppose this relationships are expressed in terms of the h parameters then we know to test the symmetry we have to calculate this V1 / I1 with I2 = 0 and V2 / I2 with I1 = 0 and I will equate them to get the desired condition.

That is what I have to do so in this case the first one if you take that is, I want to calculate V1 / I1 with I2 = 0 means that I will keep the secondary open circuit I will apply some voltage here. Therefore, from the first equation we will get V1 = h11 I1 + h12 into V2 that you will get is it not and from the second equation you will get 0 = h21 I1 + h22 into V2 this is from this equation you will get this, and I want to calculate V1/I1.

So this V2 I will express in terms of I1 therefore here I will write V2 nothing but - h21 / h22 into I1 and this I will put it here then it will be V1 = h11 I1 + h12 into V2 that means – h12 h21 / h22 and this into I1 it will be like this. Therefore, we get V1 / I1 with the condition that I2 = 0 output port open circuit you will get h11 h22 – h12 h21 divided by h22 this is what you will get I will all this is equation 1.

Similarly I will calculate a now calculate V2 / I2 with the condition I1 = 0 that is primary side there is input side is open circuited so that I10 you excite with the voltage V2 and record this quantities then you will get from the first equation since I1 = 0 V1 will be equal to I10 + h12 into V2 this is simply h12 V2 I will get the from the first equation and from the second equation you will get I2 is equal to this equation I am writing now I2 = h21 into 0 I1 is 0 + h22 into V2 this is what I will get and this is = h22 into V2 I have to calculate V2 / I2 is it not V2 / I2 I have to calculate everything is fine here.

So V2 / I2 from the second equation itself I get that V2 / I2 from this equation is 1 / h22 therefore the condition this is with I1 = 0 and I know that for these Two port network to be symmetric this one and this one this ratio this and this they must be equal therefore the condition is 1 / h22 = h11 h22 – h12 h21 divided by h22 or I will say this h22 cancels out provided it is not equal to 0, h22 cannot be 0.

Therefore, the condition is h11 h22 – h12 h21 this must be equal to 1 or I will simply say the determinant of this matrix h11 h12 h21 h22 this must be equal to 1. Determinant this determinant if it is 1, I am sure it is a symmetric 2 port network is that clear? now we will see what about A, B, C, D parameters how it will what are what is the condition when a 2 port network same Two port network is expressed in terms of A, B, C, D parameters then if that network is to be symmetric what is the condition? So let us do it once again.

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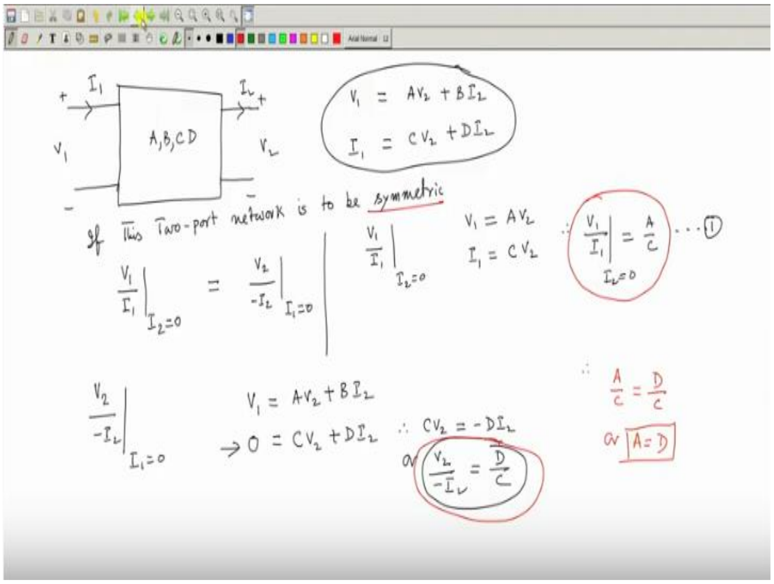

So a network is like this suppose a this network is expressed in terms of T parameters or A, B, C, D parameters this is this thing plus minus V1 and this is I1 is it not and for A, B, C, D parameters what I have assumed earlier that is will keep it, it is like this this is the convention for A, B, C, D parameters or transmission line parameters and then we know there is input voltage and current you know it is expressed in terms of AV2 + BI2 and this is equal to CV2 + DI2 these are the conditions.

But what if this network is to be symmetric if this 2 port network is to be symmetric I will demand that V1/ I1 with I2 = 0 is with I2 = 0 this is open circuited V1/I1 driving point impedance of this network they should be equal to V2 I1 = 0 is it not but what should I write here I should write minus I2 driving point admittance is this. This you must understand very clearly so in this case because of this surprise change in the direction of I2 while describing an A, B, C, D parameters this is I have to do.

But so far as driving point admittance that is the impedance is concerned Z_{22}. So the current is to be reversed because for Z parameter it enters you have got the point. Now this is the thing therefore let us first calculate first do this V_1 / I_1 with $I_2 = 0$ this thing so what we get? With $I_2 = 0$, V_1 from the first equation these are the fundamental equation for general case okay. So $V_1 = AV_2 + 0$. I am not writing that from this first equation $I_2 = 0$ from the second equation for the same condition you will get $I_1 = CV_2$ is it not that is what you will get.

Therefore, the ratio V_1 you just divide these 2 and you get your desired result that is I wanted to calculate V_1 / I_1 with $I_2 = 0$ then I will get it as this by this which will be equal to A/C I keep this result with me. Now I have to calculate $V_2/-I_2$ okay for the second now I have to calculate V_2 divided by $- I_2$ with $I_1 = 0$ I have to calculate. Now to calculate that first let us see from these equations what we get we get that $I_1 = 0$ and so the first equation is $V_1 = AV_2 + BI_2$ that is there and from the second equation I will get this equation $I_1 = 0 = CV_2 + DI_2$.

But I have to calculate V_2 / I_2 and V_2 / I_2 can be calculated from this itself is it not first equation is not necessary if wrong tell me so this is fine. Therefore, I will say that $CV_2 = - DI_2$ or I will say that $V_2 / - I_2 = D / C$ this is the thing I will get and so these two should be equated this and this should be equated? So what will be the result? therefore to be symmetric these two should we same that is $A / C = D / C$ or $A = D$ is the condition is that clear $A = D$ is the condition.

Therefore, for A, B, C, D parameters just looking at the values of A, B and C and D we can conclude whether this we can say about these Two port network whether it is symmetric or not that is the thing. So for a 2 port network to be symmetric if it is expressed in terms of impedance matrix Z_{11} should be equal to Z_{22}, Y_{11} should be equal to Y_{22} and if it is expressed in terms of h parameters then the determinant of h matrix should be equal to 1 and if it is expressed in terms of A, B, C, D or T parameters then A and D should be equal.

So this is about the symmetry in the same way we have found out the conditions for a network to be reciprocal like that several conditions we have found out. Now what I will do is this so this is the thing okay understood.

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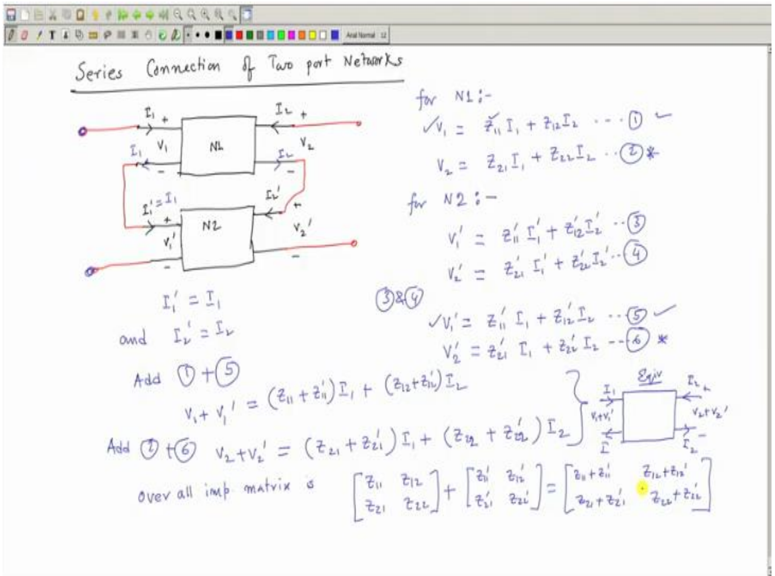

Therefore, we will know say something about series parallel connection of various Two ports network okay series connection these are very simple stuff not tough at all series connection of Two ports networks. Now suppose we have got two 2 port networks it can be extended to three 2 port networks as well only for 2 if we understand we know what to do with others, so this is 1 plus minus this is V1 this is I1 and this is the network plus minus V2 and this is I2.

Now so this is network 1 take another network so N1 Two port network another network N2 it will also have a pair of terminals here in the input port and also in the output port. Let this be called I1 dash and its input voltage be called V1 dash and this voltage is plus minus V2 dash and this is I2 dash. First of all, what do I mean by serious connection when we say that these two 2 port networks are in series it means that the input ports will be in series connected that is this connection, I will show with a red color it is connected like this and these 2 are the total input of this.

Similarly, on the output side you connect them in series and treat these 2 as output terminals. Now obviously you can easily see that suppose I am expressing this network N1 and N2 in terms of impedance parameters suppose so what we are what we will be getting? So far as network N1 network 1, I will get V1 V2 = Z11 I1 + Z12 I2 that is what I will get is Z21 I1 + Z22 into I2 this is for network. For network 2 what I will get. So far as this one is concerned, I will say all the parameters will be replaced by dash here.

So V1 dash is the input voltage for N2 input port voltage and V2 dash these are the input voltages and they are expressed in terms of Z11 dash I1 for this network parameter value maybe different we do not know I2 so I1 dash I2 dash. Similarly, Z21 dash I1 dash + Z22 dash I2 dash this will be the thing. Now this if you so this is equation 1 this is equation 2, 3, 4 okay this is how I have written it but now you see that because these 2 ports are series connected this I1 dash is nothing but I1 itself this I1 and I1 dash they are same.

Therefore, these 2 equations can be similarly so important thing to note is I1 dash = I1 and I2 dash = I2 so these 2 currents were the same if that be the case then V1 dash is equal to this 3 and 4 I am rewriting after knowing this I will say it is nothing but Z11 dash I1 + Z12 dash into I2 dash and the fourth equation that is this one will then be Z21 dash I1 + Z22 dash into I2 I am sorry this dash goes because I am using this equation.

So this is 5 and 6 same 3 and 4 equation with this 5 and 6 here. Now what do you do you add that is equation 1 and 5 you add, if you add equation 1 and 5 you will get left hand side you will get V1 + V2 dash this equation and this equation I am adding so V1 + V2 dash it will be simply Z11 + Z11 dash into I1 this term and this term added up similarly this term and this term added up so plus of Z12 + Z12 dash into I2 that is what I get is it not have I written something wrong yeah I believe.

So this is V1 + V1 dash so V1 + V1 dash similarly you add 1 + 5 I have added now I will add 2 + 6, add 2 + 6. If you add 2 + 6, you will get V2 + V2 dash to be equal to Z21 + Z21 dash into I1 + Z22 + Z22 dash into I2 Z22 + Z22 dash is it not, this is what I will get. Now this whole network this 2 series network can be thought of as single 2 port network after knowing these 2 equations I will say that this is the equivalent 2 port network is this.

These 2 are series connected where the input voltage you see what is the potential of this? These are the input ports can be considered to be the series connected 2 port networks and these two are output ports of these two series connected networks. Therefore, between these two points the voltage is how much V1 dash + V1 that is the input voltage V1+ V1 dash what is the current I1 and this side it can be thought of V2 + V2 dash is the voltage between this output port so V2 + V2 dash and what is this current I2 so equivalent.

So therefore if 2 or several Two port networks are connected in series and then the overall equivalent Two port network representation will be input port voltages will be sum of all the voltages their input current same I1 it is I2 but the interesting thing is the impedance matrix of these 2 port network will be sum of the impedance values of this network and this network. That is the overall impedance matrix is nothing but Z11 Z12, Z21 Z22 + Z11 dash Z12 dash, Z21 dash Z22 dash, is it not?

Then only you will get term by term addition these elements will be Z11 + Z11 dash this element will be Z12 + Z12 dash this will be Z21 + Z21 dash and this will be Z22 + Z22 dash got the point. Obviously you can easily you know feel any Two port network can be represented in several ways either in terms of Z parameters A, B, C, D parameters, Y parameters like that but if they are series connected I can find out the overall I can replace this series connected to port network by a single 2 port network whose parameters values if you know the impedance matrix separately here and there you can just add them and get it very simple.

Perhaps in series connected to port networks it will be easier if you represented it in the form of Z matrix. What it will turn out to be if it is an admittance or h parameter we can find it out but it will be complicated after all after you have found out this one you always know how to translate this parameter value in terms of A, B, C, D or Y parameters or so on or h parameters because each one this representation is related with others provided determinant is not equal to all these things. We will continue our discussion with this in the next class. Thank you.