

Network Analysis
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Lecture # 09
Inductor-I

So, welcome to lesson 9 and as I told you that so far we have considered networks involving only resistance has circuit elements and also sources both voltage and current sources, but they are not varying with time that is their DC quantities constant values. For example see the previous networks.

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The slide content includes the following elements:

- Circuit Diagram:** A resistor R is connected between terminals A and B . The current $i(t)$ flows from A to B . The voltage across the resistor is v_{AB} .
- Equations:**
 - $i(t) = \frac{v_{AB}(t)}{R}$
 - $= \frac{v(t)}{R}$
 - Terminal v - i relation $\xrightarrow{v_{ec}-9}$
 - $v_{AB}(t) = v(t)$
 - $i(t) = \frac{v(t)}{R}$
 - Power $= v \cdot i$
 - $= R \cdot i^2$ (will be lost)
- Graphs:**
 - A graph of voltage $v_{AB}(t)$ versus time t showing a periodic triangular wave.
 - A graph of current $i(t)$ versus time t showing a periodic triangular wave that is in phase with the voltage.
- Text:** Resistance \rightarrow is memory less element

All batteries 6 voltage and only it is resistances are present similarly, there are examples with current sources that we have done fixed current sources of certain value. But now, we will go ahead with I should be in a position that the circuit may as I told you can contain in general inductions and capacitance and also the sources may be time varying sources. So first resistive elements let us try to understand here is a resistance.

And the current in this direction which henceforward because I am not sure in general current maybe time varying quantities. So, we will indicate they any time varying current as this one it I do not know sources may be time varying And the direction of the current as I told you is my prerogative I am choosing it. But once I choose it I know the potential of V_{AB} that is if A . This is

point A resistance to terminals, then V_{AB} will be indicated by + wherever I write + in 4, it means potential of this point with respect to this.

So, you can see the voltage current relationship, of a resistance is, will be $= V_{AB}$ which may be also function of time, I do not know and that is resistance. So at any instant of time calculate potential of A with respect to B divided by R will get current in this direction that is very important. So, once you have assumed the direction of the current this will be + - and this is the, this is called the terminal v i relation.

So, small letters we generally are used for any time varying quantity I do not care so $V_{AB}(t)$ and this is also sometimes written as v instead of writing V_{AB} , you can also write it as v if you indicate only + and -. So, v is potential of this with respect to that it means that So, V_{AB} is $v(t)/R$. So, this is this thing. Now, obviously, this $v(t)$ applied voltage across the resistance As I told you could be any function of time, no restriction, I will now put that it is a cost and quantity.

For example, V_{AB} would be this ship this is my v this is how V_{AB} and this is my access time. So far I have considered for DC quantities It was like this voltage was constant t some v but am now telling these voltage could be So, this accesses voltage across time it is a special case of time varying voltage where it is constant at all the time. But here you see at this time bolded is this much is this much and so on it varies with time sometimes it becomes 0.

It may become negative also It does not matter then what will be the current in a resistance current in the resistance will be some constant in to $v(t)$. Therefore, the current if you state for this applied voltage and I will state it against time for a resistive network it will be also of same shape because $v(t)$ by R $v(t)$ is this and this is equal to $v(t)$. If you write only $v(t)$ you must show what is called + - potential of + with respect to -.

So, this is $v(t)$ so the shape of the current will be also like this This is it and this is will be $v(t)$ by R scale down by some number. So, this was this will be the simplest case. So, if the voltage is of any form current will be also of same form because $1/R$ okay. So, this is the story of a resistive

network and also we should remember that, we made resistance power loss power is voltage into current and $v = Ri$ So, this is same as Ri^2 .

Now this power will be lost and resistance all way resist power and it dissipates it in the Never node. Therefore, it will be power will be lost whatever power you input to a resistance that gets immediately dissipated as heat. Therefore, it is not an energy storing elements. And another thing I in this equation if you look at it carefully, it is needless to say that the value of the current at a given time for example, at this time at this particular time t equal to τ .

What will be the value of the current value of the current will depend only on this value of the voltage this v divided by R . R is a constant number. Therefore, a resistance current at a given time will depend only on the magnitude of the voltage a voltage at that time existing it does not depend on the previous value not on the future values. So, current value of the voltage divided by current value of the voltage divided by resistance gives you the current value of the current I mean present value of the current got the point.

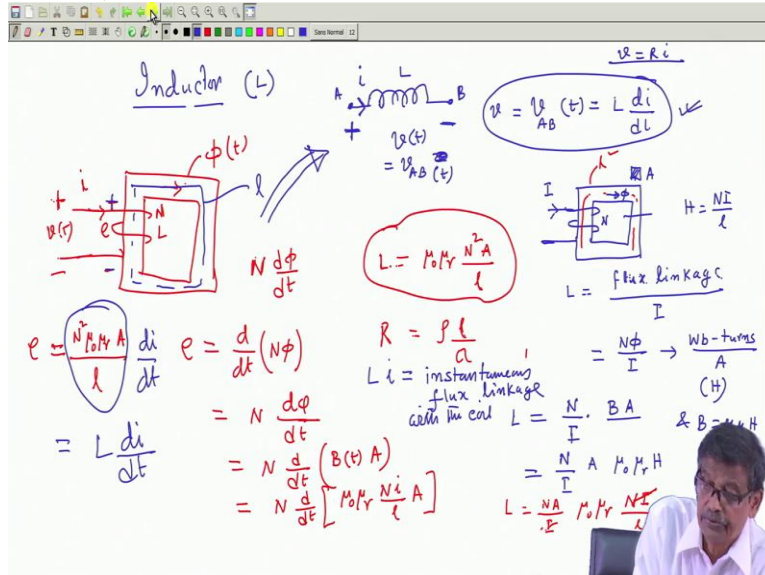
It does not depend on the previous voltage things like that what is the right now what is the voltage that voltage divided by R will give you right now what is the current that is the whole idea. So, this is about resistance that is why resistance are called a term I am writing and which we can easily appreciate resistance ah is a memory less device memory less element that is resistance does not remember how you treated that resistance with previous voltages.

Present now what current value it will return to you depend only on the what is the current or present value of the voltage it does not care what happened earlier to this resistance what kind of voltage is you applied It forgets that it will only respond what is the voltage across me right now Based on that, current value is decided. So, it is often called memory less element it does not remember and all the powers and resistance will always receive energy there will be no situations.

So, when actual value of the current will be like that, that is negative resistance will we are not dealing with that, it will never deliver current to your circuit resistance is a dissipative element

this you must remember Anyway, now, let us come to and inductive circuit. So, this is the voltage current relationship. So, this diagram is important this diagram always you keep it in your mind and this is now let us see.

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What is an inductor which is denoted by L , and now inductance will be represented by this symbol L and a value will be written which was a unit is Henry R is the known gnomes and it is in Henry. Now, if you apply a voltage across it that is V and if you want to write with respect to potential difference we understand since I have written class, we are telling that I am concerned about this voltage V_{AB} have applied and current I will always assume it is entering through the plus.

Then the voltage across this inductance V_{AB} t and small letters. Let it be time varying I do not care. So V_{AB} t at any instant what is the voltage across the inductors will be $L \frac{di}{dt}$ and the polarity of the voltage is $+$ and $-$ like that were through whichever terminal current is entering that will be the plus sign of this voltage. R if this is $+$ - you assume that is we are assuming two things. l is direction of the current have assume and $\frac{di}{dt}$ is positive. That is why this is the convention I will be following and everybody follows that.

Now the question is inductance as such, I will just tell a few words about that inductance is generally manufactured with some magnetic code and some turns are whom, like that. And if you pass some current I say DC current to pass, then there will be some flux produced in inside

the core of this magnetic material, then inductance is defined as flux linkage per unit ampere that is if you pass I ampere calculate what is the flux linkage that is if n is the number of turns it will be $n\phi/R$ if you pass one ampere current what is the flux linkage that is we get ϕ/R .

So, so, each unit basic unit is we get ϕ/R per ampere and this is called Henry one henry inductance whatever comes out that much Henry it will be that is if you pass 1 ampere current if the flux linkage is also one we get but turns then it is an 1 Henry inductor anyway this is the t and to find out and if you wish, how physically it will depend you can write it several In any book you consider ϕ is nothing but flux density into cross sectional area of this magnetic path.

So, ϕ is $B \times A/R$ then if you are assume if you assume linear magnetic circuit that is what if you want to get a constant magnitude inductance then we know so L is this and $B = \mu_0 \mu_r H$, these we know from magnetic circuit. So, put this value VAO there I wrote I^2 times. So, this is the thing and ϕ by R now this will then become $N^2 I^2 \mu_0 \mu_r A / (4\pi R)$, you can write it as $\mu_0 \mu_r N^2 A / (4\pi R)$, But you know that value of H is an I/L . What is L ? L is the meant length of the flux part.

These we know I assume will not go into detail, so, this length is L . So, write it down so, L becomes then in a NA by I then $\mu_0 \mu_r$ these are permeability is this one and this is NI/l therefore, inductance This I will then constant here and you will be left with inductance L is as $\mu_0 \mu_r N^2 A / (4\pi L)$. So, you see inductance is a value attached to a coil which does not depend upon the magnitude of the current flowing like this formula for a resistance we know whether you are passing a current or not.

The resistance value is $R = \rho L/A$ it depends upon the physical dimension of the way is not L is the length of the way A is the cross sectional ADR ρ is the resistivity. So, similar to that, this is the formula for inductance which is a constant number provided μ_r is a constant value that is VH curve is linear. Anyway, we will not go much beyond this. So, inductance can be defined as knowing the dimension of the code et cetera cross sectional area mean flux length l and the permeability of the core.

And knowing the number of turns you have whom to create this inductor, the value of the inductor will be this and it is in meter square L is in meter and $\mu_0 \mu_r$ in appropriate unit it them you can express the cell as a in terms of Henry, that that is all we need. So, this cell of course, we will not draw the core et cetera I will just draw like this it has got a finite value based on its physical dimensions, and then I am telling that the terminal relationship, see after all, if this I becomes time varying, then this flux will be also time varying.

And the flux is time varying there will be an induced voltage here. So, let me draw this inductance to remove any doubt you still might be having that is suppose, this is your inducted with magnetic code. This is the thing this is L and this is suppose i time varying current time passing if you are passing a time varying current flux produced in the code will be also time varying ϕ t is not flux in the code will be also time varying because ϕ A is after all can be related with age μ_0 .

So, i is time varying so ϕ t now if i time varying flux links this coil How much will be the induced voltage in unit across this, there will be now an induced voltage, the magnitude of that voltage is a $d\phi/dt$ is not $n d\phi/dt$ it is the magnitude of the voltage that is d/dt of $n\phi$ this will be the induced voltage between these 2 points and I am assuming this where has got no resistance pure inductance.

So, there will be induced voltage and here is your supply voltage v_t with this + this - and this is the induced voltage across the coil, the polarity of the induced voltage will be such that it will try to oppose this change in flux if flux is increasing with time if I is increasing with time ϕ is increasing with time then it must have a polarity this side + this side - so that it can opposes this supply voltage to have any tried to oppose the change in the current value.

Therefore, $n d/dt$ of with n is constant $n d/dt$ is this voltage they you write $N d/dt$ of ϕ is B into A, it will become a time varying thing like this, then you write $N d/dt$ of B is $\mu_0 \mu_r$ into age and age is Ni/l and of which if you take out all the constants so this is the induced

voltage here across the coil, so e will become is equal to this same will come out $N^2 \mu_0 \mu_r$ is can be also taken outside because they are threaten to be constant only i is the variable.

So, $e = N^2 \mu_0 \mu_r A$ divided by l what is this small l is this main length of the magnetic circuit it depends upon the physical dimension of the coil small l . So, these seem to di/dt , but we have already defined this to be my inductance look at this. So, we write it as $L di/dt$. So, mind you that L into i this quantity is the instantaneous flux linkage 10 years flux linkage with the coil flux linkage with the coil I think we have got a fair idea of an inductor.

And what is the voltage accuracy is this $L di/dt$ L is the constant property of this coil and it will be expressed in terms of that, if di/dt is positive current is increasing, then the polarity of this induced voltage will be this is + this is -. So, all these things I summarized in this diagram, while you presenting an inductor in a circuit to it do not draw this elaborate diagram to indicate inductor this by coil.

You assume current is entering through any point as I told you direction of current assumption is our priority whoever is solving this circuit is free to choose in which direction current is flowing. But once it chooses that And also assumes di/dt is positive, e has to mark the voltage across the inductor is in this way in whichever terminal current is entering that is plus that will be minus that is the whole idea of an inductor.

So, in case of resistance we have seen that $v = R i$ that was a relationship between voltage and current terminal quantities. This is how it was related. In case of inductor we find this I will also right sometimes v as I have told you v means potential of plus with respect to -. So, v find that voltage across an inductor and current flowing through it is not equal to some constant L into i , but it is L into di/dt that is most important what is the implication of that implication of this is if you know the current value of the voltage to find out the present value of the current.

It will not be of same shape as that of the voltage because there is di/dt you know from that f integrate and get the value of the current. So, let us try to see what I am.

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$i(t)$ L
 $+ v(t) -$
 Inductor is a memory element

$L \frac{di}{dt} = v$
 $\frac{di}{dt} = \frac{1}{L} v(t)$
 $di = \frac{1}{L} v(t) dt$
 $i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$

$i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$

If an inductor carries a fixed current I (which is constant) the voltage across the inductor will be $v = 0$

Trying to tell suppose, you have an inductor L and you have chosen the current direction like this and this is the voltage which varying a time varying voltage therefore I know that $L \frac{di}{dt} = V$ this is always true. My problem is I want to find out the current. So current to find out current, so what have to do I have to first get what is $\frac{di}{dt}$, it will be $= 1/L$ into V t that is all then you can ride di as 1 by L into V t dt .

Now to find out the value of the current at any time t then I have to integrate this to get the value of the current. Now the question is what should be the limit of this integration how do I integrate it look here, this point to listen carefully what I meant to say this is the time and your voltage waveform as I told you could be anything any function of time, let us say the voltage waveform was doing like this, this is your V t this is your $t = 0$ and this side it is t is equal to minus infinity and this side it is going towards the infinity positive infinity.

So, suppose let us say that I want to find out I have applied such a voltage across an inductor and I want to find out the current flowing through it at a given time t . So, at this time t what will be the current knowing the fact I have applied this sort of voltage whatever it is V t then to find out that current at any time t I have to integrate this 1 from t is equal to minus infinity to t is not this is what I have to do this is the integration I have to do.

So, what will be this integration this integration will be $i(t)$ no doubt di if you integrate $i(t)$ you get straight away. And this side you see it is equal to $1/L$ t is equal to minus infinity to t you have to integrate up to time t . So, sometimes here is t sometimes what people do is this it is a very scientific way of writing. Suppose $di/dt = 1/L \int_{-\infty}^t V \tau$ give right. You want to find out current at any time. So, $L di/dt = 1/L \int_{-\infty}^t V \tau$.

So, $di = 1/L \int_{-\infty}^t V \tau d\tau$ and then i at any time t will be $1/L \int_{-\infty}^t V \tau d\tau$ like that also you can write anyway the point I want to make it this is $V \tau d\tau$ this is the thing, there will current at this time t does not depend on the current value of $V \tau$ alone because this whole if you want to integrate these $V \tau d\tau$ it means what it means the area under discard. $V \tau$ into $d\tau$ is nothing but area under discard is not therefore, to calculate the area.

And that this voltage time card up to time t , he will require all the voltages previous voltages that you had applied across the inductor. Unlike resistance cannot remember anything present value of the current is present value of the resist voltage applied divided resistance, but for inductance it is a different volt tern altogether it says that the current at this point that is $i(t)$ will depend not only on the present value of the voltage that is this 1 but also you required to know all the voltages applied previously from t is equal to minus infinity.

And that will decide the value of the current at time t . Therefore, correctly then inductor is a memory element inductor is a memory device, memory device, wait memory element sort of memory element why, because inductor will react at time t how much current it will allow to pass through the inductor. It remembers how you treated the inductor previously with what kind of voltage for how long those integration is necessary.

Therefore, previous information is will be necessary to know the current in the inductor at a given time if you want to know the current at this time, okay all the area up to that time you have to calculate. So, it looks like a very stupendous task, I what to do in depth remembers everything I mean how we be able to the inductor with applied voltage previous to time t equal to t less than t okay that is that will not be a problem at all We will see soon, but this is the most important equation one has to remember.

So, I stop here today, please try to understand this equation and the inductors what do I mean by you and so on only one point I will tell that if at any point of time inductor suppose for a wide range of time inductor is carrying a constant current set 2 ampere current constant it is not changing with time I can at least say one thing. If the if you are allowing an inductor to carry a constant current, then voltage across the inductance will be 0 if and inductor somehow carries a fixed current.

Fixed current say I that is which is constant value which is constant then voltage across the inductor will be $L \frac{dI}{dt}$ which is constant damn telling does not change it a must be true. That means, these 2 points there will be no voltage drop, therefore inductor there will be a voltage across the inductor only when the current through it will become a time varying quantity. If this quantity is constant then I am sure there will be no voltage drop across the inductors. So more on this topic. In our next class. Thank you for listening.