

Network Analysis
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology-Kharagpur

Lecture # 08
Nodal Analysis-III

Welcome to lecture 8 and we were discussing about mesh analysis and recall that in the mesh analysis rule was like this, identify the node choose the reference node, this have chosen as the reference node and then V_{A0} , V_{B0} , V_{C0} have to be found out coefficient of the V_{A0} will be some of reciprocal of all the resistances coefficient of the V_{B0} will be negative of reciprocal of the resistance line between A and B nodes and so on and right hand side will be sources.

(Refer Slide Time: 01:11)

KCL dec-7
 At node c

$$-\frac{1}{R_4} V_{A0} - \frac{1}{R_4} V_{B0} + \left(\frac{1}{R_4} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_{C0}$$

$$= -\frac{E_2}{R_3} - \frac{E_4}{R_5} \quad \text{--- (3)}$$

If $n = \text{total no. of nodes}$
 • solve $(n-1)$ eqn's.

at node (A):

$$\frac{V_{A0} - E_1}{R_1} + \frac{V_{A0} - (V_{B0} + E_2)}{R_2} + \frac{(V_{A0} - (E_6 + V_{C0}))}{R_6} - I_s = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_6}\right) V_{A0} - \frac{1}{R_2} V_{B0} - \frac{1}{R_6} V_{C0} = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_6}{R_6} + I_s \quad \text{--- (1)}$$

at node (C):

$$-\frac{1}{R_2} V_{A0} + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) V_{B0} - \frac{1}{R_4} V_{C0}$$

$$= \frac{E_3}{R_3} - \frac{E_2}{R_2} \quad \text{--- (2)}$$

So, this is how one can really write it down and then i took a situation like this and I decided to solve it by nodal method. Then once again here I follow the rules we have learned that that coefficient of V_{A0} will be $1/4 + 1/2$ no resistances + $1/0$ and the moment $1/0$ comes in we are at a loss, because it is infinite. So what to do with that similarly coefficient of the V_{C0} will be infinite, similarly the sources if you want to write at node A it will be $-2/2$ that is fine. But here, this $4/0$ so all these things complicate the matter. One way of doing this I am telling you, but you really do not do that to avoid such a complication that point I will take later.

But now, let us see then what to do here as I told you see there are 4 nodes you can choose any of the nodes as your reference, here comes the point. So, I have planned now, I will choose C as my reference plan between the nodes where an ideal voltage source is present better choose one of them either A and C as your reference because there is a freedom there I will not stick because if I choose a reference node as O I face this difficulty so, I will not choose. So, I will choose my reference this is I will choose as the reference node.

Then so, this is useless I mean no point in writing this one. So, we will be doing it a like this then so, choose C as your reference node as reference node then there are altogether 4 nodes then which in node voltages have to be found out have to find out V_{AC} , V_{BC} and V_{OC} with respect to C all the potentials then I will define these have chosen as a reference node. So, these voltages are to be found out now.

So, reference node with respect to that point all the other node voltages are now specified with respect to see that double suffix we assured about it what does it mean? Now if you look at this 3 unknowns we immediately say there is no resistance in this branch. V_{AC} potential of A with respect to C start from C try to reach A, and then is -4 volt. So, V_{AC} is known, that is the most interesting point V_{AC} is known. So, out of these 3 unknowns, we note that V_{AC} is known and that is equal to -4 volt.

Therefore, I will not attend to write KCL at A, it is not necessary, so not necessary to write KCL at node A. After all there are now 2 unknowns V_{BC} and V_{OC} . So, we have to write down the KCL at node B and that node O and I will generate those 2 equations from which I will be able to calculate V_{BC} and V_{OC} . Now let us write that. So, not necessary to write KCL at node A. In fact if you want to write it down there will be difficulty, but nonetheless V_{AC} is already known because there is a fixed battery here now KCL at node B.

Let us write is that coefficient of V_{B0} will be some of reciprocal of all the resistances that is $1/6 + 1/8 + 1/2$, 6, 8 and 2 are connected to B so, this will be the coefficient of V_{B0} coefficient of V_{C0} will be common resistance $-1/8$ into V_{C0} and coefficient of V_{A0} will be $-1/2$ no V_{A0} I

am Please forgive me, I mean I have made a mistake because my reference node is these, VBC I should write potential of B with respect to C.

Similarly this one at nodes the coefficient of BC will be $1/6$, $1/8$, $1/2$ and I am writing here. So, this term so VBC and the coefficient of VOC minus got the point $-1/6$ into VOC because my unknown said this VBC and VOC are the unknowns VBC, VOC and coefficient of A will be there that is $-1/2$ into VAO, VAC is that but VAC is -4 volt. So, this is you got the point so this into -4 volt.

This will be the thing and this is plus and the right hand side I will write down the sources they must be dimensionally ampere. So, at node B this 2 volt is connected. So, it will be $2/2$ and it will be plus because plus is connected to B similarly $+5/6$ and of course, in these brands there is no source. So, this is 1 equation and I will write down the KCL at node O, because O is not reference node mind you, you can call it ABCD how does it matter so, at node A if you want to write down KCL let us see what you should relate us.

So, VOC so, coefficient of VOC will be some of reciprocal of all the resistances connected to O that is $1/6 + 1/4$, this one $+1/10$. VOC coefficient of VOC, then coefficient of the VAO will be $-1/4$ into VAC because the reference point is C now, VAC this plus $-1/4$ into VAC and then finally, the contribution of VBC here. So, $-1/6$ into VBC and they should be equal to on the right hand side sources at node O we are writing.

So, it will be as you can see $-5/6$, because minus is connected to $-10/10$ minus is connected to and for this no sources connected so, that is all these are the 2 equations. Operate VAC is known, so, this is equal to minus 4 volt I would right. So two equations to unknown and after you solve then, he will be able to calculate all the branch currents. This branch currents I can calculate, because we $VAB - 2/2$ will be this branch current how to calculate this branch current, you then apply KCL between these two points to get this correct got the idea.

Therefore, remember that it is it may be situations like this therefore, like I told you that if a number of equations to be solved by nodal method is $n-1$ for example $4-1$ is 3 , but if between 2

nodes A an ideal voltage source is connected like this, then number of equations to be solved these one less further one less because 1 noded voltage will be known provided a present one of the 2 nodes across each volt is ideal volt source is connected as your definition.

So, this example tells you now that is it is not necessary. 2 things it educates me 1 thing is in this 1 any 1 of them I could choose B as my reference, I could choose B as my reference in general problem as well, then your unknown will be VOB, VAB and VCB like that. So, I think you have got the point these are the points to be noted that is always it is not necessarily all the below points he chooses a reference although people say it is visually attractive.

Otherwise, there is search no restriction in that, in fact that has been applied here to solve this problem. But the point now, so, number of equations to be solved here these only 2. So, if you are lucky between 2 nodes and ideal voltage sources connected perhaps to be solve 1 equation less therefore, given a network. Look at the network carefully and see whether current sources are present in the outer loop.

And there ideal voltage sources present between 2 nodes whether you can exploit a number in terms of number of equations to be solved if you can reduce that you achieve something less time will be required to solve the network. Now, I will take another example.

(Refer Slide Time: 14:30)

V_{A0}, V_{B0}, V_{C0} ex 8
 $V_{A0} = V_{C0} - 4 \quad \text{--- (1)}$
 at node A
 KCL
 $\left(\frac{1}{4} + \frac{1}{2}\right)V_{A0} - \frac{1}{2}V_{B0} = -\frac{2}{2} - x \quad \text{--- (2)}$
 KCL at node B
 $\frac{V_{A0}}{4} + \frac{V_{A0} - (V_{B0} - 2)}{2} - \frac{1}{2}V_{A0} + V_{B0}\left(\frac{1}{6} + \frac{1}{8} + \frac{1}{2}\right) - \frac{1}{8}V_{C0} = \frac{2}{4} + \frac{5}{8} \quad \text{--- (3)}$
 KCL at node C
 $-\frac{1}{8}V_{B0} + V_{C0}\left(\frac{1}{10} + \frac{1}{8}\right) = \frac{10}{10} + x \quad \text{--- (4)}$

Now, suppose another way of handling this problem could be in this way, same problem why I am telling in various ways so that you really understand fundamentally what you are doing KVL, KCL, but now apply your intelligence to solve the network quickly that intelligence level I am addressing to for example, this could be also solved in this way, let us see. Suppose I copy this 1 this network and I go to next page and I paste it. Okay say the same network is here.

Another way you could do it you could solve this problem in this way. So I will it is this C business another way altogether I am doing these things have done, the advantage of that I told you this is 10 ohm is not this is plus minus. I think all the components have been copied. Now, another way of handling this situation could be okay choose this as your reference node okay and this is ABC this is your this voltage.

Now, another way of looking at it is that the potential of the VAO if I have chosen O as a reference is nothing but VCO suppose we have got this potential with respect to O and then try to reach point A and there you will find - 4 got the point this is the thing. So, this potential is then not A another variable, but it depends on VCO by a constant add or subtract by a constant. So, VAO is this one if that be the case.

So, this is 1 equation involving VAO, VB0 here what is the target I have chosen this has made a difference, VAO and VB0 and VCO these are my equations to be written. So, KCL is to be written to generate the equations, but 1 equation is gifted by the very definition of the problem because VAO is nothing but VCO -4. So 1 equation I have got then I have to write down 2 more equations is not so, 2 more equations, how can I write for example, at node A KCL I will now write KCL fundamentally, because that formula one by resistance in this brand will result into a infinitely large number.

So, at node A when I am writing it, you see it is a source I will assume but at the end when you solve the circuit, there will be a finite current flowing either this way or that way that you could solve this problem in the earlier method. So, suppose, I assume this current to be except here in this direction it is free and it is related with this 4 volt source. Now, let us see KCL at a what it will be, it will be the sum of all the resistances $\frac{1}{4} + \frac{1}{2}$, no doubt of sorry $\frac{1}{4} + \frac{1}{2}$ will be this

1 and this 1 I will treat it as a current some current takes is flowing out of it and that value is known.

Like that I can do 1/0 I am not going to write I also there is a source and some x ampere will eventually flow through it and that current is flowing out. So, and these I will show it so, far as these 2 currents are concerned $1/4 + 1/2$ will be involved and this will be the coefficient of V_{A0} minus coefficient of V_{B0} at node A will be $-1/2$ into V_{B0} correct $-1/2$ into V_{B0} . So at node A this current is V_{A0} by this one.

So, all the currents have taken this current and this current is only involved V_{C0} because that branch here that current tab assume to be exempt here. So this one I will write it as the sources connected at node A that is -2 divided by 2 and then another current is going away from node A. So, $-x$ this way I will write, then to avoid that problem, so this is an unknown mind do unknown have assumed.

Similarly at KCL at node B, coefficient of V_{B0} will be $1/6 + 1/8 + 1/2$ coefficient of V_{A0} will be $-1/2$ V_{A0} . Common resistances is 2 and coefficient of V_{C0} is $-1/8$ V_{C0} and this will be equal to the sources of which a V_{A0} is nothing but $V_{C0} - 4$ is not V_{A0} is nothing but $V_{C0} - 4$ again. So, another $-V_{C0}$ by 8 and this will be called 2 sources connected at node B.

So that is 2 volt is connected $2/2$ and 5 volt is connected $+5/6$ and that is no source. So, at node A, B written all together how many unknowns are there once again actually here 4 unknowns V_{A0} , V_{B0} , V_{C0} and x. So, I have written this is equation 1, this is equation 2, this is equation 3 is not 123 another equation I need, so I will write KCL at node C and that will be how much it will be coefficient of V_{C0} he is $1/10 + 1/8$ and I will not going to write 1/0 because these have assume to be some current source fitting x ampere current and this I will right on the right hand side.

So, this is coefficient of V_{C0} coefficient of V_{B0} will be $-1/8$ into V_{B0} and coefficient of V_{A0} will not come because it is a current source as if fitting that point x. So, this plus this must be called sources, sources are what 10/10. Let see, this is nothing and there is another current source

coming in plus 6. So, this is the equation 4 and there are 4 unknowns. So, in summary what I am telling that to solve this network by nodal method it is always better to choose or not as a reference because then you are multiplying equation number of equations becomes now 4 to be solved.

In general as I told you in nodal method you have to solve $n - 1$ equations, that is the usual case when there is no such ideal source voltage source present between 2 nodes then if that be the case, then better choose one of those 2 nodes as your reference node, then you really get benefit number of equation is to be solved as I have chosen here C as my difference is only 2 equations are to be written and you have to solve that and so, that is 1 option and the 2 option is you could choose O as your reference then what would do with this branch? Because I cannot do some voltage by resistance here although 4 volt is present.

So, what you can do is this, you assume whatever current it will be flowing is x ampere that I can always assume okay this part is x ampere if that is x ampere and treat it as because the current in the circuit will be finite thing like that. So, finally x will be a finite number whatever that finite number will be, after you solve the network that variable to assume x then apply KCL at A then it will KCL at A.

It will be I mean if you want to write from fundamental $V_{A0}/4$ this way, then $+ V_{A0}$ minus potential of this point that is $V_{A0} + 2$ here coefficient of at 10 at this current time I calculate so V_{A0} , minus the potential of this point, it is V_{B0} . Then to reach from this to this $- 2$ volt divided by 2 this will be this current. So, this current and going here this current is $+ 6$ that is equal to 0. So, coefficient of V_{A0} at node A will be, you see, $1/4 + 1/1$ from this really have got this coefficient of V_{B0} will be $- 1/2$ and on the right hand side what it will be $+ 2/2$ here, take it on the right hand side $- 2/2$ and this $+ x$ comes here, get $- x$.

So, these are the things see network solving problem is I feel it is sort of an art, it will vary which method you will choose from person to person, but always try to see that you have to adopt a method where you have to level less to get the solutions for the given network. So you should be

given a network just did not jump that I will solve it by nodal method. Of course, there will be problems where you will be asked to solve by a particular method.

Then of course, you do not have any other choice you have to solve by that method only if your examiner ask, but if you are given a free hand, apply your intelligence whether should I solve it by mesh analysis or should I solve it by nodal analysis by looking at the topology of this circuit by looking at the various sources which are present in this circuit and so on. I hope you understood this point.

This will be the basis of your so, far what I have done, I have considered only constant value sources, sources are not time adding and only resistance as your other elements present in this circuit no energy storing elements. So, situations look like a bit simpler I have done, but will now soon go into these circuit, where there will be also energy storing elements will be present and also the sources could be any function of time. That will start from our next class. Thank you.