Network Analysis
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Lecture – 78
Gyrator

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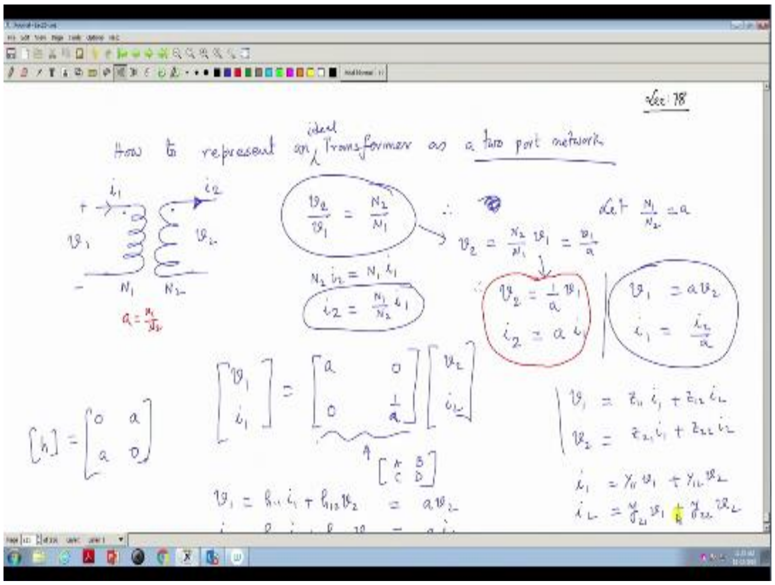

We were discussing about representing an ideal transformer in terms of two port network, and last time we arrived at this condition. Now the way I have assumed the direction of i1 and i2, this parameters are A, B, C, D parameters where is the number of trans of primary to secondary. And I know what to do if I have been asked to find out the Z parameters or Y parameters that I was telling to you. Because to find out Z and Y parameters current should be; i2 should be reversed in that way. Now the interesting thing is about this transformer once you have written a 1 / a.

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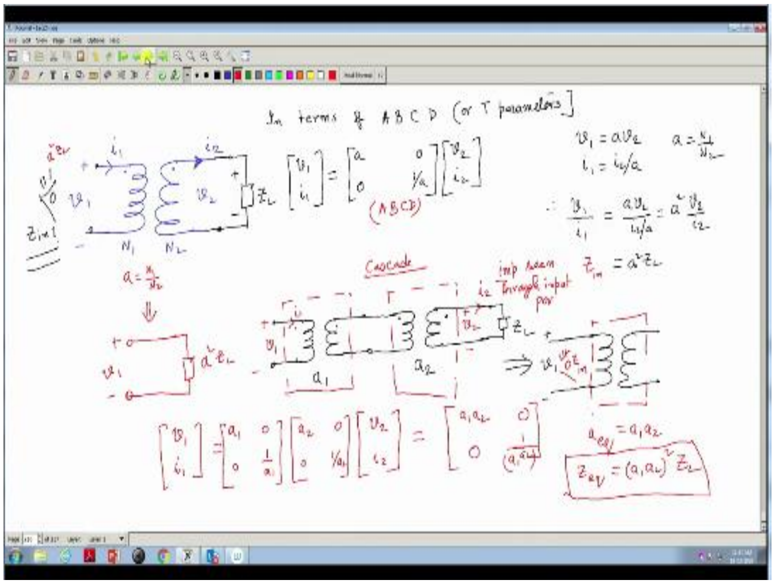

So this is the transformer and; we say that in terms of A, B, C, D parameters or sometimes called T parameters transmission lane parameters. It will be that is the transmission lane parameters means V1 i1 should be there that is the input voltage and current is represented in terms of output voltage and current that is V2 i2 and this we got a 0 and this we got 0 1 / 8 because V1 = V2 and i1 = i2 / a, ideal transformer. a is N1 / N2. That is fine. This is the thing.

Now suppose this two terminals of this is terminated across and impedance say ZL, secondary side if you terminate and impedance then what impedance will be seen from the input port, that is we want to find out when ZL connected what is the impedance seen from looking through the input ports, Z in is how much of this network, that can be also easily calculated and many of us know the result but nonetheless let us calculate from here.

So you know V1 = a V2, i1 = i2 / a therefore V1 / i1 the impedance seen from the primary side will be equal to aV2 / i2 / a which is equal to square V2 / i2. But V2 / i2 is nothing but ZL that is equal to a square into ZL. So impedance seen from this side which will connect ZL is a square into ZL. So sometimes transformer people say it transfers the value of the impedance. But one thing should be understood impedance seen through input port that is ZL. It will be like this.

But a is a number without any dimension and things like that. Therefore, although the value of the impedance will be changed looking from this primary end, but quality of the impedance remain same. That is, if I it is inductive this will be also inductive seen by the source that is this circuit is equivalent to this circuit that is V1 and here as if you have connected an impedance which is a square ZL. So if ZL 3 + Z (()) (05:05) inductive this will also remain inductive.

Although the magnitude of the impedance etcetera will change, therefore, a transformer transfers the value of the impedance from the secondary to the primary side this is Z in. Quality of the impedance does not change but it changes. And obviously suppose you have a circuit like this. Suppose you have a transformer like this with input port and this is with output port and you have another transformer with input, output port.

And suppose the trans ratio of this is a1 and trans ratio of this is a2, both of them are ideal transformer. Then what I will be doing is I will cascade them. And here I will connect an impedance ZL across the secondary. So this is one transformer. And this is another transformer. What is the trans ratio of this transformer? Small a1. Trans ratio of this transformer this turn is to this turn a2. Now these two are then connected in cascade, cascaded.

In fact, cascade connection of transformers are very common. In power system also you step down the voltage then once again step up the voltage then step down the voltage and so on. But in any case this can be represented by this whole thing can be represented by a single transformer where you give this as voltage V1, so equivalently it can be represented by a single transformer. We want to find out what will be the trans ratio of this equivalent transformer, very simple.

Because this two are cascaded and if they are cascaded it is better you deal with A, B, C, D parameters. Then if you say in this network not be; suppose this is i1 and you say that this is i2 and this is V2 then we know from our previous knowledge of A, B, C, D parameters is that this V1 i1 can be represented in terms of this overall output voltage V2 i2. Now this V2 i2 if you pre-multiply with this A, B, C, D parameters that is a2 0 0 1 / a2 then you will get voltage current here.

This column will give you voltage and current in this. Then you pre-multiply this. A, B, C, D parameters gets multiplied if things are connected in and this will become a1 0 and 0 1 / a1, this is the thing. And if you multiply this two this will simply become a1 a2; a1 0 so this is 0 then first row second column a1 into 0 and 0 into 1 / a2 this will give 0 and finally the last one it will become 1 / a1 a2, is not. This will be the thing.

Therefore a equivalent will be just product of a1 a2. Therefore, any impedance you connect to this side will get multiplied by Z equivalent Z in, in this case in this equivalent network Z equivalent will be then a1 a2 square into ZL. Or you can think ZL is first converted to a2 square ZL and then it is multiplied by a1 square to get the input impedance here, got the idea? So this is how; although the quality of the impedance does not change in case of impedance matching circuit transformers are used in amplifier circuit like that in electronic.

Therefore, this is what is called impedance matching. If your source impedance is to be matched with the load impedance for maximum power transfer to take place, then this is how and if there is an mismatch between source impedance and load impedance you then can use a transformer of appropriate trans ratio to keep to make them same so for maximum transfer takes place, you know that. But this is how a two port network can be very easily applied in order to transfer the impedance. Although, the quality of the impedance will not change. If it is ZL is inductive Z equivalent will be also inductive. If ZL is capacitance it will be capacitance and so on.

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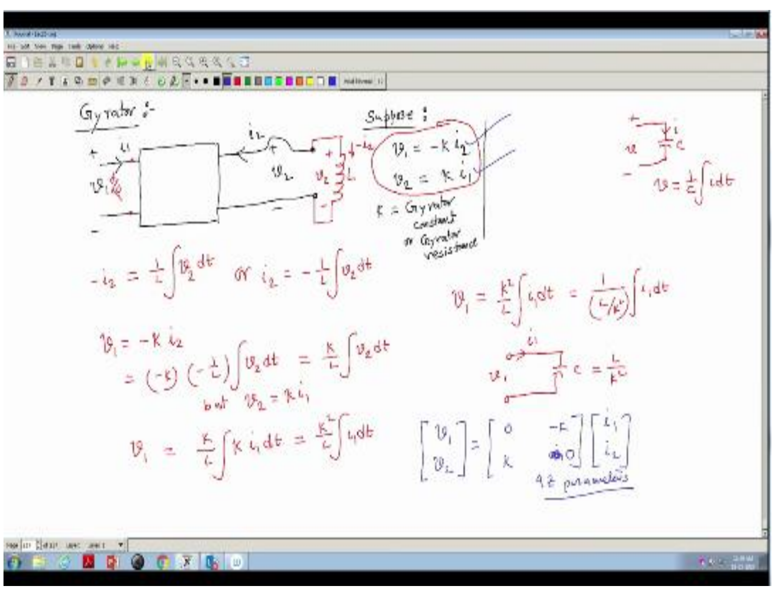

Now we will discuss about an interesting two port network called Gyrator. That is, I will say that this is V1, this is i1 and this one I will write i2 and this is + -V2 two port network. Now suppose I say, suppose the relationship between input voltage current with output voltage and current is like this that is V1 is equal to suppose minus k into i1 where k is a number k into i2. Okay. Suppose we say that. And the second equation is V2 = k into i1.

Suppose this is the relationship between the input voltage and output current that is this input voltage is such that it is proportional this current i2 voltage; I should not say input voltage, voltage input port voltage and similarly output port voltage depends on this current. Suppose such a relationship exists between input port quantity and output port quantities. Now obviously if; and this relationship can be written in various form. We will come to this slightly later.

Now you see, first thing is the interesting point is that suppose I have connected, so a gyrator is what a two port network, the relationship between input voltage and current are related by this where k is called gyrator constant or sometimes called gyrator resistance because the dimension of k is voltage y current, so gyrator or gyrator resistance. This is thing. Suppose on this side I will connect and inductor. What is the voltage of current in this inductor? Same as this.

What is the current flowing through the inductor minus i2? And we know that minus i2 the current through the inductor is nothing but 1 / L into voltage across the inductor V2 into dt. Is not? (()) (15:46) by L we have studied in; while defining inductor voltage relationship essentially means V2 = L di to dt, that is fine. So this is the relationship –i2 = 1 / L or or i2 = -1 / L into V2 dt. That is fine. Now what we will be doing we will be calculating what happens to V1.

V1 if it is a gyrator is minus k into i2. If any mistake, just point out. So V1 = -k into i2, that is fine. Therefore, for i2 I will write –k is there then -1 / L substitute this into integral V2 dt, this will be the thing which is equal to k / L into integral V2 dt. But V2 = k into i1. Therefore, V1 = k / L into integral for V2 you write k i1 dt which is equal to k square by L into i1 dt. Therefore, input will say that a voltage has been applied or if you pass a current source k square by L into i1 dt.

So input will say the integration of current i1, whatever impedance the input fellow will see, he will see this equation, okay voltage; so it is just like an capacitor, because in a capacitor what happens you know that if this is i then voltage across the capacitor V, we have seen earlier that if current flows when the voltage across the capacitor is 1 / c idt. So it is exactly of this form. So applied voltage V1 will see the integration of the input current divided by some constant.

That is equal to 1 / L by k square into i1 dt. Therefore, you see the interesting part of it. For this two port network if you connect an inductor across the output input will see a capacitor as if we have connected a capacitor unlike a transformer. Transformer will manipulate the magnitude of the impedance by some a square turn that is okay but the quality of the impedance remain same. Here quality of the impedance changes.

Similarly, you can show that if you connect a capacitor here to this input terminal the impedance will appear to be inductive. This is the beauty of this gyrator. Of course these circuits are not developed at high power level circuit like in power system. Transformers are used in power systems but gyrator in control system or at a low power electronic devices you can easily make this input supply terminal say fuel by connecting an inductor.

He will always interpret okay as if a capacitance has been connected to; so this one through this input port it will look like V1 i1 and there is as if somebody has connected an inductance capacitance, what is the value of the capacitance 1 / c that is L / k square. So for a gyrator network if you know the value of k inductance value the magnitude of the capacitance that this input port we will see can be easily calculated. Okay.

So gyrator if you can achieve this input voltage, output voltage relations you can do it like that, cleared? Now if suppose somebody wants to, so what are this parameters, see first let me write in terms of now metrics. So what is the relationship? Relationship is V1 V2, is not? Is equal to in terms of metrics I would like to write it in terms of i1 and i2 like this. So V1 = - k i2 so 0 - k this equation, the first equation and V2 = k into i1.

And this k into i, 0 into i2. This equation. These are this two equations in metrics. What is this metrics? This metrics is impedance metrics Z11, Z12, Z21, Z22. Is it a symmetric metrics? For symmetric metrics these two value should be same. It is in fact anti-symmetric metrics. These two are magnitude same that opposite side. So it is anti-symmetric metrics that can be noted. And this is the Z parameters.

Because it is consistent with the deduction of current and voltages I have assumed for this two port network called as the Z parameters of this two port network. Now obviously if I want to connect two gyrators in cascade then like the two transformers in cascade we found it is equivalent to another equivalent transformers of some trans ratio a1 into a2. Now if you have two gyrators connected in series, I want to find out what is the input output port voltage.

And at the end, what is the output voltage output current that relationship. Suppose I want to find out then what should I do? The moment things are connected in cascade. Always try to write down this relationship in terms of A, B, C, D parameters. So easy type is are they A, B, C, D parameters? No. Because if they are written in terms of A, B, C, D parameters then those two metrics are simply to be multiplied to get the overall relationship between input voltage current and output voltage and current, that is the whole idea. Now suppose, so I want to write down the same equation in terms of A, B, C, D parameters.

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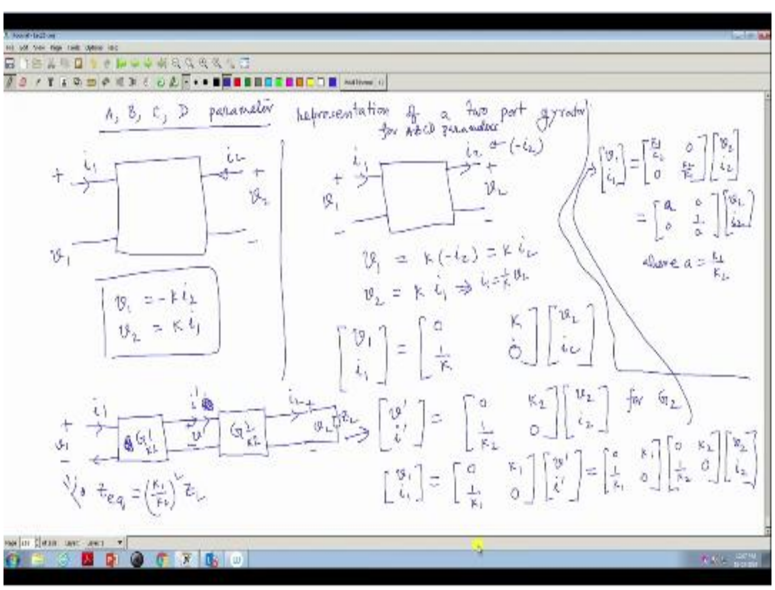

Parameter representation of a two port network, two port gyrator, I want to do it. So the thing is like this. This is + -V1. This is i1, but here I had assumed it to be i2 to be this way. And we got this relationship that is V1 = -k i2 and V2 = k i1 and this is too V1 = - k i2 and V2 = k i1 that is the thing, usual thing. But if I want to represent it in terms of what is called the A, B, C, D parameters you know what we do is this, this direction of the current of i2 is to be reversed.

That is, we generally as I have been telling many a times that to find out A, B, C, D parameters for A, B, C, D parameter convention is to assume the direction of voltages and currents in this way. Therefore, what should be how these equations will be modified? It will be simply; this is i2 means –i2 is entering, good. Therefore, V1 should be k into –i2 which is equal to k i2 for this particular case and V2 = k into i1.

Therefore, A, B, C, D parameters to find it out V1 i1. And i must represent it in terms of this deduction of the current V2 i2 and we get this way. i2 is 1 / k V2. V1 is k i2, V2 is k i1, fine. So V1 is k i2, that is also fine and i1 is 1 / k into V2 that is i1 is 1 / k into V2. So it will be 1 / k and this is 0. This will be the thing, V2 / k2. So this will be the thing. Now suppose if two gyrators you connect then the; if two gyrators, gyrator 1 and gyrator 2, this is gyrator G1 and G2.

And this is the input voltage V1. This is i1. And you connect another and you get the overall output is V2 and this current is i2. Then what I am telling this voltage if you say this is some V dashed and i dashed here, this voltage is V dashed, this voltage is i dashed. Then first thing is for this gyrator it should be V dashed, i dashed is the input voltage and input current. And it will be equal to 0. Suppose this gyrator constant is this K2 and this should be equal to into V2 into i2.

This voltage and current are related with this. That is this gyrator has got a constant k1 and this gyrator has got a constant k2, so V dashed i dashed and V2 i2 are related with that. Similarly, for gyrator 1 V dashed i dashed is the output and V1 i1 is the input. So for the; this is for G2. And for G1 this gyrator the input voltage is V1 i1. And this will be equal to 0 k1 and 1 / k1 and 0. And this will be equal to V dashed i dashed.

And then this; for V dashed i dashed I will substitute this in this equation. So it will be 0 k1, 1 / k1 0. And for V dashed, i dashed from this equation I will put this thing that is 0 k2 and 1 / k2 and 0 and this V2 i2. This is the thing. Therefore, the overall relationship of these two gyrators will be like this, say if I write it from there, if you go so I will say V1 i1 is equal to from this, I will say it is a metrics multiply 2 / 2 metrics, very easy to multiply 0 to 0 + k1 first row first column k1 / k2.

So 0, second term 0 0 + k1 / k2. Correct. So this will be k1 / k2, so this into this I have written. Now this into this, it will be 0 0 so this is 0. Then second row first column this 0, this 0 so this is 0. And second row second column it will be k2 / k1. And this will be your output voltage and current. Therefore, if two gyrators are connected in cascade and this is the overall relationship between V1 i1 and this V2 i2 there were two gyrators.

Does it remain a gyrator now? The answer is no, because for a gyrator the form of the metrics should be like this. What it is then? It is a transformer. Because for a transformer we are seeing that a / 1 / a and the diagonal (()) (33:36) these elements are present, so for this one you will see it becomes is equal to of this form a 0, 0 1 / a, sorry this is a where a is k1 / k2. And this relationship suggest that two gyrators in cascade will behave like an ideal transformer.

Just to explain to you what to do with this relationship. Therefore, a single gyrator although changes the impedances quality of the impedances changed looking from the single a gyrator will do that; a gyrators will not be doing that. It will once again, if you connect an ZL here, suppose some ZL you connect here what is the impedance that will be seen by this fellow. ZL equivalent seen from the input side will be a square, a is k1 / k2 whole squared into this ZL, because in equivalent it looks like a single transformer. Thank you for watching this. I will continue some more points about gyrator in the next class. Thank you.