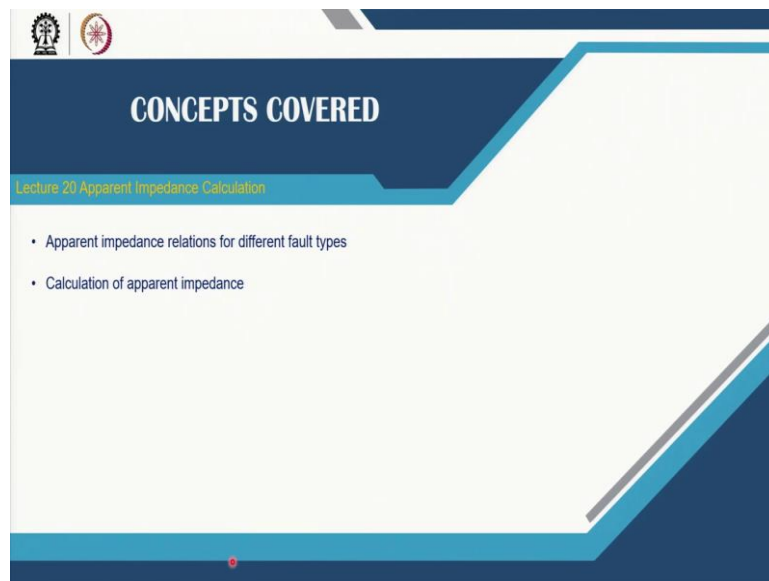


**Power System Protection**  
**Professor. A. K. Pradhan**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture No. 20**  
**Apparent Impedance Calculation**

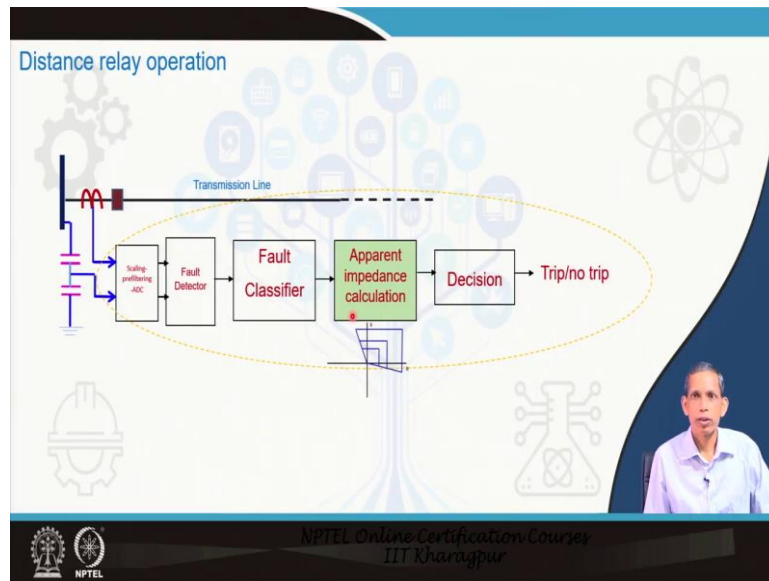
Welcome to NPTEL course on Power System Protection. Today we will continue with Distance Relaying lecture on Apparent Impedance Calculation.

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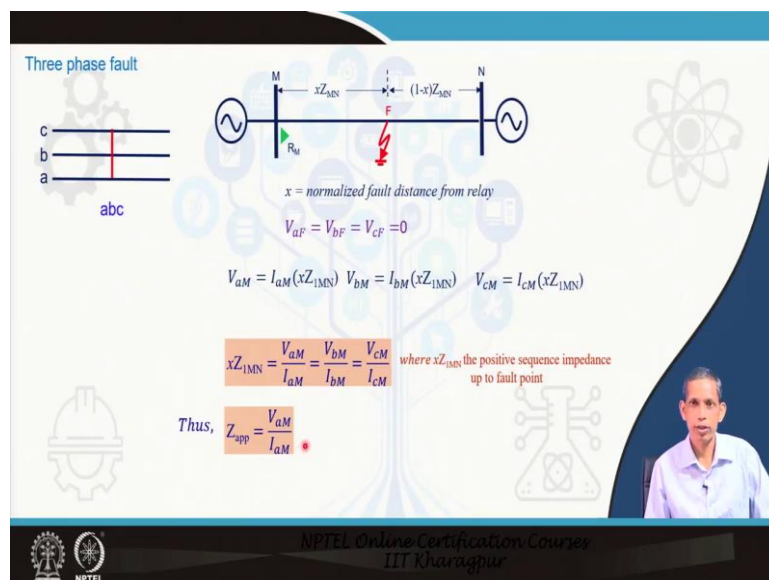
So, our focus will be on apparent impedance relations for different faults, calculation of apparent impedance.

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So, last class, if you remember, we have this basic configuration of a distance relay having different steps. Fault detection, fault classifications, we discuss in detail for the Fault Classifier. And then the step is apparent impedance calculations, which we will be dealing with today. And the rest is the decision part. So, our focus will be here on today on apparent impedance calculations which the relay does before checking whether this impedance is inside the characteristics or not for the final decision.

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So, let us see for different types of fault, how the apparent impedance calculation will be related. 3-phase fault, balanced, so we have 3-phase fault. So, let us say this is the 2-bus system. We have a transmission line between M and N's buses, and our distance relay is at  $R_M$ . We

know distance relay uses local voltage and current signals. So, we will be using only the signals available at bus M. A Fault happens to be there in the line, and the category of fault is 3-phase fault. Let us assume that  $x$  corresponds to here, the normalized fault distance. So, here  $x$  implies 0 to 1 for this line.

So, we have 2 sections from Z, M to F and from there Z, F to N. So, for that corresponding section, we have the corresponding impedance happens to be  $xZ_{MN}$  and  $(1 - x)Z_{MN}$ . Now, for a 3-phase fault, we know at the fault point, this point, F point, the 3-phase voltages become 0. So, therefore,  $V_{af} = V_{bf} = V_{cf} = 0$  voltage. Now, with this relation, for a 3-phase fault, we have, we can write down,  $V_{aM} = I_{aM} \cdot xZ_{MN}$ , this impedance because this point voltage is 0.

So, this voltage becomes equals to  $V_{aM}$ , corresponding current in phase a becomes  $I_{aM}$  and the corresponding impedance, and similarly,  $V_{bM} = I_{bM} \cdot xZ_{MN}$ , this corresponding impedance and  $V_{cM} = I_{cM} \cdot xZ_{MN}$ , this impedance. What do you mean by that, that the corresponding voltage here is 0, so voltage at this point you can say is nothing but the corresponding current here and the related impedance, the impedance is  $xZ_{MN}$ . So, that is what we say for each phase, there are, the relations are similar.

So, therefore, to, let, here the corresponding positive sequence /  $I_{aM}$  or  $V_{bM} / I_{bM}$  impedance  $xZ_{1MN}$  from any relation for out of these 3, we can say  $V_{aM}$  or  $V_{cM} / I_{cM}$ . Note that for, if you go to the impedance corresponding to here, we are talking about a positive impedance because for the 3-phase fault only positive sequence network remains. So thus, the corresponding impedance from the relay bus to the fault point, is  $xZ_{1MN}$  positive sequence impedance can be obtained from the relations, voltage/current, any phase voltage by corresponding phase current.

So, these fault, the corresponding  $Z_{1F}$  or the fault impedance, positive sequence fault impedance up to the fault point, which the relay calculates can be obtained from the voltage and current of any phases any phase. So, that therefore the  $Z$  apparent, which the relay has to calculate is nothing but this part, so that can be obtained  $V_{aM}/I_{aM}$  or  $V_{bM}/I_{bM}$  or  $V_{cM}/I_{cM}$ . So, the relations for apparent impedance, which the relay will calculate the relay will calculate at bus M becomes  $V_{aM}/I_{aM}$  or  $V_{bM}/I_{bM}$  or  $V_{cM}/I_{cM}$  in case of it 3-phase fault.

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Example: Three phase fault at 30 km from bus M

Fault current and voltage data at Relay ( $R_M$ )

Currents		Voltages	
$I_a$	$0.79 \angle -151.73^\circ$ kA	$V_a$	$6.79 \angle -65.81^\circ$ kV
$I_b$	$0.79 \angle -88.26^\circ$ kA	$V_b$	$6.79 \angle -174.18^\circ$ kV
$I_c$	$0.79 \angle -31.74^\circ$ kA	$V_c$	$6.79 \angle -54.19^\circ$ kV

Calculate the apparent impedance seen by the relay at M.

Solution-

$$Z_{app} = \frac{V_{aM}}{I_{aM}} = \frac{6.79 \angle -65.81^\circ}{0.79 \angle -151.73^\circ} = 0.6148 + j8.6044 \Omega$$

Check-

$$\text{Fault distance} = \frac{0.6148 + j8.6044}{0.02 + j0.28678} = 30 \text{ km}$$

Note for protection decision,  $Z_{app} < Z_{1MN}$

Line impedance data:  
Positive sequence impedance per km  $z_1 = 0.02 + j0.28678 \Omega/\text{km}$   
 $Z_{1MN} = 120 (0.02 + j0.28678 \Omega) = 2.4 + j34.41 \Omega$

Now, we will go to an example for such a system, 230 kV, system 120 kilometre line length, and we have the positive sequence impedance per kilometre small  $z_1 = (0.02 + j0.28678) \Omega$  per kilometre. So, this is nothing but from the resistance and the corresponding inductance of the line available. So therefore, the corresponding total positive sequence impedance of the line  $Z_{1MN}$ ,  $Z$  is  $120 \times$  per kilometre impedance. So, that gives us  $Z = (2.4 + j34.41) \Omega$ .

Out of this total impedance, a fault is created at 30 kilometres from this 3-phase fault. And the fault current recorded at the relay bus are like this, 0.79, 0.79, 0.79 each phase. And then we have the corresponding angles for phase a, phase b phase c. Similarly, phase a, phase b phase c voltages are having 6.79, much down because of the 3-phase fault close by to the relay bus, and the corresponding angles are there. So, this is a 3-phase balanced fault case, so that is why we are observing similar values for all the 3 phases.

Now, the question is, calculate the apparent impedance seen by the relay at bus M during this 3-phase fault. So, relay  $R_M$  obtains this phasors, voltage and currents. And now it will calculate the  $Z_{app}$ . So, from the earlier discussion, the relations we have that  $Z_{app} = V_{aM}/I_{aM}$  or  $V_{bM}/I_{bM}$  or  $V_{cM}/I_{cM}$  at the relay bus. So therefore, the corresponding relay, we are using  $V_{aM}/I_{aM}$  corresponding  $V_{aM}/I_{aM}$ , and this you can say that ratio gives us  $(0.6148 + j8.6044) \Omega$ .

So, this is the apparent impedance which can be calculated by a relay, which will be subsequently, will be used for relay decision. The relay will afterwards check whether this impedance is inside the trip boundary or not. If it is inside, it goes for a trip decision, otherwise it remains silent. Now to check whether our calculation is okay or not, I will say that the fault

distance fault distance means from here to here, the physical distance in terms of kilometre can be obtained from these, the corresponding apparent impedance, this apparent impedance in nothing but the positive sequence impedance up to the fault point calculated from these available voltages and currents.

So, this value divided by the corresponding total impedance, the total impedance total impedance divided by this will be nothing but the corresponding, the kilometre length of the fault point. So therefore, this impedance of divided by the per kilometre impedance  $Z_{app}/z_1$ ,  $z_1=0.02 + j0.28678$  that gives us 30 kilometre length fault point, which is correct because we have simulated the case for the 30 kilometre distance from the relay bus. So, the fault distance calculated from this, the apparent impedance computed by the relay is in agreement with the actual fault created position.

So, that says that you can say that the Z apparent impedance is correctly calculated by the relay. Now, note that the apparent impedance which is seen by the relay, the apparent impedance which is seen by the relay for this fault is smaller than the Z positive impedance of the line  $Z_{1MN}$ . So,  $Z_{1MN}$  is  $2.4 + j34$ , so which is much higher than this Z apparent ( $0.6 + j8.6$ )  $\Omega$ . So, that implies that, this is, the relay will go for a decision to trip.

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**Phase-to-ground fault**

ag-type

$V_{a1M} = V_{a1F} + I_{a1M}(xZ_{1MN})$     $V_{a2M} = V_{a2F} + I_{a2M}(xZ_{2MN})$     $V_{a0M} = V_{a0F} + I_{a0M}(xZ_{0MN})$

$V_{aM} = V_{a1M} + V_{a2M} + V_{a0M}$

$= V_{a1F} + I_{a1M}(xZ_{1MN}) + V_{a2F} + I_{a2M}(xZ_{2MN}) + V_{a0F} + I_{a0M}(xZ_{0MN})$

$= I_{a1M}(xZ_{1MN}) + I_{a2M}(xZ_{2MN}) + I_{a0M}(xZ_{1MN}) - I_{a0M}(xZ_{1MN}) + I_{a0M}(xZ_{0MN})$

*for line  $Z_{2MN}=Z_{1MN}$*

$= xZ_{1MN}(I_{a1M} + I_{a2M} + I_{a0M}) - I_{a0M}(xZ_{1MN}) + I_{a0M}(xZ_{0MN})$

$= xZ_{1MN}I_{aM} + I_{a0M}(xZ_{1MN}) \left( \frac{Z_{0MN} - Z_{1MN}}{Z_{1MN}} \right)$

$= xZ_{1MN}I_{aM} + K_0 I_{a0M}(xZ_{1MN})$

$xZ_{1MN} = \frac{V_{aM}}{I_{aM} + K_0 I_{a0M}}$

Thus  $Z_{app} = \frac{V_{aM}}{I_{aM} + K_0 I_{a0M}}$

where  $K_0 = \left( \frac{Z_{0MN} - Z_{1MN}}{Z_{1MN}} \right)$

zero sequence compensation factor

Complex number

?? for cg fault type

We will continue to other types of fault. For the same system, we will now consider for the phase-to-ground fault, and will consider for simplicity phase A to ground, which you can extend to other faults also like bg-type and the cg-type. So, Phase A to ground fault at F happens to be there. So, we know for such a system, the positive sequence and

0 sequence networks, they will be connected in series. Here we are for derivations, we are considering  $R_F$  to be 0, otherwise there should have been a  $3R_F$  here.

Now, we see here now, the data which will be processed by the relay is only this voltage and current data. So, this is the M bus, the corresponding currents are positive sequence current, and the positive sequence voltage by the seen by the relay and negative sequence and the corresponding current, negative sequence current and 0 sequence voltage and 0 sequence current. Now, for this case we know this is a line to ground fault case, so therefore, phase a because ag-type fault, so phase a at F point will be grounded, means voltage is 0.

So therefore, we know,  $V_{aF}$  is,  $V_{aF}$  consists of positive sequence negative sequence and zero sequence voltages. So, these relations are from the sequence component analysis.  $V_{aF} = V_{a1F} + V_{a2F} + V_{a0F}$ , the corresponding positive negative and 0 sequence components of  $V_{af}$ , so therefore, this will be also 0. So, this relation is for the a to ground fault, phase a to ground fault. So, from these relations, we will proceed further things.

Now, from this sequence network, we know  $V_{aM}$ , we know we can say that the corresponding  $V_{aM} = V_{a1M} + V_{a2M} + V_{a0M}$ , but  $V_{a1M}$ , it is nothing but this voltage  $V_{a1F}$  plus this drop  $V_{a1F}$  plus this drop, so  $V_{a1F} + I_{a1M} (xZ_{1MN})$  impedance up to this point, positive impedance up to this point. So, this is positive sequence network. So, this voltage is  $V_{a1F} + I_{a1M} (xZ_{1MN})$ . So, this is the part about this  $V_{a1M}$ .

Similarly, for  $V_{a2M}$ , we have  $V_{a2F}$  plus this drop, so the  $V_{a2F} + I_{a2M} (xZ_{2MN})$ . And similarly, for the 0 sequence,  $V_{a0F}$  plus this drop, so  $V_{a0F} + I_{a0M} (xZ_{0MN})$ . So, this is what, is represented by this apparent sequence for  $V_{aM}$ , the fault point voltage at phase a during that time. So, if we see, here, we say that  $I_{a1M} (xZ_{1MN})$ ,  $I_{a2M} (xZ_{2MN})$ . And the other part here,  $V_{a1F} + V_{a2F} + V_{a0F}$  as you have seen becomes 0. So, they in total become 0.

So, only remaining first this term, this term and the third term. So, this is this term, this is this term. So, what we have done now, that this term is this one, so, we have added now 2 terms,  $I_{a0M} (xZ_{1MN}) - I_{a0M} (xZ_{1MN})$ . Only for the further analysis. So, we see here, these 2 terms are cancel each out, they are same terms, so we have added for you can say the further analysis. So now, from this you can see here that  $xZ_{1MN}$ ,  $xZ_{2MN}$   $xZ_{1MN}$ , but we know for a transmission system, positive sequence impedance equals to negative sequence impedance. So, this becomes also  $xZ_{1MN}$ .

So,  $xZ_{1MN}$  becomes a common, so  $I_{a1M} + I_{a2M} + I_{a0M}$ . So, these 3 part gives us these. Now, these 2 considered parts becomes this ( $-I_{a0M} \cdot xZ_{1MN} + I_{a0M} \cdot xZ_{0MN}$ ). What we will see now, that this part  $I_{a1M} + I_{a2M} + I_{a0M}$  as you see in the sequence component analysis, nothing but this  $I_{aM}$ , what is  $I_{aM}$ , the phase a current at relay  $R_M$ , as seen by the relay  $R_M$  and the fault is also phase a fault. So, that is why this part becomes  $I_{aM}$ , so we have  $xZ_{1MN}$ .

Now, if we see these 2 components, so if we take this is  $I_{a0M} \cdot xZ_{1MN}$  common, if we see, we can see that  $I_{a0M} \cdot xZ_{1MN}$  common, so that becomes equals to your  $(Z_{0MN} - Z_{1MN}) / Z_{1MN}$ . So, this relates, that if you take  $I_{a0M}$ , x you can say this, so that is why I can say that this becomes minus. So, this becomes  $(Z_{0MN} - Z_{1MN}) / Z_{1MN}$ . Therefore, we see here, that this  $xZ_{1MN} \cdot I_{a0M} +$  we define a term now  $K_0 I_{a0M} \cdot xZ_{1MN}$  and then you can see that we see here the corresponding relations where  $K_0$ , we have defined for this term equals to  $(Z_{0MN} - Z_{1MN}) / Z_{1MN}$ .

This is called the zero sequence compensation factor, a complex number here, because they had  $Z_0$   $Z_1$  and  $Z_1$  are complex numbers, so this  $K_0$  comes out to be a complex number. We call it 0 sequence compensation factor, because we see here in this term, if we see this, in this term, we have  $I_{a0M} I_{a0M}$  and that is why  $Z_0$ ,  $Z_1$  and  $Z_0$ . So, that you can say that the corresponding 0 sequence has a different terms in terms of  $Z_0$ , and note that  $Z_0$  is not same as  $Z_1$  unlike  $Z_2$  equals to  $Z_1$  for transmission line, it has a wide variations, it can go up to as 6 time also from the  $Z_1$  depending upon the configurations, which we have already discussed in the initial classes.

So, what we learn from here, that the  $V_{aM}$  is expressed in terms of this relation. So, therefore, from there we can calculate  $xZ_{1MN}$ , which is nothing but the positive sequence impedance up to the fault point equals to this  $V_{aM} / (I_{aM} + K_0 I_{a0M})$  and which are, all the measurements can be obtained by the relay at the local bus, M bus.  $V_{aM}$  measured at phase a,  $I_{aM}$  measured by at phase a, and  $K_0$  depends upon the line data, and  $I_{0M}$  is nothing but, we can find  $(I_a + I_b + I_c) / 3$ .

So, these are 0 sequence component during the fault situations for this case as seen by relay  $R_M$ . So therefore, we say that the corresponding positive sequence impedance up to the fault point which the relay likes to calculate for the decision that is what  $Z_{app} = V_{aM} / (I_{aM} + K_0 I_{a0M})$  for which category of fault? For phase a to ground fault. Now, if you go to any other category, let us say cg fault, phase to is to g fault, then the corresponding relations will start from these  $V_{cF} = 0$  and so, and in which you can say that Z apparent equals to  $Z_{app} = V_{cM} / (I_{cM} + K_0 I_{c0M})$ , same.



So, that gives us the way to relate the corresponding  $Z$  apparent in terms of these corresponding faulted phase currents and the 0 sequence current. And we found another term here, the  $K_0$ , which happens to be the 0 sequence compensation factor that is having a complex number.

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Example: Simulated ag-type fault at 30 km from bus M

Fault current and voltage data at Relay ( $R_M$ )

Currents		Voltages	
$I_a$ : 0.68∠-150.23° kA	$V_a$ : 10.46∠-64.84° kV	$I_b$ : 0.13∠-81.32° kA	$V_b$ : 114.97∠139.05° kV
$I_c$ : 0.12∠171.63° kA	$V_c$ : 120.49∠28.19° kV		

Calculate the apparent impedance seen by the relay at M.

Solution-

$$K_0 = \frac{Z_{0MN} - Z_{1MN}}{Z_{1MN}} = \frac{z_0 - z_1}{z_1} = 1.9310 - j0.1668$$

$$Z_{app} = \frac{V_{aM}}{I_{aM} + K_0 I_{0aM}} = 0.6024 + j8.5947 \Omega$$

Check-

$$\text{Fault distance} = \frac{0.6024 + j8.5947}{0.02 + j0.28678} = 29.98 \text{ km}$$

Note for protection decision,  $Z_{app} < Z_{1MN}$

230 kV transmission system, 120 km

sequence impedances per km

$$z_1 = 0.02 + j0.28678 \Omega/\text{km}$$

$$z_0 = 0.10645 + j0.836989 \Omega/\text{km}$$

$$Z_{1MN} = 120 (0.02 + j0.28678 \Omega) = 2.4 + j34.41 \Omega$$

Now, let us see an example for the same system 120 kilometre to 230 kV line. And have now, positive sequence impedance already known and we required 0 sequence impedance for this case. So, per kilometre 0 sequence impedance even here, you see, here it is higher in the x part and so also the resistance part also compared to positive sequence impedance. The  $Z_{1MN}$  positive sequence impedance can be computed in terms of  $Z_{1MN} = 120 \times$  positive sequence per km impedance and the total 0 sequence impedance of the line can be also  $120 \times z_0$ .

Now, let us see this data, which is available for during the fault and the fault is carried out again at the same point 30 kilometres from the relay bus  $R_M$  for this 120 kilometre line. And in this case, we see that the  $I_a$  phase a to ground fault this is already mentioned, phase a to ground fault case simulated, so  $I_a$  is substantially high, as compared to  $I_b$  and  $I_c$ ,  $V_a$  is substantially down as compared to  $V_b$  and  $V_c$  phases. So, the question is calculate the apparent impedance seen by the relay at  $R_M$ .

So, we know that for phase a to ground fault, we need the 0 sequence compensating factor  $K_0$  ( $Z_{0MN} - Z_{1MN} )/Z_{1MN}$  , having a complex number. But note that this  $Z_{0MN}$  is nothing but the length of the line into the impedance,  $Z_{1MN}$  is nothing but length of the line times the small  $z_1$  that is a per kilometre  $z_1$ . So therefore, if we see this relation, each one being in length of the line component multiplied to the corresponding per kilometre impedance, so therefore, the



length of the line cancels out and we get a relation in terms of  $(z_0 - z_1)/z_1$ , where these  $z$ , small  $z$  corresponds to the per kilometre impedance of the line.

So, that means that if you substitute, we can say that these value of per kilometre impedance here, then we get the corresponding  $K_0$  to be  $(1.9310 - j0.1668)$  unit-less. Note, if you have the total impedance of the line, 0 sequence and negative sequence, 0 sequence and positive sequence, you can compute also or if you have per kilometre length, you can say that per kilometre impedance of the line for the 0 sequence and positive sequence, you can compute from that relation also, because the length of the line cancels for this ratio.

Now, we will go for the  $Z$  apparent using the relations because this is line to ground fault in phase a, so  $Z_{app} = V_{aM} / (I_{aM} + K_0 I_{a0M})$ , so  $I_{a0M} = (I_a + I_b + I_c) / 3$  and  $I_{aM}$  is  $0.68 \angle -150.23^\circ$  kA. Substitute the of value  $V_a$ , this  $I_a$  and  $I_{a0M}$ , and the  $K_0$  is already calculated. So, we got to be consider  $0.6024 + j8.5947 \Omega$ . So, then this apparent impedance will be seen by the relay. So, that corresponds to the positive sequence fault impedance up to the fault point.

Now, we will check whether the apparent impedance calculated is correct or not. For that, again, we will calculate the fault distance as in the earlier example for the 3-phase fault case. So, this impedance whatever is being calculated divided by this positive sequence per kilometre impedance of the line,  $Z_{app} / z_1$  if we divide that, you are getting 29.98 actually created the fault at 30 kilometre, so, this is really close to that. So, that means that  $Z$  apparent is calculated by the relay is correct accurate.

Furthermore, we can conclude from this data obtained for the  $Z$  apparent that this value is much smaller than the  $Z_{1MN}$ , so therefore, the fault is inside the line between M and N, so the relay will go for a trip decision from protection point of view.

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phase-to-phase fault

bc-type

$R_F = 0$

$V_{a1F} = V_{a2F}$

$$V_{a1M} = V_{a1F} + I_{a1M}(xZ_{1MN}) \quad (1)$$

$$V_{a2M} = V_{a2F} + I_{a2M}(xZ_{2MN}) \quad (2)$$

$$V_{a1F} = V_{a2F} \quad (3)$$

(1) - (2) and using (3)

$$V_{a1M} - V_{a2M} = I_{a1M}(xZ_{1MN}) - I_{a2M}(xZ_{2MN}) = xZ_{1MN}(I_{a1M} - I_{a2M}) \quad \text{for line } Z_{2MN} = Z_{1MN}$$

$$\text{Thus, } xZ_{1MN} = \frac{V_{a1M} - V_{a2M}}{I_{a1M} - I_{a2M}}$$

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Now, we will go to the other categories of fault for the same system. So, fault at F, the type is phase-to-phase fault. Let us consider bc fault phase b to phase c short circuit with  $R_F = 0$ . So, for such a situation, such we know, you can say that the corresponding sequence diagram will live for the phase a, positive sequence diagram, negative sequence diagram, no 0 sequence diagram, ground is not involved. So, therein connected like this. And we see here,  $V_{a1F} = V_{a2F}$  at the fault point. So, that is what the relations for the bc fault,  $V_{a1F} = V_{a2F}$ . So, this relation is obtained from this sequence diagram for this case of bc type fault.

Now, we see here, the corresponding voltage current relations to obtain the apparent impedance,  $V_{a1M} = (V_{a1F} + I_{a1M} \cdot xZ_{1MN})$ , this voltage plus this drop. So, this is what this drop, impedance drop. Similarly, in the negative sequence component  $V_{a2M}$  at the relay bus will be equals to  $V_{a2M} = (V_{a2F} + I_{a2M} \cdot xZ_{2MN})$ , plus this drop, this one, and the third equation you got  $V_{a1F} = V_{a2F}$  for this situation of phase b to c type fault. So, if you subtract 2 from 1 and this relations for this  $V_{a1F} - V_{a2F}$ , in that case that cancels out.

So therefore, we got  $V_{a1M} - V_{a2M} = (I_{a1M} \cdot xZ_{1MN} - I_{a2M} \cdot xZ_{2MN})$ . So,  $Z_{2MN} = Z_{1MN}$  for transmission line. So, that we got  $xZ_{1MN}$  common  $(I_{a1M} - I_{a2M})$ . So therefore, the  $xZ_{1MN}$ , the positive impedance up to the fault point from the relay  $x \cdot Z_{1MN} = (V_{a1M} - V_{a2M}) / (I_{a1M} - I_{a2M})$ . The positive sequence voltage minus negative sequence voltage divided by the positive sequence current minus negative sequence current at the relay bus.

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phase-to-phase fault

General expression

$$V_{bM} - V_{cM} = \alpha^2 V_{a1M} + \alpha V_{a2M} + V_{a0M} - (\alpha V_{a1M} + \alpha^2 V_{a2M} + V_{a0M}) = V_{a1M}(\alpha^2 - \alpha) + V_{a2M}(\alpha - \alpha^2) = (V_{a1M} - V_{a2M})(\alpha^2 - \alpha)$$

similarly,

$$I_{bM} - I_{cM} = (I_{a1M} - I_{a2M})(\alpha^2 - \alpha)$$

Thus for bc - type fault

$$xZ_{1MN} = \frac{V_{a1M} - V_{a2M}}{I_{a1M} - I_{a2M}} = \frac{V_{bM} - V_{cM}}{I_{bM} - I_{cM}}$$

$Z_{app} = \frac{V_{bM} - V_{cM}}{I_{bM} - I_{cM}}$  ?? for ab fault type

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From the same diagram, which we earlier seen, we will continue how we can represent the corresponding sequence, voltages and current for the, in terms of the phase, voltages and currents. We know that the corresponding phase voltage  $V_{bM}$  can be expressed in terms of the positive sequence voltage and negative sequence voltage and 0 sequence voltage. And this is what the expressions, we know from the sequence component. And then we know for the phase c also, this is a  $V_c$  fault. The corresponding relations becomes  $\alpha \cdot V_{a1M} + \alpha^2 \cdot V_{a2M} + V_{a0M}$ . So, this is the general expressions for phase b and phase c.

So, if we subtract  $(V_{bM} - V_{cM}) = V_{a1M}(\alpha^2 - \alpha) + V_{a2M}(\alpha - \alpha^2) = (V_{a1M} - V_{a2M})(\alpha^2 - \alpha)$ . So,  $(V_{a1M} - V_{a2M}) = (V_{bM} - V_{cM}) / (\alpha^2 - \alpha)$ , that is one relations. Similarly,  $(I_{bM} - I_{cM})$ , from the current relations from these general relations for the sequence component, we can compute  $(I_{bM} - I_{cM}) = (I_{a1M} - I_{a2M})(\alpha^2 - \alpha)$ .

And from here, you can relate  $(I_{a1M} - I_{a2M}) = (I_{bM} - I_{cM}) / (\alpha^2 - \alpha)$ . So,  $xZ_{1MN}$  and the the fault impedance up to the fault point from the relay bus, which you have already derived,  $(V_{a1M} - V_{a2M}) / (I_{a1M} - I_{a2M})$  in the last slide. So, that becomes equal to from these two relations of voltage and current can express  $(V_{bM} - V_{cM}) / (I_{bM} - I_{cM})$ . So, what we say that that the fault impedance up to the fault point from the relay bus can be obtained from the phase voltage differences, difference and upon current subtraction.

So, we see here for the bc fault, the general expression for the Z apparent because this corresponds to our Z apparent here, it is required by the relay, positive sequence impedance for the fault point can be obtained from the measurements by the relay for  $(V_b - V_c) / (I_b - I_c)$ , phase b voltage minus phase c voltage upon phase b current minus phase c current Note, the fault

type is bc fault. So, we have, we are using b and c, voltages and current for the expression for Z apparent. So, if you go for any other categories of in a similar way like let us say, ab fault, for ab fault, the corresponding Z apparent will be equal to at that time  $(V_a - V_b) / (I_a - I_b)$ .

And for ca fault, we will be having  $(V_c - V_a) / (I_c - I_a)$  at the measurement bus by the relay. So, this gives for the phase-to-phase fault, the corresponding Z apparent expression. So, we found that for 3-phase fault, it is  $V_a / I_a$ . And for line to ground fault, ag fault, we got  $V_a / (I_a + K_0 I_0)$ . And for bc fault, we got  $Z_{app} = (V_b - V_c) / (I_b - I_c)$ , kind of thing.

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phase-to-phase-to-ground fault

bcg-type

$R_f = 0$

Thus for bcg - type fault

$$Z_{app} = \frac{V_{bM} - V_{cM}}{I_{bM} - I_{cM}}$$

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Now, we will see the fourth category, the last one that let us say that fault is bcg-type in the same system at F, and in this case the sequence diagram will becomes 3 parallel things, networks, positive, negative and 0, so we know this. And our measurement bus is at M position as positioned by the relay. So, we have, we see here that for this case again the positive sequence and the negative sequence are in parallel line. So therefore, we know that this voltage  $V_{a1F} = V_{a2F}$ . So, this we saw in the phase b to phase c also in earlier phase-to-phase fault also, same relation.

And this was our starting relations, and now if you see this release also here also,  $V_{a1M} = (V_{a1F} + I_{a1M} \cdot xZ_{1MN})$ , and  $V_{a2M} = (V_{a2F} + I_{a2M} \cdot xZ_{2MN})$ . So, these 3 relations are same relations, what we saw for the phase b to phase c fault also. So therefore, we can conclude that for this kind of fault also, bcg-type also, the final relations for the apparent impedance will be  $(V_{bM} - V_{cM}) / (I_{bM} - I_{cM})$ , that we mean that the type of fault bc and bcg, they have the same apparent

impedance relation of the  $Z_{app} = (V_b - V_c) / (I_b - I_c)$ , . And similar also, you can conclude for a cag fault or abg fault for the apparent impedance calculation.

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Example: abg-type fault at 30 km from bus M

Current and voltage data at Relay ( $R_M$ )

Currents	Voltages
$I_a: 0.76 \angle -145.05^\circ$ kA	$V_a: 8.99 \angle -83.37^\circ$ kV
$I_b: 0.73 \angle 82.51^\circ$ kA	$V_b: 8.74 \angle -166.50^\circ$ kV
$I_c: 0.13 \angle 166.34^\circ$ kA	$V_c: 121.54 \angle 23.18^\circ$ kV

Calculate the apparent impedance seen by the relay at M.

$$Z_{app} = \frac{V_{aM} - V_{bM}}{I_{aM} - I_{bM}} = 0.6158 + j8.6032 \Omega$$

Check-

Fault distance =  $\frac{0.6158 + j8.6032}{0.02 + j0.28678} = 30.01$  km

Note for protection decision,  $Z_{app} < Z_{1MN}$

230 kV transmission system

Positive sequence impedances per km  
 $z_1 = 0.02 + j0.28678 \Omega/\text{km}$

$Z_{1MN} = 120 (0.02 + j0.28678 \Omega) = 2.4 + j34.41 \Omega$

Now, let us see an example now, an abg fault same point 30 kilometres and the corresponding fault data provided here and voltages are also provided here. So, fault is an ab, so that is why the voltage in a and b gone down and currents are up. Now, you will calculate the apparent impedance for this case as seen by the relay. So, data per the line are provided as we have seen earlier. Z apparent equals to, this is ab fault, So, corresponding voltage  $(V_a - V_b) / (I_a - I_b)$ . So, you substitute the data and then got the corresponding apparent impedance to be  $0.6158 + j8.6032 \Omega$ .

So, we can check, we can say that what is the fault distance here. So, this impedance upon the, per kilometre impedance of the positive sequence impedance, so that divided, we got this corresponding kilometre to be 30.01. So, created fault at 30 kilometre and we found this distance to be of similar length. So, that means that Z apparent seen by the relay is correct accurate. Furthermore, we see that this impedance Z apparent is smaller than  $Z_{1MN}$ , which is having  $2.4 + j34.41 \Omega$ . So therefore, the relay will find this fault inside the line and go for a trip decision.

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Apparent impedance calculation

Three phase fault

$$Z_{app} = \frac{V_{aM}}{I_{aM}} = \frac{V_{bM}}{I_{bM}} = \frac{V_{cM}}{I_{cM}}$$

abc

phase-to-ground fault

$$Z_{app} = \frac{V_{aM}}{I_{aM} + K_0 I_{a0M}} \quad \frac{V_{bM}}{I_{bM} + K_0 I_{a0M}} \quad \frac{V_{cM}}{I_{cM} + K_0 I_{a0M}}$$

ag                      bg                      cg

phase-to-phase fault or phase-to-phase-to-ground fault

$$Z_{app} = \frac{V_{aM} - V_{bM}}{I_{aM} - I_{bM}} \quad \frac{V_{bM} - V_{cM}}{I_{bM} - I_{cM}} \quad \frac{V_{cM} - V_{aM}}{I_{cM} - I_{aM}}$$

ab                      bc                      ca  
abg                      bcg                      cag

• Need of fault classification

So, in summary, with that different kinds of fault, we can say that 3-phase fault,  $Z_{app} = V_{aM} / I_{aM}$  or  $V_{bM} / I_{bM}$  or  $V_{cM} / I_{cM}$ . Phase to ground faults ag, we know for the  $Z_{app} = V_{aM} / (I_{aM} + K_0 I_{a0M})$ , bg:  $Z_{app} = V_{bM} / (I_{bM} + K_0 I_{a0M})$ ,  $Z_{app} = V_{cM} / (I_{cM} + K_0 I_{a0M})$ , for this cg: type of fault. And for phase-to-phase fault or phase-to-phase to ground fall ab abg bc bcg and ca cag, the Z apparent becomes equals to for ab and abg,  $Z_{app} = (V_a - V_b) / (I_a - I_b)$  and for bc and bcg correspondingly  $Z_{app} = (V_b - V_c) / (I_b - I_c)$  and for ca cag,  $Z_{app} = (V_c - V_a) / (I_c - I_a)$ .

So, this is how, the corresponding, relay will calculate the Z apparent for different types of faults. What we found from this that, that the relation for you can say that different types of fault, the corresponding calculation of Z apparent is different. It means that they relay needs say fault classifier, which you have already seen, the, our module of the distance relay, fault classification subsequently apparent impedance calculation. So, this reveals that we require a fault classifier to accomplish the apparent impedance calculation.

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Apparent impedance calculation: field data from India Grid

	Currents	Voltages
Fault current and voltage data at Relay ( $R_M$ )	$I_a$ : 967.4 $\angle$ 5.8° A	$V_a$ : 5.832 $\angle$ 76.4° kV
	$I_b$ : 92.9 $\angle$ 171.82° A	$V_b$ : 73.76 $\angle$ -42.1° kV
	$I_c$ : 68.9 $\angle$ 38.3° A	$V_c$ : 73.66 $\angle$ -162.5° kV

50Hz, 132 kV transmission system, 73 km

Line ( $Z_{LMN}$ ) impedance:  
 $z_1$ : 0.024 + j0.264  $\Omega$ /km  
 $z_0$ : 0.098 + j0.837  $\Omega$ /km  
 $Z_{1MN} = 73 (0.024 + j0.264 \Omega) = 1.752 + j19.27 \Omega$

Step 1: Fault classification

$I_{a1}$ : 336.17  $\angle$  -3.22° A  
 $I_{a2}$ : 325.26  $\angle$  11.60° A  
 $I_{a0}$ : 312.41  $\angle$  9.43° A

$\delta_1^+$ : 14.82°  
 $\delta_1^0$ : 2.17°

It is an ag fault

Step 2: Apparent impedance calculation

$$K_0 = \frac{Z_{1MN} - Z_{0MN}}{Z_{1MN}} = \frac{Z_1 - Z_0}{Z_1} = 2.178 - j0.08$$

$$Z_{app} = \frac{V_{aM}}{I_{aM} + K_0 I_{a0M}} = 1.21 + j3.32 \Omega$$

$Z_{app} < Z_{1MN}$

Now, we will see a practical field data in India Grid. So, 132 kV transmission system having 73 kilometre length 50 hertz system. So, we have positive sequence parameter, 0 sequence parameter per kilometre given here. We can find the  $Z_{1MN}$  total impedance of the, positive impedance of the line  $1.752 + j19.27 \Omega$ . For this situation, fault data are available, the relay recordings provides. So, the fault current at  $R_M$  is a  $I_a$  being  $967.4 \angle 5.8^\circ$ ,  $I_b = 92.9 \angle 171.82^\circ$  and  $I_c = 68.9 \angle 38.3^\circ$ .  $V_a = 5.8$ , pretty small,  $V_b$  and  $V_c$  are having 73.76 or so.

So, this if we see this fault data, then it reveals that is a very likely phase a to ground fault, but the relay has to decide based on the data. So, now for the apparent impedance calculation and subsequently for the relay decision, before apparent impedance calculation the relays used to know that which type of fault it is. Accordingly, the corresponding voltage current relations will be used. So, we use the current based fault classifier, already learned in earlier lectures. So, we compute the corresponding  $I_{a1}$ ,  $I_{a2}$  and  $I_{a0}$  and then compute the  $\delta_1^+$  and  $\delta_1^0$ . So,  $\delta_1^+$  is nothing but  $\delta_1^+ = \angle I_{a2} - \angle I_{a1}$ , and  $\delta_1^0 = \angle I_{a2} - \angle I_{a0}$ .

Using these 2, if we find that the, it positions in this ag, and here also it ensures that this is in ag. And you see that 0 sequence component is significant, all are almost of similar order, that ensures that this is a phase a to ground fault. So, this ensures us (( ))(36:28) on phase a-to-ground fault now will activate the corresponding apparent impedance calculation. So, this means that ag fault, so, we know that the corresponding voltages, voltage and currents will be used for that  $V_a / (I_a + K_0 I_{a0})$ . So, for that we require  $K_0$ , so  $K_0$  can be obtained from the data from here. So, we got to consider these to be  $2.178 - j0.08$ , zero sequence compensating factor.



$Z_{app} = V_a / (I_a + K_0 I_{a0})$ . So, substituting with the  $V_a$ , we got these be  $1.21 + j3.32 \Omega$ . Note that these  $Z$  apparent impedance, it is much smaller than the positive impedance of the line, that means that the fault is inside the line, the relay will go for a trip decision.

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Remarks-

Apparent impedance  $Z_{app}$

- Calculated using currents and voltages
- In case of phase-to-ground fault, zero sequence compensation factor  $K_0$  is used in the calculation

Is  $Z_{app} = V/I$  ?

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So, in summary,  $Z$  apparent calculation is integral part of this distance relay. It uses the local currents and voltage for this. For that, the corresponding relation, proper relation of the  $Z$  apparent will require a fault classification, fault type must be known. In case of phase to ground fault, 0 sequence compensation factor is being used only for only for phase to ground fault, not for any other types of fault.  $K_0$  happens to be complex also and depends on the line parameters.

The last question comes is that “Is  $Z$  apparent equals  $V$  by  $I$ ?” Not, because in case of, we are not that  $V_a/I_a$  or so. For phase to ground, phase to phase faults, it is  $(V_b - V_c) / (I_b - I_c)$  relation. And particularly for phase to ground fault, it is  $V_a / (I_a + K_0 I_{a0})$  kind of relations. So therefore, it is not simply (phase voltage)/(phase current) for  $Z$  apparent calculation. So, we saw that you can see that how the  $Z$  apparent can be calculated by relay successfully. Thank you.