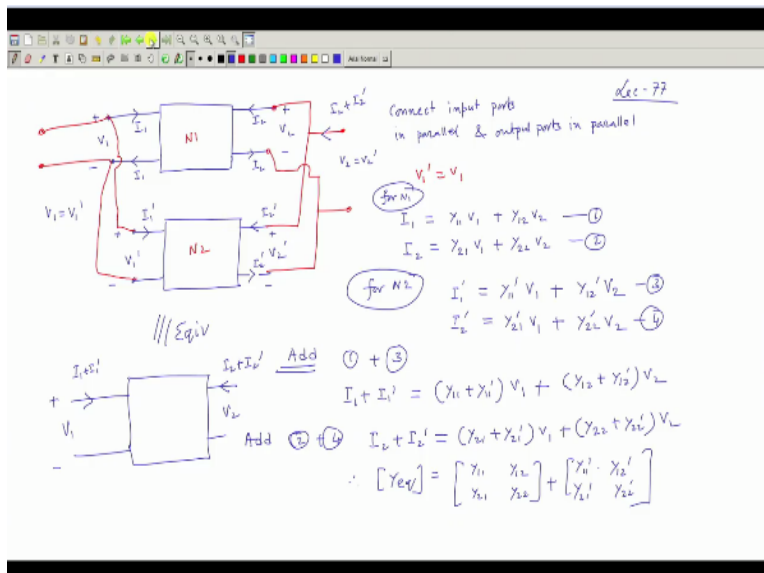


Network Analysis
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Lecture – 77
Two Port Network - VII

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This is lecture 77. So we discussing about series connections. Similarly, if you can connect this one in parallel also. For example, if you have a two port network like this, this is suppose V_1 and this is I_1 and this suppose V_2 and this is suppose I_2 these are two port networks okay. Now here another network which are having things like this. This is plus, minus V_1 dash, this is plus minus V_2 dash and this I_1 dash and this is I_2 dash, this is the thing. Okay.

This two networks when we say connect them in parallel we mean that; of course two sources can be connected in parallel, for example this input ports should be connected in parallel plus with plus, minus with minus and that is connect input ports in parallel and output ports in parallel. So that means I take a piece of wire I connect this point with this and I connect this point with this and this, obviously I connect these two in parallel.

If I connect it in parallel V_1 dash must be equal to V_1 , nothing doing. Similarly, the output ports I will connect the terminals in parallel that is this plus with plus and this minus with minus and

this and this are the overall output terminals, this is input terminals. Now in this case as it will turn out to be; if you experience this network 1 and network 2 in terms of admittance parameters then the equivalent parameter values will be some of the admittances, that is the whole idea.

So first of all I will write down the equation as in terms of admittance parameters, that is suppose I_1 for this two port network $I_1 = Y_{11} V_1 + Y_{12} V_2$ and $I_2 = Y_{21} V_1 + Y_{22} V_2$. This basic equation for network 1. For network 2, this time I will straightaway write down, that is I_1' in this case, I_1' will be equal to Y_{11}' for this network which; whose parameters I am marking with prime numbers, so Y_{11}' into V_1 .

But $V_1' = V_1$, so I am straightaway writing this as V_1 instead of skipping a step plus Y_{12}' into V_2' , but $V_2' = V_2$ are same, so I am writing it as V_2 like this. Similarly, I_2' ; I am sorry; I_2' this is I_2' . Similarly, for this I_2' for this concentrate on this, it will be equal to Y_{21}' into V_1 , but V_1' and V_1 are same. So mind you, $V_1 = V_1'$. Similarly, $V_2 = V_2'$. Therefore, $I_2' = Y_{21}' V_1 + Y_{22}' V_2$. Okay.

Now what you do if you call 1, 2, 3, 4 then add 1 + 3, it will give you $I_1 + I_1'$ that is this equation and this equation I am adding 1 + 3. So $I_1 + I_1'$ is equal to, it will be equal to $Y_{11} + Y_{11}'$ into $V_1 + Y_{12} + Y_{12}'$ into V_2 . And similarly add 2 + 4, equation 2 and 4 you add, if you add you will get $I_2 + I_2' = Y_{21} + Y_{21}' V_1 + Y_{22} + Y_{22}' V_2$.

So this is the thing. Therefore, this whole network can be considered to be an equivalent; and equivalently you can think of this network to be a single to port network whose input voltage is either of this voltage is $V_1 + V_1'$. But in input currents of this is $I_1 + I_1'$ is what, is this input current, $I_1 + I_1'$. Similarly, this side it will be equal to V_2 . And you can see this current is $I_2 + I_2'$. I_2 is going, I_2' is going here so $I_2 + I_2'$.

Therefore; so output port current is this one $I_2 + I_2'$. And the equivalent the admittance parameter of this network will simply with the sum of the y parameters. Therefore, if several two port networks are connected in parallel you simply go on adding the parameter values of each one them therefore equivalent Y metrics will be equal to obviously $Y_{11} + Y_{11}'$, $Y_{12} + Y_{12}'$, $Y_{21} + Y_{21}'$, $Y_{22} + Y_{22}'$.

dash Y12 dash Y21 dash and Y22 dash, that is what we have seen. Now there is another interesting connection which is called cascade connection.

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The slide shows two two-port networks, N_1 and N_2 , connected in cascade. Network N_1 has parameters A_1, B_1, C_1, D_1 and network N_2 has parameters A_2, B_2, C_2, D_2 . The input to N_1 is V_1, I_1 and its output is V_2, I_2 . The output of N_1 is connected to the input of N_2 , so $V_2 = V_1'$ and $I_2 = I_1'$. The final output of N_2 is V_2', I_2' .

For N_1 :

$$V_1 = A_1 V_2 + B_1 I_2$$

$$I_1 = C_1 V_2 + D_1 I_2$$

For N_2 :

$$V_2 = V_1' = A_2 V_2' + B_2 I_2'$$

$$I_2 = I_1' = C_2 V_2' + D_2 I_2'$$

The overall relationship between the input $[V_1, I_1]^T$ and the output $[V_2', I_2']^T$ is given by the product of the ABCD matrices:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

The equivalent network is represented by the cascaded ABCD parameters.

Two port networks in cascade. In cascade means the output of one port is the input; the output whatever you get from the first port is connected, suppose this network 1 it is plus minus V_1 , this is I_1 you connect it and here another network into whose input you connect to this port, get the point. This is plus minus; this is V_2 . And say, so V_1 it will have its terminal and you consider this to be your V_1 dash.

And it will be shown here, if this two networks are connected in cascade the representing each of this two port network in terms of its ABCD parameters is beneficial. Then the product I think it will come of this two metrics will give you the overall two port network thing. For example, let us try to do that. What; let us show that. This is I_1 . For ABCD parameters representation I will do represent each two port network in terms of ABCD parameters.

That is network 1 is suppose its ABCD parameters are A_1, B_1, C_1, D_1 . Similarly, for this network ABCD parameters are A_2, B_2, C_2, D_2 like that we will represent. Now so far as the first network is concerned we know that for ABCD parameters we have $V_1 = V_1$ input parameter V_1 I_1 . And since I will be writing ABCD representation so I_2 deduction I have reversed it is like this as we have been doing all the time.

So $V_1 = A_1 I_1 + B_1 I_2$ for this network. This is V_2 , this is I_2 and $I_1 = C_1 V_2 + D_1 I_2$, this will be the thing. Now for this second; this is for N_1 , this will be the thing. For this second network this is the input current of the second network which is suppose prime I_2 , I_1 dash is the input current. And input voltage is V_1 dash and output voltage is V_2 dash and output current as I_2 dash. This is what I will do.

Therefore, for this network the relationship will be V_1 dash equal to $A_2 V_2$ dash + $B_2 I_2$ dash that is what I will do for this. And in the second equation that is I_1 dash is equal to what, $C_2 V_2$ dash + $D_2 I_2$ dash, that is what I will do. Is not? But you see that V_1 dash and V_2 are same, this V_2 . So this is nothing but V_2 itself. Similarly, I_1 dash is nothing but I_2 itself. So here first I write it is $V_1 I_1 = A_1 B_1 C_1 D_1$ and here we write $V_2 I_2$ this is the thing.

And here we write; we will also get this as $V_2 I_2$ its input quantity V_1 dash V_2 because of this parallel connection. And $V_2 I_2 = A_2$; sorry $A_2 B_2 C_2 D_2$ and V_2 dash and I_2 dash. This way I will write it. This is the thing. So if you want to get the overall structure, suppose I want to replace this whole thing by a simple two port network with this thing that is; this is I_2 dash, this is + $-V_2$ dash and here you have applied V_1 and this is current. This is the equivalent.

That is, I can represent this two cascaded; two port networks as a single two port network equivalent. Now what you can see from this equation $V_1 I_1$ or, this equal to $A_1 B_1 C_1 D_1$ is this thing into $V_2 I_2$. But this column vector $V_2 I_2$ is nothing but this; so substitute this column vector in terms of the product of $A_2 B_2 C_2 D_2$ and V_2 dash I_2 dash. So this can be written as $A_2 B_2 C_2$ and D_2 . And this will be V_2 dash and I_2 dash.

So the overall, the ABCD parameters of this; ABCD parameters of this equivalent two port network will be product of the, this two metrics because this is product. So this is equivalent to; it will be also a 2/2 metrics whatever it will be that is $A_1 A_2 + B_1 C_2$ etc. So this will be A equivalent from whatever you get you say B equivalent, C equivalent and D equivalent. So this equivalent ABCD metrics will be just product of this two $A_1 B_1, C_1 D_1$ into $A_2 B_2$ and $C_2 D_2$.

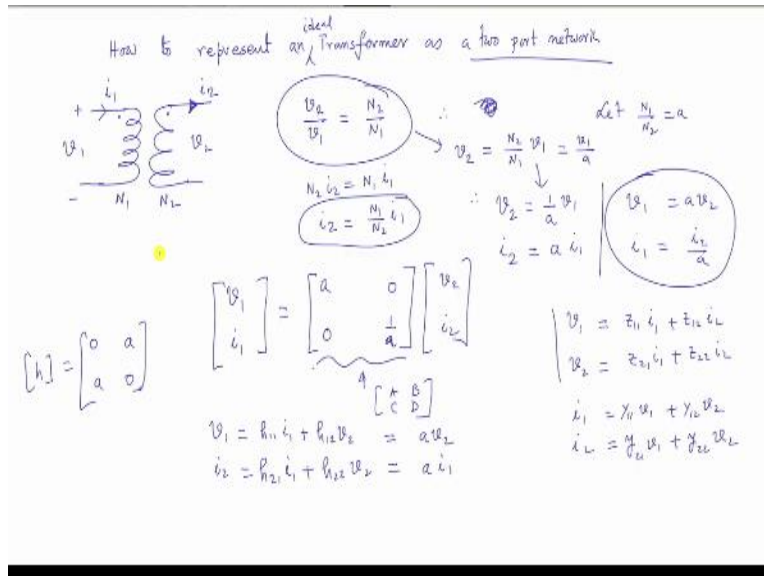
So very easy. Therefore, if several two port networks are connect; cascade connection means what then, the input voltage of this second network is the output voltage of the first and so on you can connect several in cascade. So the product of the ABCD parameters will give you the equivalent representation ABCD parameters of the network that is what I want to tell you. Is that clear? Therefore, see it therefore depends.

If it is transmission line parameter and there will be several network connected one after another that is in cascade, it is better you deal with ABCD parameters equivalent ABCD parameters will be simply multiplication of this 2×2 metrics. As many stages are there, so many, multiplication you have to do. Similarly, if the networks are in series connected mode and by series connection I mean the way it was connected input ports are series connected, output ports are series connected.

In that case if each one of these ports are represented in Z parameters equivalent Z parameters can be easily calculated. It does not mean that this cannot be represented in ABCD parameters, it can be. But then the equivalent Z will come in a complex way. And we already know that if you have represented the network in any form either H or Y or Z parameters terms then you can find out the parameter values for other representation.

We have to do a little bit of work algebra there that is all. So this is how things will go around. Anyway so these are the three connections I told you. And you go through it and if you find any further conditions should be put you think about that. But I have assumed here that the networks each one of them are two ports and $I_1 I_1$ dash etc., or same and so on. Now I will tell you some about one two port; so far I was telling that two port networks maybe; anyway let us start without much talking.

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Suppose, how to represent a transformer and ideal transformer as a two port network. We know what a transformer is. Suppose we have a ideal transformer here. These are the dot terminals of the transformer. Ideal transformer means no resistance, no linkage flux etcetera. And if you apply a voltage here we will get some induced voltage there, this is number of trans N1, this is number of trans N2, and if it delivers a current I2 in this direction.

Oh, two port networks, so I will; okay I2 and this as I1. Let us write. The deduction it is my thing. Okay, if you write like this then I will say we know that $V_2 / V_1 = N_2 / N_1$. This is one equation and second equation is $N_2 i_2$, it is called MMF balance equation is equal $N_1 i_1$. So from this say that, this is the thing and $i_2 = N_1 / N_2 i_1$. This way I can write. Similarly, I can write from this as $V_2 = N_2 / N_1 i_1$.

So N_1 / N_2 is the ratio. Let N_1 / N_2 be a some number. Therefore, I will say $V_2 = a V_1$. This is one equation from this; "Professor - student conversation starts" $1 / a$. $1 / a$. $1 / a$. $V_2 = 1/a$. "Professor - student conversation ends". So this is equal to V_1 / a . Is not, if I define like this. So N_2 / N_1 is $1/a$. So this is one thing and second thing is $i_2 = a i_1$. Or I can write it down suppose $V_1 = a V_2$ etc., in same equations I am; and it is i_2 / a .

Now the way I have assumed the deduction of the current i_1 and i_2 like this. This one, this equation I am writing in metrics form $V_1 i_1$ that is input voltage and current in terms of the

output voltage and current and the deduction of i_2 I have assumed like this. So what will be this one? $V_1 = a V_2$. So here it will be V_2 , no i_2 and $i_1 = i_2 / a$ so this is 0. This is $1 / a$. So it is also a two port network but for terminal two port network, not that this two (()) (27:45) when if it is join you can. But what I am telling this is a. What do you think these parameters? Is it Z, is it ABCD? Is it H parameters?

It is ABCD parameters, because I have assumed like i_2 like this and input voltage and current is represented in terms of this. So this one can be termed as an ABCD parameters of the network. So this is nothing but ABCD parameter of the transformer. So a transformer can be considered to be a what is called a two port network whose ABCD metrics is this. Can this be represented as an impedance metrics?

To represent in terms of impedance metrics this transformer what you have to do is this, V_1 and V_2 you have to write in terms of i_1 and i_2 . Is not? This is what $Z_{11} i_1 + Z_{12} i_2 + Z_{21} i_1 + Z_{22} i_2$. This is how you have to write it. Question is can it be written like this? So V_1 cannot be expressed in terms of i_1 i_2 . Similarly, V_2 cannot be expressed in terms of that. That is it looks like impedance metrics does not exists there.

I will ask you to see carefully that whether ABCD parameters if there known how they are related Z parameters. And whether it is possible to obtain the impedance parameters or admittance, similarly admittance parameters, is it possible that is $i_1 i_2 = y_{11} V_1 + y_{12} V_2$ and $i_2 = y_{21} V_1 + y_{22} V_2$. Can I write i_1 in terms of V? No. i_2 in terms of V? No. I cannot write. Can I write it in H parameters known?

For example, is it possible to write it $V_1 i_2$ you are writing. V_1 is equal to what, $h_{11} i_1 + h_{12} V_2$. Is not? And this is equal to $h_{21} i_1 + h_{22} V_2$. The question is $V_1 = a V_2$. So this one looks like; there is no i_1 term so this one is a into V_2 , is not? $V_1 = a V_2$. And $i_2 = a$ into i_1 . So it looks like it can be written. Is not? There is no H parameters exists. What will be h_{11} ? 0. So this same transformer as a two port network has a H parameter it looks like.

What its value will be? h_{11} that is 0 into i_1 , so 0. h_{12} into V_2 that is a . Is not? Then $i_2 = h_{21} i_1$ that is a ; and of course there is no contribution for V_2 . 0. So you please; behind all these representation slight manipulation of the equation in most of the cases, you will be able to conclude several things whether can I represent it in H parameters or Y parameters or Z parameters things like that. We will continue with this idea and tell you about another two port network which is an interesting network called Gyrator in my next class.