Power System Protection
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Lecture No. 10
In the Presence of Decaying DC

Welcome to the NPTEL power system protection course. So, we are in module 2 on phasor estimation.

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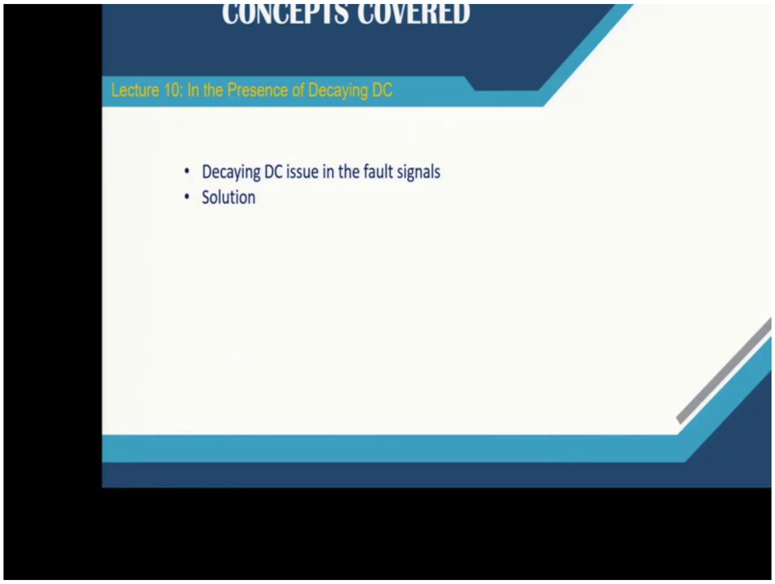

In this lecture we will address how decaying DC becomes an issue to phasor estimation techniques and what are the mitigation strategies for that.

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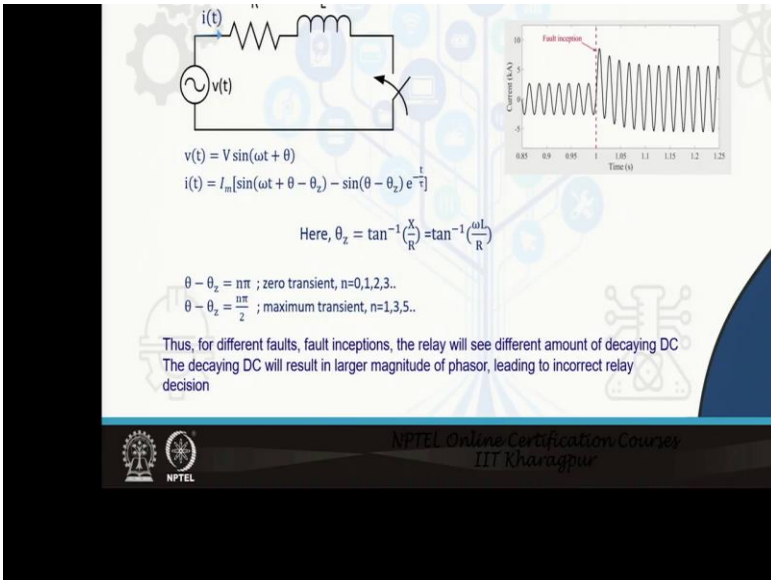

So continuing with that, let us take a, take an equivalent system, we have a voltage source and a transmission line that can be an approximate model have resistance and inductance only and then a fault is triggered. This is RL transient situation, in that case the corresponding equation of system voltage being sinusoidal can be like this.

$$V(t) = Vsin(\omega t + \theta)$$

And the corresponding i(t) during this fault period will be

$$i(t) = I_m[\sin(\omega t + \theta - \theta_z) - \sin(\theta - \theta_z)e^{-\frac{t}{\tau}}]$$

Where, τ is the time constant that is L/R of the circuit and θ_Z which you have in these two expression is obtained from,

$$\theta_z = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\omega L}{R}$$

ω, corresponds to the angular frequency of fundamental components. Note that in this transient current, we have an exponential term which is exponentially decaying and a steady sinusoidal current. So, this is voltage sinusoidal, this is current sinusoidal. So, in this part θ is fixed and θ_Z is also fixed depending upon the L and R. So, this part is a constant term and this is an exponential term, so this part is nothing but that decaying DC part. So, we see that for a sinusoidal voltage signal if a fault happens to be there, the current is expected to have sinusoidal plus decaying DC part. This is a typical signal here, this is the before fault current, pre-fault current, and fault is incepted at one second in this case and then we have a current signal like this steady state sinusoidal current and in this transient period, it has a decaying DC component and that is what this component is doing in addition to the sinusoidal part. So, this with the large time T, this part vanishes, this becomes 0. Therefore, this part becomes no more contributing to the signal. Here in this case, how these component influencing we will see that

$$\theta - \theta_z = n\pi \text{ ; Zero transient, n=0, 1, 2, 3}$$

$$\theta - \theta_z = \frac{n\pi}{2} \text{ ; Maximum transient, n=1, 3, 5}$$

In the first case there will be zero transient for n= 0, 1, 2, 3, only sinusoidal current signal presents. On the other hand when $\theta - \theta_z$ will be $\frac{n\pi}{2}$ the sinusoidal component becomes 1 and transient becomes maximum for n equals to 1 3 5.

However, the switching event, θ will depend upon that what instant it is being switched on. Accordingly the magnitude of decaying DC will come into the picture to this corresponding current signal. Note that this decaying DC part is an exponential part what we have discussed in frequency response is either harmonics, inter-harmonics or complete only DC not exponential term not decaying DC. So, how good is the corresponding phasor estimation technique to this challenge that you have to investigate? This is what we will see from here that depending upon the fault inception, fault location and different types of fault, the corresponding relay will be observing decaying DC component, and this is an additional component to the sinusoidal component. Our target is to capture the fundamental from this part from this modulated signal, therefore, what we see here is that to use the corresponding things in that manner. Hence, the purpose is that how influencing this part is to the phasor estimation technique. These decaying DC become significant for this $n\pi/2$ perspective, then we see that the current magnitude will be affected and therefore over current relay, distance relay and so, which use the corresponding fundamental component current are expected to malfunction, to avoid that we need solution, mitigation strategy to eliminate the corresponding decaying DC part from the fundamental phasor estimation process.

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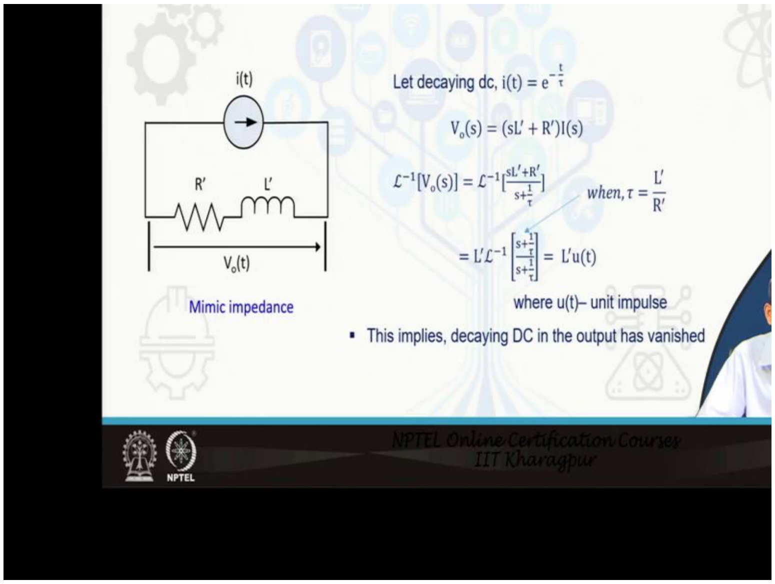

For that, we have a concept called mimic filter. Let us see what happens using this approach. First we will talk about analog filter and then we will go to the digital mimic filter. So, this is the current which is there during the transient process, this is the fault current in the primary side and it is injected to R' and L' combinations. So, this fault current is being injected to an R' and L' combinations and we are taking the output of this $V_0(t)$ to the relay site. This part is called the mimic impedance. Now, how this corresponding circuit is useful we will see here.

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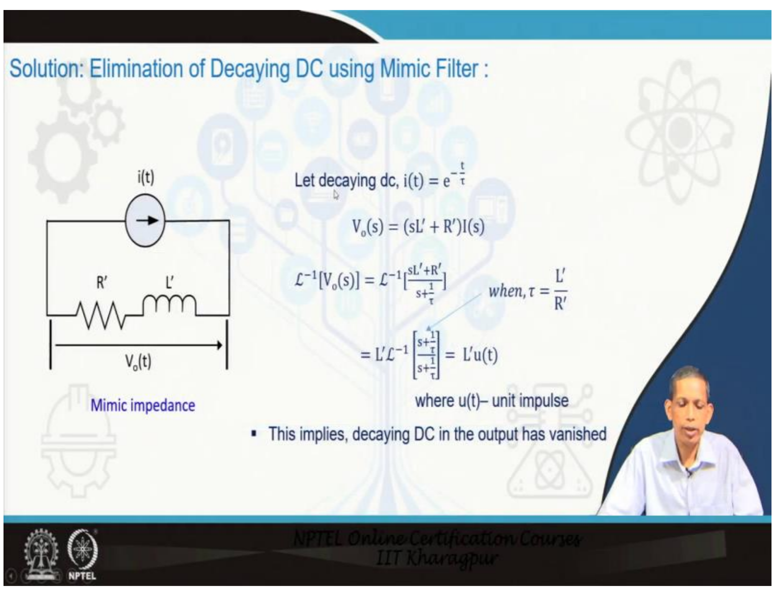

So, let the decaying DC current is

$$i(t) = e^{-\frac{t}{\tau}}$$

This τ is the time constant L/ R of the power system circuit as you have seen in the earlier slide. Therefore, $V_0(s)$ the Laplace transform of output voltage will be

$$V_0(s) = I(s)(sL + R)$$

The Laplace inverse of this voltage which will give us the

$$V_0(t) = \mathcal{L}^{-1}[\frac{sL' + R'}{s + \frac{1}{\tau}}]$$

Now, when

$$\tau = \frac{L'}{R'}$$

This is the filter we have put in the circuit connecting to the relay, when $\tau,$ the time constant of the power system matches with the time constant of this additional circuit which you have put here; in that case we see from this expression,

$$\mathcal{L}^{-1}\left[\frac{sL' + R'}{s + \frac{1}{\tau}}\right] = L'u(t)$$

Where u(t) is the unit impulse. So, the output in i(t) that consists of decaying DC in the power system, but if we connect the corresponding signal to R' and L' with condition that is $\frac{L'}{R'}$ equals to the time constant of the power system circuit, then the output of this in terms of voltage does not contain any decaying DC; that is the beauty of this filtering process. This part you can say is called the mimic impedance, mimicking the power system circuit in terms of the perspective of $\tau = \frac{L'}{R'}$ and this process eliminates the decaying DC part.

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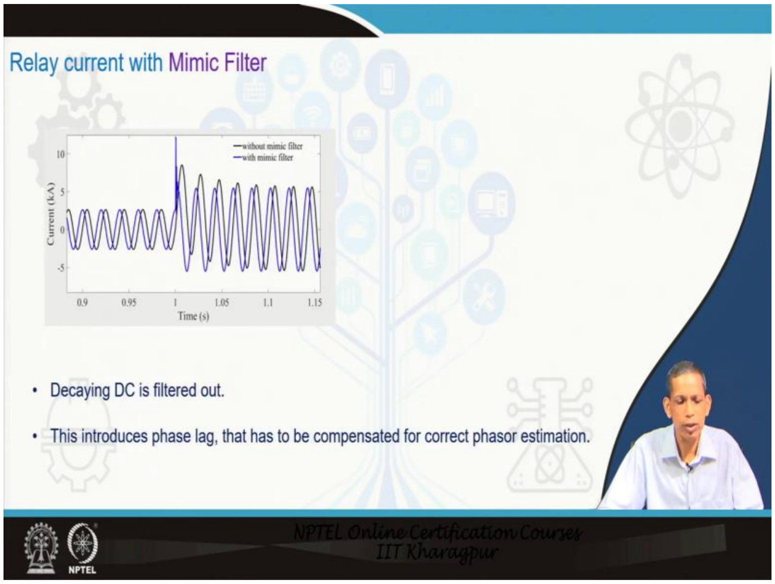

Now, if you see this as performance wise, the results what you see is without mimic filter and with mimic filter. Mimic filter introduces a phase lag perspective because this is a RL combinations. So, we have these corresponding things, without mimic filter we have the black one that having the decaying DC, with mimic filter we have the blue one where the corresponding decaying DC part is almost overcome, but there is a phase displacement due to the presence of the L in the circuit that need to be compensated in the phasor computation techniques.

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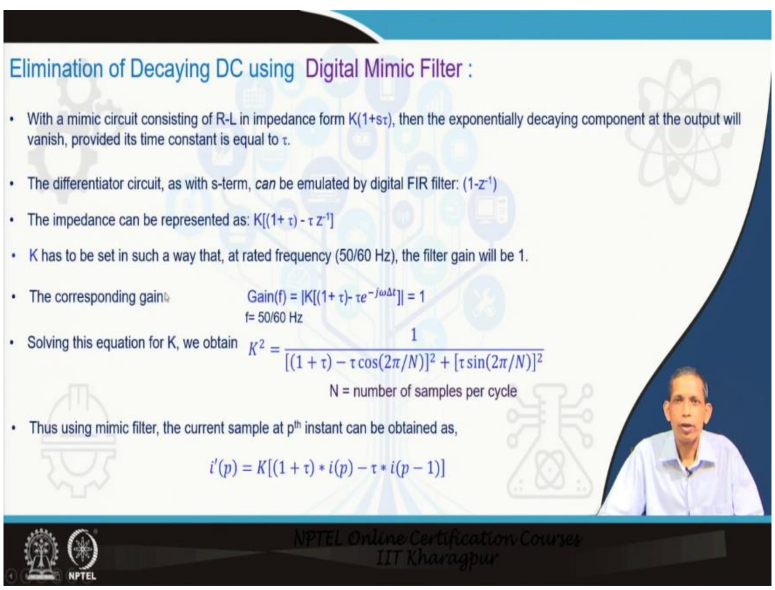

Now, extending this idea of putting and R' and L' in this circuit that is a mimic filter, you can emulate this to digital filter also. Now, let us see with mimic filter consisting of RL combinations and which is nothing but represented in terms of Laplace transform into K(1+sτ), τ the time constant of the circuit, then the exponentially decaying component of the output will vanish, that is what you observed and with the time constant τ matching with the L' and R'.

So, the 's' term in this is nothing but actually a differentiator circuit and that with a digital filter can be represented by$(1 - z^{-1})$. Therefore, substituting 's' by$(1 - z^{-1})$. This impedance becomes $K[(1 + \tau) - \tau z^{-1}]$. K has to set such that the filter gain has to be 1 for the 50 or 60 Hz nominal frequency. In that case, we have from these relations,

$$G(\mathrm{f}) = |K[(1 + \tau) - \tau e^{-j\Delta\omega t}| = 1$$

For the corresponding 50 or 60 Hz frequency. In our earlier examples, we are considering 50 Hz nominal frequency. Thus, solving this equation, we got

$$K^2 = \frac{1}{[(1 + \tau) - \tau \cos\left(\frac{2\pi}{N}\right)]^2 + [\tau \cos\left(\frac{2\pi}{N}\right)]^2}$$

Where N corresponds to the number of samples per cycle. Thus using the mimic filtering concept, the signal output for a current samples at p$^{\text{th}}$ instant is obtained from

$$i'(p) = K[(1 + \tau)i'(p) - \tau i'(p - 1)$$

So, for the p$^{\text{th}}$ instance the corresponding output in the mimic filter against that signal will have no more decaying DC component. This differential current of present sample and just past sample with this weightage will leads to elimination of the decaying DC component successfully. This is what the digital mimic filtering approach is.

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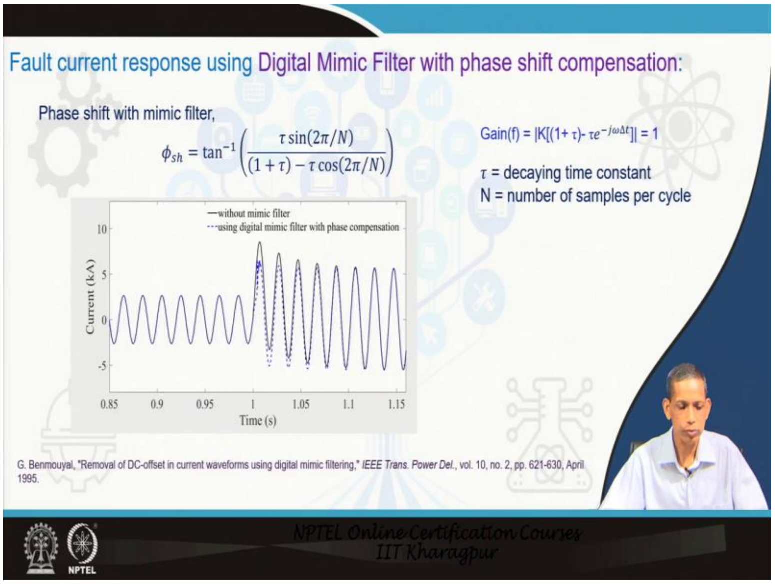

However, if you see this gain, then for a particular nominal frequency the gain becomes 1 it has an imaginary and real component. So, the phase shift by the mimic filter, which we see in the analog domain also, that becomes equal to

$$\varphi_{sh} = \tan^{-1}[\frac{\tau\sin(\frac{2\pi}{N})}{(1+\tau) - \tau\cos(\frac{2\pi}{N})}]$$

This phase shifting can be obtained and that can be compensated in the digital platform. So, by doing that compensating we have two plots; without mimic filter and using a digital mimic filter with phase compensation, we say that the black one is without mimic filter; it has a decaying DC component and with digital mimic filter and compensating this phase compensation, you get the corresponding blue one with the dotted points. Therefore, we see that it has overcome the decaying DC part and also the phase is being compensated; and the current signal does not require any further change in angle as compared to the voltage or so.

Note that such exponential decaying part that we have demonstrated here for the current can be applicable to voltage also, particularly when it is using the capacitor voltage transformer, when the fault happens to be close to that, the corresponding output signal contains exponential decaying part. So, in that case also we can put such perspective to overcome the issue to extract the fundamental component successfully.

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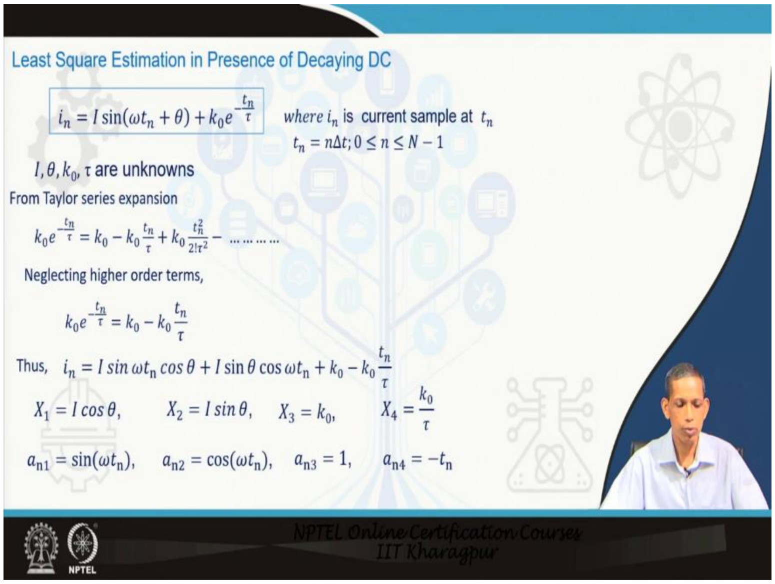

Now, from the above discussion we see that the 1 cycle DFT which is influenced by the decaying DC part can integrate with the digital and analog mimic filter before processing for phasor estimation technique. Now, we will see how the corresponding decaying DC can be incorporated in the least square estimation modeling perspective and how it can successfully eliminate the decaying DC part.

So, we know from our earlier discussion that the decaying DC signal is an exponential term and have a sinusoidal component. Therefore, for the nth sample of the corresponding signal can be written as

$$i_n = Isin(\omega t_n + \theta) + k_0 e^{-\frac{t_n}{\tau}}$$

So, we can model the corresponding fault current component with a decaying DC and a sinusoidal component as you see.

Now, in this case the corresponding I, θ, k_0, and τ are the unknowns in this signal, which need to be estimated to get the fundamental correctly. Now, if we expand this using the Taylor series, then

$$k_0 e^{-\frac{t_n}{\tau}} = k_0 - k_0\frac{t_n}{\tau} + k_0\frac{t_n^2}{2!\,\tau^2} - \cdots$$

So, approximating for this series and neglecting higher order terms we first only consider these first two. If you require more accuracy, you can extend it or take it to further terms also. Thus, considering only first two terms these decaying DC part can be

$$k_0 e^{-\frac{t_n}{\tau}} = k_0 - k_0 \frac{t_n}{\tau}$$

Therefore, expressing this corresponding current

$$i_n = Isin(\omega t_n)\cos(\theta) + Icos(\omega t_n)\sin(\theta) + k_0 - k_0 \frac{t_n}{\tau}$$

If you remember in least square estimation technique, we use the same in this part as earlier. Now, we will set the unknown vector and the corresponding A matrix. The unknown vectors, as like to the earlier case become

$$X_1 = Icos\theta \ ; X_2 = Isin\theta \ ; X_3 = k_0 \ ; \ X_4 = \frac{k_0}{\tau}$$

So, these are the 4 unknowns in this signal that we like to estimate in our least square formulation and the corresponding small a coefficients for this signal model are

$$a_{n1} = \sin(\omega t_n) \ ; a_{n2} = \cos(\omega t_n) \ ; a_{n3} = 1 \ ; \ a_{n4} = -t_n$$

So, to multiply these a and the corresponding X then you will get the original signal.

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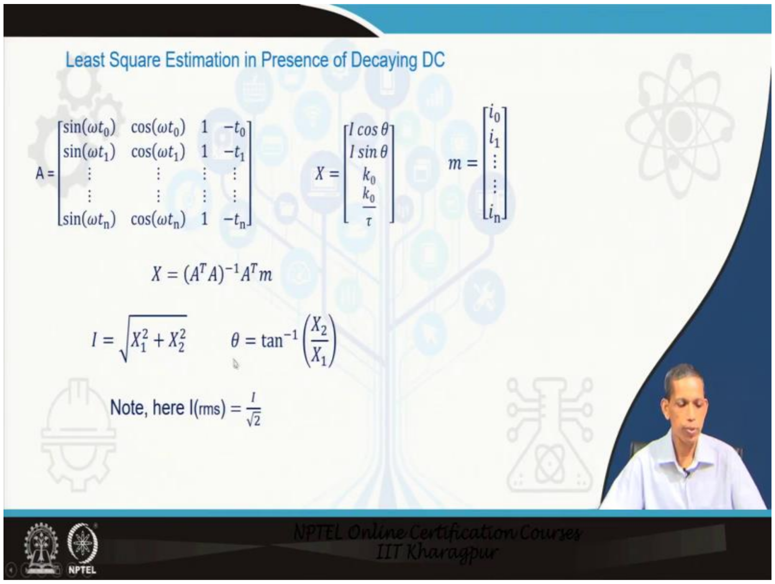

In that fashion, for the four unknowns that are 1 2 3 and 4 and we have 1 2 3 4 in the A matrix and like that to be taken and from 0^{th} samples, then you will multiply this A into X, you get i_0 to i_n the n+1 samples in the measurement process. So, we say AX = m and this is the corresponding signal model perspective for the system.

$$A = \begin{bmatrix} \sin(\omega t_0) & \cos(\omega t_0) & 1 & -t_0 \\ \sin(\omega t_1) & \cos(\omega t_1) & 1 & -t_0 \\ . & . & . & . \\ . & . & . & . \\ \sin(\omega t_n) & \cos(\omega t_n) & 1 & -t_0 \end{bmatrix} \quad X = \begin{bmatrix} I\cos\theta \\ I\sin\theta \\ k_0 \\ \frac{k_0}{\tau} \end{bmatrix} \quad m = \begin{bmatrix} i_0 \\ i_1 \\ . \\ . \\ i_n \end{bmatrix}$$

Here A is available to us like we did for the phasor estimation using least square estimation technique, X is to be found out and measurements are the samples of the currents that are also available to us. Therefore, the X the unknown vectors become

$$X = (A^T A)^{-1} A^T m$$

So, then from there you can find out the

$$I = \sqrt{X_1^2 + X_2^2} \quad ; \quad \theta = \tan^{-1}\frac{X_2}{X_1}$$

Note that the corresponding RMS value of current is

$$I(rms) = \frac{I}{\sqrt{2}}$$

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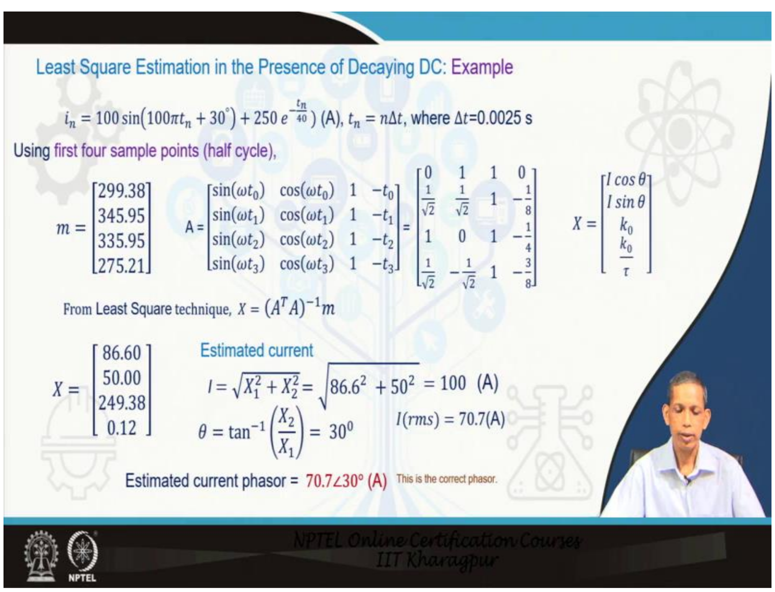

For example, in the least square in presence of decaying DC how good is the least square estimation technique in eliminating the decaying DC, we can evaluate here. Let us take a signal

$i_n = 100\sin(\pi t_n + 30^0) + 250e^{-\frac{t_n}{40}}$; $t_n = n\Delta t,$ Where $\Delta t = 0.0025s$

Using first 4 samples points only that is half cycle least square part, because we have 4 unknowns. So, at least we require 4 samples. We consider the 4 measurements for this signal, and then the corresponding *A* and *m* matrix we have defined are expressed as

$$A = \begin{bmatrix} \sin(\omega t_0) & \cos(\omega t_0) & 1 & -t_0 \\ \sin(\omega t_1) & \cos(\omega t_1) & 1 & -t_1 \\ \sin(\omega t_2) & \cos(\omega t_2) & 1 & -t_2 \\ \sin(\omega t_3) & \cos(\omega t_3) & 1 & -t_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 & -1/8 \\ 1 & 0 & 1 & -1/4 \\ 1/\sqrt{2} & -1/\sqrt{2} & 1 & -3/8 \end{bmatrix}; m = \begin{bmatrix} 299.38 \\ 345.95 \\ 335.95 \\ 275.21 \end{bmatrix}$$

Using the least square techniques, the estimated values of *X* to be considered from this relation is given by

$$X = \begin{bmatrix} I cos\theta \\ I sin\theta \\ k_0 \\ \frac{k_0}{\tau} \end{bmatrix} = (A^T A)^{-1} A^T m = \begin{bmatrix} 86.60 \\ 50.00 \\ 249.38 \\ 0.12 \end{bmatrix}$$

From the first two elements of *X* matrix, we get the corresponding estimated value of *I* and θ are given by,

$I = \sqrt{X_1^2 + X_2^2} = \sqrt{86.6^2 + 50^2} = 100$ (A) ; $\theta = \tan^{-1}\frac{X_2}{X_1} = 30^0$

Finally, RMS value of current will lead to be

$$I(rms) = \frac{100}{\sqrt{2}} = 70.7 \text{ (A)}$$

Therefore, we say that the estimated current phasor we consider $70.7\angle 30^0$ is the correct phasor. Because, if you see here this current signal which you have considered here is $100\sin(\pi t_n + 30^0)$ is the fundamental part and this is decaying DC part. So, even the corresponding signal is being contaminated with this decaying DC, this 100 /√ 2 is nothing but 70.7 that we correctly got from the phasors by using this least square technique, where we have model the decaying DC component in the least square approach. Thus we see least square estimation technique is able to eliminate the decaying DC component successfully.

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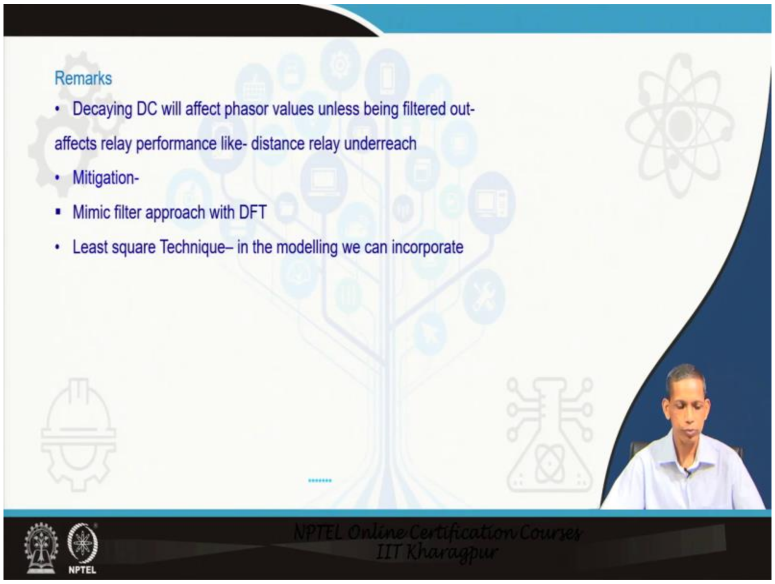

So, as a remarks, the decaying DC components can remain in the current signals and which may affect the performance of the phasor estimation technique unless it is being filtered out. If it is not being filter out and affects the phasor estimation technique, then the relay performance will be affected like distance relay can see with higher current underreach issue over current may find CT saturation issue, and also the over current may have a coordination problem and so. Therefore, to mitigate such things, we have a mimic filter, both analog and digital type along with the DFT, if we use it then decaying DC can be eliminated. The cosine filter has its inherent capability for suppressing the decaying DC component or so, which we have already talked earlier. In least square technique with proper modeling, we can overcome the decaying DC component. So, this is all for the phasor estimation techniques. Thank you.