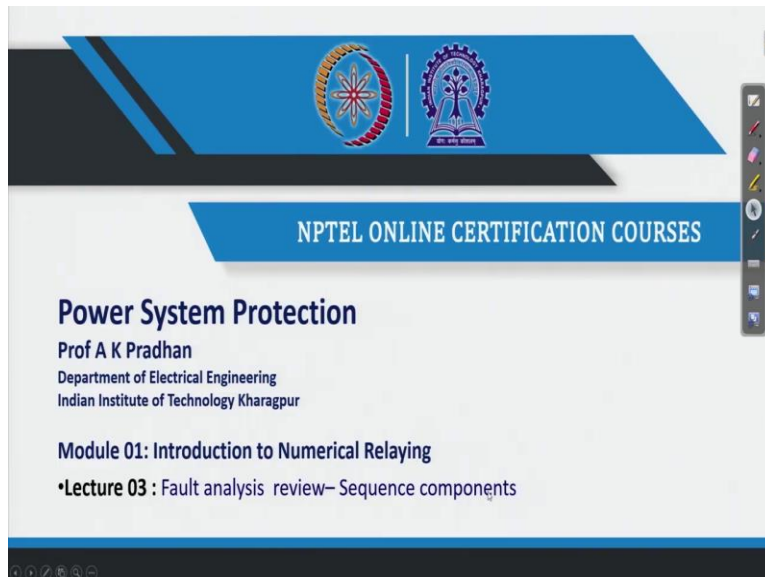


Power System Protection
Professor A K Pradhan
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture 03
Fault Analysis Review – Sequence Components

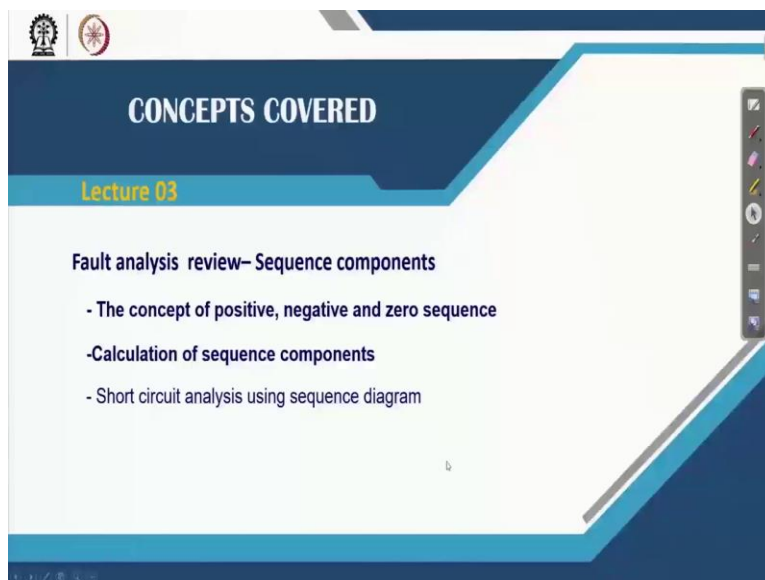
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The slide features a blue header with the NPTEL logo and the text "NPTEL ONLINE CERTIFICATION COURSES". Below this, the course title "Power System Protection" is displayed, followed by the instructor's name "Prof A K Pradhan" and his affiliation "Department of Electrical Engineering, Indian Institute of Technology Kharagpur". The slide also lists "Module 01: Introduction to Numerical Relaying" and "Lecture 03 : Fault analysis review– Sequence components".

Welcome, in this third lesson, we will address on the fault analysis perspective, a review on this, also includes the sequence components or commonly called as fault analysis using different sequence components.

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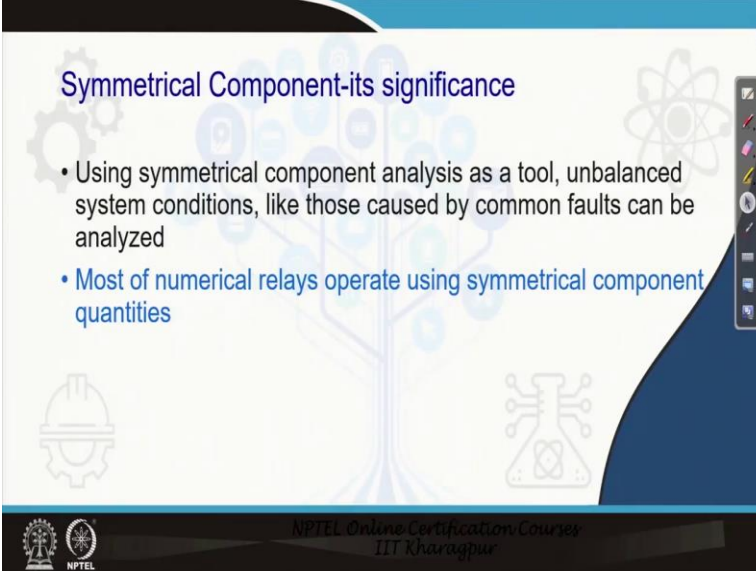


The slide is titled "CONCEPTS COVERED" and lists the topics for "Lecture 03": "Fault analysis review– Sequence components". The concepts covered are:

- The concept of positive, negative and zero sequence
- Calculation of sequence components
- Short circuit analysis using sequence diagram

In this we will cover the sequence component aspects. We will introduce all the three sequence components: positive, negative and zero sequence components. We will learn how to calculate sequence components from the phase components and also we will see how these sequence diagrams can be drawn, and then from there we consider how the different faults can be analyzed; that we call short circuit analysis and so. In addition, we will see how this will be beneficial in understanding different numerical relaying principle including different relay settings.

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The slide is titled "Symmetrical Component-its significance" and contains two bullet points. The background features a stylized tree with gears and a circuit diagram. The NPTEL logo and "NPTEL Online Certification Course IIT Kharagpur" are visible at the bottom.

- Using symmetrical component analysis as a tool, unbalanced system conditions, like those caused by common faults can be analyzed
- Most of numerical relays operate using symmetrical component quantities

So, in this lesson we are emphasizing on the symmetrical components or so-called sequence components. There are two main important reasons: that if you go with the phase quantities, then fault analysis for a large system become tedious, very complex, and symmetrical components analysis is used as a tool to simplify the analysis process so that different faults in a system can be analyzed and in particular the unbalanced fault, which is commonly found in the power system.

Furthermore, it is a found that most of the numerical relays today use symmetrical components for decision-making process and also as I mentioned that such fault analysis is very useful while considering the circuit breaker rating or different relay settings in the system.

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Symmetrical Components

- Unbalanced phasors of a three phase system can be resolved into three balanced phasors called the symmetrical components of the original phasors.
- The set of balanced components are- (i) positive (ii) negative and (iii) zero sequence

Positive sequence -consisting of three phasors equal in magnitude and separated by 120° to each other and having the same phase sequence as the original phasors of the system (*abc* in this case)

For *abc* phase sequence of a system

$V_{b1} = \alpha^2 V_{a1}$
 $V_{c1} = \alpha V_{a1}$
 $\alpha = 1 \angle 120^\circ$

'1'-Positive sequence

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Now, let us come to different symmetrical components, any unbalanced set of phasors in a three-phase system can be resolved into three balanced phases that we call symmetrical components. These symmetrical components or the balanced phasors are three types: positive, negative and zero sequence.

So, coming to the positive sequence, it consists of three equal phases, equal in magnitude and with a phase separation of 120° to each other and it maintains the original phase sequence of the system like here. We see here these three components of the positive sequence phases V_{a1} , V_{b1} , and V_{c1} , are with same magnitude and separated by 120° from each other. The phase sequence here in this case V_{b1} follows V_{a1} and then V_{c1} follows V_{b1} . Therefore, the phase sequence is *abc* that is same as the phase sequence of the system. Furthermore, if you see here the magnitude of each phasor being same and they are 120° apart, so if you can write on V_{b1} in terms of V_{a1} , then from V_{a1} , 120° and another 120° , we reached to the V_{b1} . That means V_{b1} can be expressed in terms of $V_{b1} = \alpha^2 V_{a1}$, where $\alpha = 1 \angle 120^\circ$. Similarly, for representing V_{c1} in terms of V_{a1} , then from V_{a1} if we proceed by 120° we reach to V_{c1} . So, $V_{c1} = \alpha V_{a1}$, thus we can express V_{b1} and V_{c1} in terms of V_{a1} with the operating parameter α that equals to $1 \angle 120^\circ$. The '1' in this representation corresponds to positive sequence component.

(Refer Slide Time: 5:42)

The slide is titled "Symmetrical Components". It is divided into two main sections. The top section is for the negative sequence, and the bottom section is for the zero sequence. A small video inset of a speaker is visible in the bottom right corner of the slide.

Symmetrical Components

- Negative sequence- consisting of three phasors equal in magnitude and displaced by 120° to each other with phase sequence opposite to the original phasors (abc in this case) of the system

Diagram for Negative sequence: Three phasors V_{a2} , V_{b2} , and V_{c2} are shown. V_{a2} is at the top, V_{b2} is at the top-left, and V_{c2} is at the bottom. A red arc indicates a 120° angle between V_{a2} and V_{b2} . The sequence is labeled "2'-Negative sequence".

Equations for Negative sequence:
 $V_{b2} = \alpha V_{a2}$
 $V_{c2} = \alpha^2 V_{a2}$
 $\alpha = 1 \angle 120^\circ$

- Zero sequence- consisting of three phasors equal in magnitude and with zero phase displacement from each other (no rotation sequence)

Diagram for Zero sequence: Three phasors V_{a0} , V_{b0} , and V_{c0} are shown, all pointing in the same direction. The sequence is labeled "0'-Zero sequence".

Equation for Zero sequence:
 $V_{a0} = V_{b0} = V_{c0}$

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Similarly, for the negative sequence components, it again consists of 3 phasors having same magnitude 120° separated, but here the phase sequence is opposite to the phase sequence of the system. So, it is in opposite to the positive sequence what we saw earlier. In this case you see here V_{a2} corresponds to the negative sequence of phase a , V_{b2} corresponds to that of the phase b and the negative sequence for phase c is V_{c2} . Now, if you see here it is 120° they maintain with each other, also they are having same magnitude and the phase sequence if we see here is nothing but acb ; whereas, the system phase sequence is still you can say that abc , and if we represent the V_{b2} and V_{c2} in terms of V_{a2} , then we see here that 120° proceed from V_{a2} , you get V_{b2} ; therefore, $V_{b2} = \alpha V_{a2}$ and $V_{c2} = \alpha^2 V_{a2}$ with $\alpha = 1 \angle 120^\circ$.

Now, next we will go to the zero sequence, this is something different. It consists of three equal phasors and they are considered as equal in magnitude and they maintain zero phase displacement; that is no rotational sequence. Therefore, no rotational sequence implies zero sequence. And here we see $V_{a0} = V_{b0} = V_{c0}$.

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Symmetrical Components

- Unbalanced 3 phase system to 3 sets of balanced system

For *abc* phase sequence of phasors of a system

$\alpha = 1\angle 120^\circ$

Positive sequence

$$V_{b1} = \alpha^2 V_{a1}$$
$$V_{c1} = \alpha V_{a1}$$

Negative sequence

$$V_{b2} = \alpha V_{a2}$$
$$V_{c2} = \alpha^2 V_{a2}$$

Zero sequence

$$V_{a0} = V_{b0} = V_{c0}$$

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So, in summary we see that, the symmetrical components, positive, negative and zero, can be expressed like this, that in positive sequence; $V_{b1} = \alpha^2 V_{a1}$ and $V_{c1} = \alpha V_{a1}$ and in case of negative sequence $V_{b2} = \alpha V_{a2}$ and $V_{c2} = \alpha^2 V_{a2}$. So, if we see the components of positive sequence it is *abc*, for negative sequence it is *acb*, and there is no sequence among the zero sequence components.

Now, in this perspective we see that any three phasors V_a , V_b , V_c can be decomposed into three sets of positive, negative and zero sequence phasors. Note that, in case of positive sequence, each component V_{a1} , V_{b1} , V_{c1} have same magnitude; similarly, in case of negative and zero sequence also. Whereas, it is not necessary that V_{a1} magnitude and V_{a2} magnitude will be same or V_{a2} magnitude and V_{a0} magnitude will be same. Therefore, the components in this positive, negative or zero sequence depends upon the different situations in the power system.

(Refer Slide Time: 8:54)

Symmetrical Components

• Each of the original unbalanced phasors is the sum of its sequence components

For *abc* phase sequence of the system

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{b0} + V_{b1} + V_{b2} = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$

$$V_c = V_{c0} + V_{c1} + V_{c2} = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

$\alpha = 1 \angle 120^\circ$

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In addition to the other important aspect is each of the original unbalanced phasors is the summation of its sequence components. That means if we have V_a for the system, V_a corresponds to phase a voltage; that can be written as $V_a = V_{a0} + V_{a1} + V_{a2}$. So we mean to say that if this is V_{a1} taken from this positive sequence component and V_{a2} taken from this negative sequence component and then we can say that V_{a0} here. Now if we see in terms of magnitude and angle, and substitute here, then summation of V_{a1} , V_{a2} and V_{a0} gives us the V_a . So, that is what we see here that each phasor V_a , V_b or V_c can be represented by a summation of their corresponding positive, negative sequence and zero sequence quantities, that is we can express V_b in terms of V_{b0} , V_{b1} and V_{b2} as $V_b = V_{b0} + V_{b1} + V_{b2}$; but you know $V_{b0} = V_{a0}$, $V_{b1} = \alpha^2 V_{a1}$ and $V_{b2} = \alpha V_{a2}$. Therefore,

$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$

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Symmetrical Components

For *abc* phase sequence of the system

$$\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} \\ V_c &= V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2} \end{aligned}$$

$$\alpha = 1 \angle 120^\circ$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \Rightarrow [V^{abc}] = [T][V_a^{012}]$$

$$[V_a^{012}] = [T]^{-1}[V^{abc}] \Rightarrow \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Similarly for currents,

$$[I^{abc}] = [T][I_a^{012}] \quad [I_a^{012}] = [T]^{-1}[I^{abc}]$$

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Likewise,

$$V_c = V_{c0} + V_{c1} + V_{c2} = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}.$$

So, these unbalanced phasors V_a , V_b , V_c can be represented in terms of summation of the positive, negative and zero sequence components of the corresponding phase quantities. Now, from the above discussions we can write the V_a , V_b , V_c in terms of V_{a0} , V_{a1} and V_{a2} and thereby this V_a , V_b , V_c can be put into a matrix form in terms of like this

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

where, $\alpha = 1 \angle 120^\circ$. Now, we see here this in the matrix notation, we can put

$$[V^{abc}] = [T][V_a^{012}].$$

But this T is nothing but the corresponding matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$. This leads to other way, from this

matrix representation we can write

$$[V_a^{012}] = [T]^{-1}[V^{abc}]$$

where $[T]^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$, if we see the relation between this $[T]$ and $[T]^{-1}$ we find $[T]^{-1}$ is

nothing but inverse of the earlier matrix. From this we see that there is a clear relations between phase components to the sequence components and also you can relate the sequence components to the phase components by multiplying the corresponding matrix into the system. Consequently, we can express the corresponding phase component of currents in terms of the sequence components or you can obtain the considered sequence components from the phase components using the same $[T]$ and $[T]^{-1}$ likewise

$$[I^{abc}] = [T][I_a^{012}]$$

$$[I_a^{012}] = [T]^{-1}[I^{abc}]$$

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Symmetrical Components

Example : Calculate the sequence components for the given phase voltages for a system with *abc* as phase sequence

$V_a = 110\angle 0^\circ$ kV, $V_b = 88\angle -100^\circ$ kV, $V_c = 80\angle 175^\circ$ kV

Solution:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 110\angle 0^\circ \\ 88\angle -100^\circ \\ 80\angle 175^\circ \end{bmatrix} = \begin{bmatrix} 25.99\angle -11.74^\circ \\ 27.03\angle -79.32^\circ \\ 85.65\angle 21.83^\circ \end{bmatrix} \text{ kV}$$

$\alpha = 1\angle 120^\circ$

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Now, let us take an example. Calculate the sequence components for the given phase voltages for the system with *abc* as phase sequence, $V_a = 110\angle 0^\circ$ kV, $V_b = 88\angle -100^\circ$ kV, $V_c = 80\angle 175^\circ$ kV.

Solution: So, from these unbalance phase voltages to get the corresponding sequence components we will multiply the phase voltage matrix with $[T]^{-1}$.

$$\text{So } \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 110\angle 0^\circ \\ 88\angle -100^\circ \\ 80\angle 175^\circ \end{bmatrix} = \begin{bmatrix} 25.99\angle -11.74^\circ \\ 27.03\angle -79^\circ \\ 85.65\angle 21.83^\circ \end{bmatrix} \text{ kV}$$

As $\alpha = 1\angle 120^\circ$. So the zero, positive and negative sequence components of voltages are $V_{a0} = 25.99\angle -11.74^\circ$ kV, $V_{a1} = 27.03\angle -79^\circ$ kV and $V_{a2} = 85.65\angle 21.83^\circ$ kV respectively. We see here that for these unbalanced situations the magnitude of V_{a0} , V_{a1} and V_{a2} are different and they have their own phasor position on the phasor diagram plot. So, this clearly shows how to compute the corresponding sequence components from the phase quantities.

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Symmetrical Components

- **Example** : A balanced load condition with *abc* phase sequence has,
 $I_a = 100\angle 0^\circ \text{ A}, I_b = 100\angle -120^\circ \text{ A}, I_c = 100\angle 120^\circ \text{ A}$

Obtain sequence components.

Solution:

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}(100\angle 0^\circ + 100\angle -120^\circ + 100\angle 120^\circ) = 0\text{A}$$

$$I_{a1} = \frac{1}{3}(100\angle 0^\circ + (1\angle 120^\circ)(100\angle -120^\circ) + (1\angle 240^\circ)(100\angle 120^\circ)) = 100\angle 0^\circ \text{ A}$$

$$I_{a2} = \frac{1}{3}(100\angle 0^\circ + (1\angle 240^\circ)(100\angle -120^\circ) + (1\angle 120^\circ)(100\angle 120^\circ)) = 0\text{A}$$

Note- Only positive sequence component available and is same as I_a .

In the next example, we look into the corresponding sequence components for a balance loading condition and perspective

Example: A balanced load condition with *abc* phase sequence has $I_a = 100\angle 0^\circ \text{ A}, I_b = 100\angle -120^\circ \text{ A}, I_c = 100\angle 120^\circ \text{ A}$. Obtain sequence components.

Solution: Now, this is clearly seen that the current magnitudes are same and they are 120° apart, so this is a clear balanced load condition in a system. Now the sequence components of currents

are obtained from
$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}; \alpha = 1\angle 120^\circ$$

$$I_{a0} = \frac{1}{3}(100\angle 0^\circ + 100\angle -120^\circ + 100\angle 120^\circ) = 0\text{A}$$

$$I_{a1} = \frac{1}{3}\{(100\angle 0^\circ) + (100\angle -120^\circ)(100\angle 120^\circ) + (100\angle 120^\circ)(100\angle -120^\circ)\} = 100\angle 0^\circ \text{ A}$$

$$I_{a2} = \frac{1}{3}\{(100\angle 0^\circ) + (100\angle -120^\circ)(100\angle -120^\circ) + (100\angle 120^\circ)(100\angle 120^\circ)\} = 0 \text{ A}$$

So, what it can be said from this one that the zero sequence and negative sequence components is not available; whereas, the positive sequence component is same as that the I_a phasor. So, you can conclude from this observation that for a balanced condition only positive sequence component remains and the other two are not available.

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Symmetrical Components

Example : Calculate the line currents, if the obtained sequence currents for a system with abc as phase sequence are:

$$I_{a0} = 0 \text{ A}, I_{a1} = 6.2\angle -30^\circ \text{ A}, I_{a2} = 6.2\angle 45^\circ \text{ A}$$

Solution:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 6.2\angle -30^\circ \\ 6.2\angle 45^\circ \end{bmatrix} = \begin{bmatrix} 1.61\angle 172.53^\circ \\ 9.82\angle 7.49^\circ \\ 11.44\angle -172.53^\circ \end{bmatrix} \text{ A}$$

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Now, in the other way given the sequence components you can calculate the phase quantities also using the T matrix.

Example: Calculate the line currents, if the obtained sequence currents for a system with abc as phase sequence are $I_{a0} = 0 \text{ A}$, $I_{a1} = 6.2\angle -30^\circ \text{ A}$, $I_{a2} = 6.2 \angle 45^\circ \text{ A}$.

Solution:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = [T] \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

After putting the values of [T] and substituting the sequence components I_{a0} , I_{a1} , I_{a2} , the phase currents are obtained from

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 6.2 \angle -30^\circ \\ 6.2 \angle 45^\circ \end{bmatrix} \text{ A}$$

With $\alpha = 1 \angle 120^\circ$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1.61 \angle 172.53^\circ \\ 9.82 \angle 7.59^\circ \\ 11.44 \angle -172^\circ \end{bmatrix} \text{ A}$$

So, we get here $I_a = 1.61 \angle 172.53^\circ$ A, $I_b = 9.82 \angle 7.59^\circ$ A and $I_c = 11.44 \angle -172^\circ$ A

(Refer Slide Time: 18:02)

Sequence impedance

For a transposed transmission line,

$$[Z_L^{abc}] = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$$

$$[V_{23}^{abc}] = [Z_L^{abc}] [I^{abc}] \Rightarrow [T] [V_{23}^{012}] = [Z_L^{abc}] [T] [I^{012}]$$

$$[V_{23}^{012}] = [T]^{-1} [Z_L^{abc}] [T] [I^{012}]$$

$$[V_{23}^{012}] = [Z_L^{012}] [I^{012}]$$

$$[Z_L^{012}] = [T]^{-1} [Z_L^{abc}] [T]$$

$$[Z_L^{012}] = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

$$Z_0 = (Z_s + 2Z_m)$$

$$Z_1 = (Z_s - Z_m)$$

$$Z_2 = (Z_s - Z_m)$$

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From the above we will learn how to calculate the sequence components from the phase quantities and how the phase quantities can be derived from the sequence components perspective. Now, I will go to the sequence impedances from where to the next level to sequence diagram and this sequence diagram will be useful in the fault analysis as a tool to the power engineering communities.

Now, for $V = IZ$ is the corresponding relations between the phase voltages and phase currents where, Z becomes the phase impedances. Similarly, when you go to the sequence quantities in terms of the voltage, that is V and the corresponding sequence quantities of current is I then Z must also be in terms of the sequence components perspective.

So, that gives us a platform to use the corresponding sequence components for the fault analysis purpose. See this is a system where we have a source G_1 and G_2 , they are connected by a network. This network is having two transformers in the left and right hand side respectively and both of them are connected by a transmission line. Then let us focus now for this transmission line which we assume to be a transposed one. For this transmission system the Z matrix, the corresponding impedance matrix for the transmission system can be represented in terms of the self-impedance of the line (Z_s) and the mutual impedance of line (Z_m). This representation is due to phase a self-impedance, phase a to b mutual and phase a to c mutual components. Therefore, this comes out to be a 3×3 symmetrical matrix for this purpose and can be represented as

$$[Z_L^{abc}] = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$$

Here L corresponds to line, and Z_L^{abc} corresponds to the 3×3 matrix for the abc phases of the three-phase transmission line. Consider a voltage V_{23}^{abc} between bus 2 and 3 at an instant of time and the associated current flowing is I^{abc} . So, this V_{23}^{abc} voltage drop across the transmission line from bus 2 to 3 can be represented as

$$[V_{23}^{abc}] = [Z_L^{abc}][I^{abc}] \quad (1.1)$$

So, this is what we know that from the voltage current relation for any transmission line. Now, we know that the abc components of the voltage and currents can be expressed in terms of sequence components as

$$[V_{23}^{abc}] = [T][V_{23}^{012}] \quad (1.2)$$

$$[I^{abc}] = [T][I^{012}] \quad (1.3)$$

Therefore considering the relation between phase and sequence components described in (1.2) and (1.3), expression mentioned in (1.1) can be written as

$$[T][V_{23}^{012}] = [Z_L^{abc}][T][I^{012}]$$

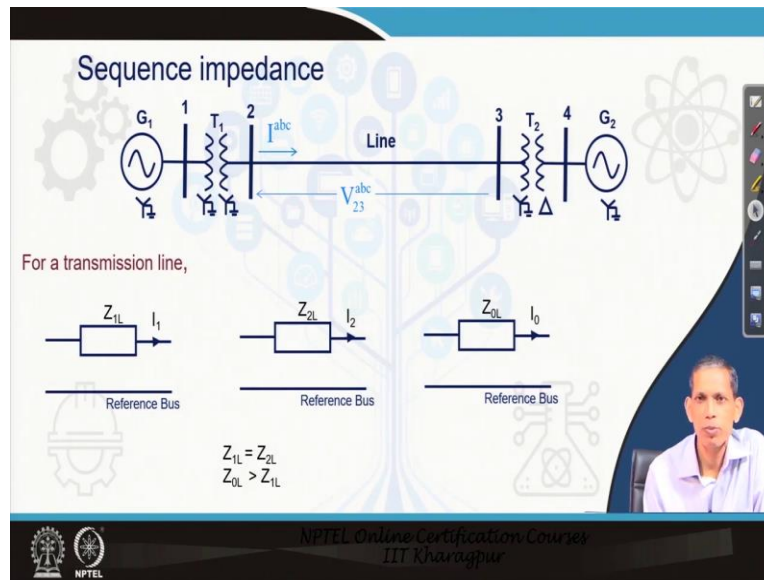
$$[V_{23}^{012}] = [T]^{-1}[Z_L^{abc}][T][I^{012}] = [Z_L^{012}][I^{012}]$$

where, $[Z_L^{012}] = [T]^{-1}[Z_L^{abc}][T]$. So, this gives us a platform that the sequence components of the transmission line can be computed from these transmission line impedance matrix multiplying $[T]^{-1}$ and $[T]$ as in this relation. So, going by this if you substitute the values of $[T]^{-1}$ and $[T]$ and also the corresponding $[Z_L^{abc}]$ the corresponding sequence component matrix of the transmission system comes out to be

$$[Z_L^{012}] = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

From the above expression the corresponding relation of the zero, positive and negative sequence impedances of the transmission line are given by $Z_0 = Z_s + 2Z_m$, $Z_1 = Z_s - Z_m$ and $Z_2 = Z_s - Z_m$ respectively.

(Refer Slide Time: 22:42)




Now, coming to sequence the impedance of transmission line, so we say that the sequence impedances of transmission line having positive sequence, negative sequence and zero sequence as we have seen in the earlier slide. So, this can be represented in terms of Z_{1L} , 1 for positive, and L for the line. Similarly, negative sequence component can represent in this fashion Z_{2L} , 2 for the negative sequence and L for the line and zero sequence component as Z_{0L} .

And the associated current which is flowing through this individual component is nothing but I_1 for positive, I_2 for negative and I_0 for the negative sequence quantity in this case. Transmission system being passive, so positive sequence voltage or negative sequence voltage lead to same impedance; therefore, Z_{1L} equals to Z_{2L} for the transmission line.

Zero sequence component of impedance is associated with the corresponding zero sequence current and zero sequence induced voltage in the line. In this case the flux becomes additive unlike the flux in case of positive and negative sequence components, where it becomes balance; being balanced the summation of current becomes 0. On the other hand, in case of zero sequence component the current is no more 0 and the associated flux becomes additive. Therefore zero sequence component in the line is higher than the Z_{1L} or Z_{2L} and typically for a transmission system depending upon the configurations including double circuit line, it varies from 2 to 6 times of the positive sequence quantity.

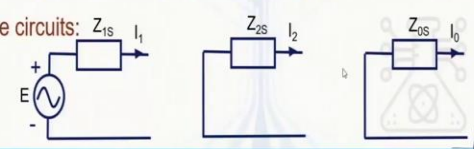
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Sequence impedance




- For a synchronous machine (generator):
 - Positive sequence component of a synchronous machine can be **subtransient**, transient or synchronous reactance
 - The zero sequence component depends on the stator winding connection; star grounded (solidly or with impedance), ungrounded or delta

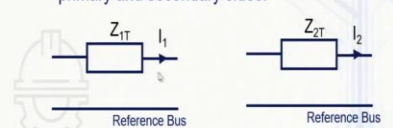
The sequence circuits:



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- For transformer:
 - positive sequence and negative sequence impedances are same. $Z_{1T} = Z_{2T}$
 - zero sequence impedance Z_{0T} depends on the type of connection of the primary and secondary sides.



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The next element in the system we are considering is the synchronous machine. For synchronous machine also we have three sequence components: positive sequence, negative sequence and zero sequence. This is being the source; the model is a voltage source. Therefore, in positive sequence component we have this corresponding voltage and an impedance associated with it. This gives us the positive sequence perspective and in a negative sequence we simply put in terms of the Z_{2s} only because we consider that the generator produce balanced voltage only; therefore, negative sequence or zero sequence diagram do not have any source voltages along with only they are being represented by sequence impedance perspective.

Z_{1S} in this case can be represented in terms of sub transient, transient or synchronous reactance depending upon the period of observation in time frame and that we can say is nothing but concerned with the relay design and decision-making process; that is fast decision making process. Therefore, in our perspective we will consider Z_{1S} to be the sub transient reactance.

The Z_{2S} is determined by the average of sub transient component of the d-axis and q-axis impedances of the synchronous machine and zero sequence current as we did that gives you corresponding zero sequence impedance perspective.

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Zero Sequence Circuit of Transformer

Case	Connections	Zero-sequence Equivalent Circuits
1		
2		
3		
4		
5		

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Similarly for the transformer we have positive sequence and negative sequence components and being this is passive, the negative sequence and positive sequence components are in general same.

Coming to the zero sequence component this is different for this transformer case, it depends upon the connections, if the connection is star grounded- star grounded (Y_g-Y_g) at both the sides the zero sequence component finds a path to be flowing through the line to the ground in both these sides. Therefore, we can say that the associated zero sequence impedance is being connected to the line side in both the perspective. For ungrounded star (Y_g-Y) then it does not find path to the ground; therefore, the zero sequence current does not flow in the line and the right hand side in this case is no more connected the system, when it happens to be delta (Δ) connected in one of these sides, the sequence component flows in the transformer windings and there is no scope for the zero sequence current to flow in the line side. Unlike we see here the three phase components

I_{a1} , I_{b1} , I_{c1} they become 0, but in case of I_{a0} , I_{b0} , I_{c0} , they are co phasors, so they can flow in the windings of the transformer. That leads to the circulating current inside the windings of the transformer and therefore this finds a path like this. Similarly, you can extend the idea to the other connections perspective also for delta-delta (Δ - Δ) or Y- Δ connections. So, depending upon the connection the zero sequence impedance of the transformer is being incorporated in the system.

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Sequence Network for different types of fault

(i) Three phase fault

$I_{a1}^f = I_a^f$ and $I_{a2}^f = I_{a0}^f = 0$

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Now, we will go to the classification of faults and see what the corresponding sequence diagram and sequence network for that. This is a three phase fault, let us first consider a three-phase system. So, the three phases are short circuited by a fault resistance in this case. This is a balanced condition as you know; therefore, the corresponding current also becomes balanced like this with magnitude same and 120° apart. Now, if you find the corresponding sequence components 0 1 2 for the fault conditions that can be expressed by multiplying the $[T]^{-1}$ with the phase components I_a^f , I_b^f , and I_c^f

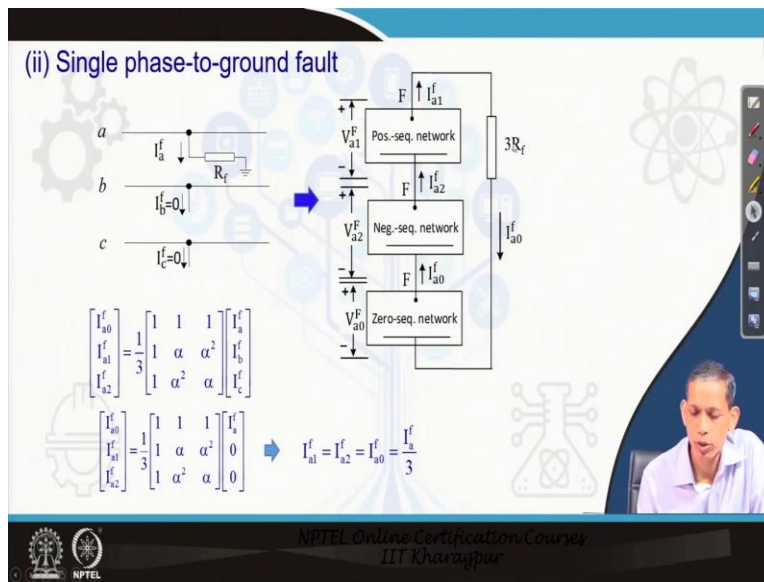
$$\begin{bmatrix} I_{a0}^f \\ I_{a1}^f \\ I_{a2}^f \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 0 \\ I_a^f \\ 0 \end{bmatrix}$$

So we see here this zero sequence component becomes I_a^f plus I_b^f plus I_c^f , that becomes 0; similarly, the negative sequence component also becomes 0; only positive sequence component remains in this case.

So, $I_{a1}^f = I_a^f$ and $I_{a2}^f = I_{a0}^f = 0$

That implies the corresponding sequence network for three phase fault condition becomes nothing but only contains positive sequence component; no more negative and zero sequence quantities. That is why it only contains positive sequence diagram and from where we can analyze only the positive sequence current perspective.

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Extending this concept for the single phase to ground fault, let us say phase a to ground fault as shown here happens to be there and fault resistance R_F and the I_b and I_c components are 0 in this case. So, if we go to the sequence components by multiplying the T inverse matrix with the phase current quantities; that is

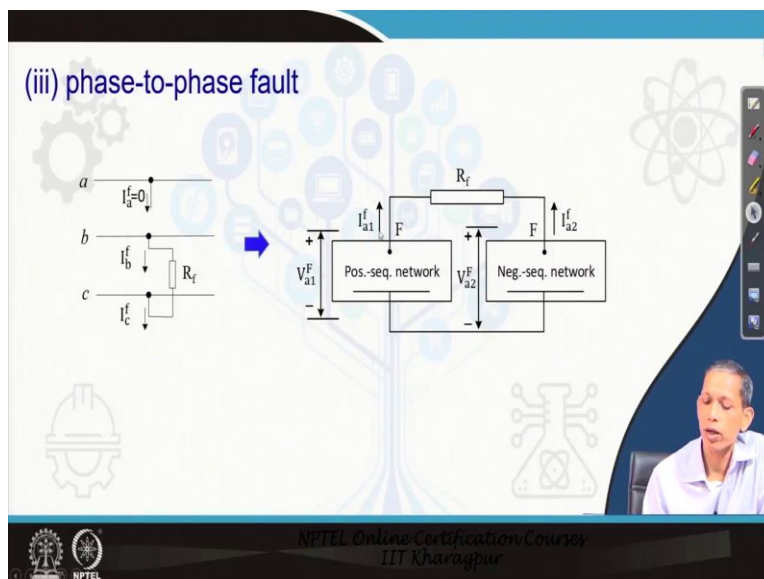
$$\begin{bmatrix} I_{a0}^f \\ I_{a1}^f \\ I_{a2}^f \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix}$$

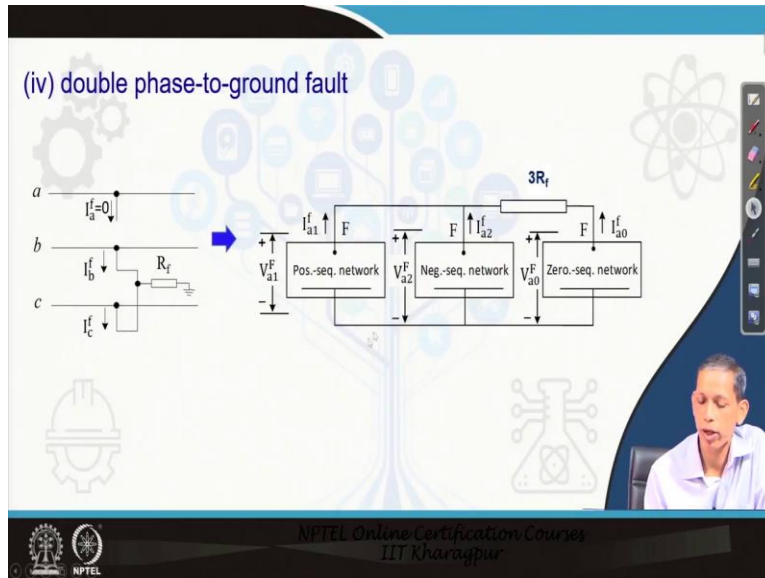
Substituting I_b^f and I_c^f being 0 in the expression; the sequence components are expressed by

$$\begin{bmatrix} I_{a0}^f \\ I_{a1}^f \\ I_{a2}^f \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a^f \\ 0 \\ 0 \end{bmatrix}$$

Which provides $I_{a1}^f = I_{a2}^f = I_{a0}^f = \frac{1}{3}I_a^f$. This implies that I_{a1}^f , I_{a2}^f and I_{a0}^f during the fault condition becomes same and each one is nothing but one third of the fault current which is flowing to the R_F in this case. So, that leads to a situations that all the three components are same, both magnitude and phase; therefore, they must be connected in series. Positive sequence network, negative sequence network and zero sequence network are being connected in series and the corresponding current through them becomes $\frac{1}{3}I_a^f$. Therefore, this connection diagrams becomes this and note that the corresponding zero sequence current component happens to be in all the three phases. In order to compensate this, corresponding R_F is represented as $3R_F$ in the sequence diagram. This is what the corresponding sequence diagram for phase a to ground fault.

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For Phase to phase fault, similarly you can extend the idea and in this case no more ground is involved and so that $I_a^f = 0$ and $I_b^f = -I_c^f$ in this case, this is a bc type fault, but the corresponding sequence diagram we are shown as per the relation for the phase a positive and negative sequence components are connected in parallel as shown here, and in this case

$$I_{a1}^f = -I_{a2}^f.$$

The fourth category of fault which the power system encounters double line to ground fault or double phase to ground fault, here phase b and phase c grounded by a fault resistance. In this case both positive, negative and zero sequence networks all are in parallel and in this case

$$I_{a1}^f = I_{a2}^f + I_{a0}^f$$

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• After obtaining sequence components from sequence diagram, Phase quantities can be obtained

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0}^f \\ I_{a1}^f \\ I_{a2}^f \end{bmatrix}$$

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So, we see different kinds of faults having the connectivity of the sequence diagrams in terms of positive, negative and zero depending on the availability. Now, for any system like this if you want to analyze, then with the mentioned procedure you get the corresponding zero sequence, negative sequence and positive sequence, currents and voltages at different points. In addition, the phase quantities can be computed from the T matrix multiply by the sequence component matrix perspective that is by

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0}^f \\ I_{a1}^f \\ I_{a2}^f \end{bmatrix}$$

So, from the above discussions we see that any set of phasors V_a, V_b, V_c or I_a, I_b, I_c can be decomposed into three sequence components 0 1 2 respectively. Simultaneously, from the sequence components you can get the corresponding phase quantities and also we have seen the sequence impedances of the corresponding transmission line, transformer or generator.

In next one lesson we will see how we can draw the sequence diagram for the systems and use it for the fault analysis perspective and how sequence components in the system is being useful for relay applications. Thank you.