Power System Protection
Professor A K Pradhan
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture 06
Discrete Fourier Transform

Welcome to Module 2, lecture number 6 and this is on Phasor estimation technique and we continue with the Discrete Fourier Transform.

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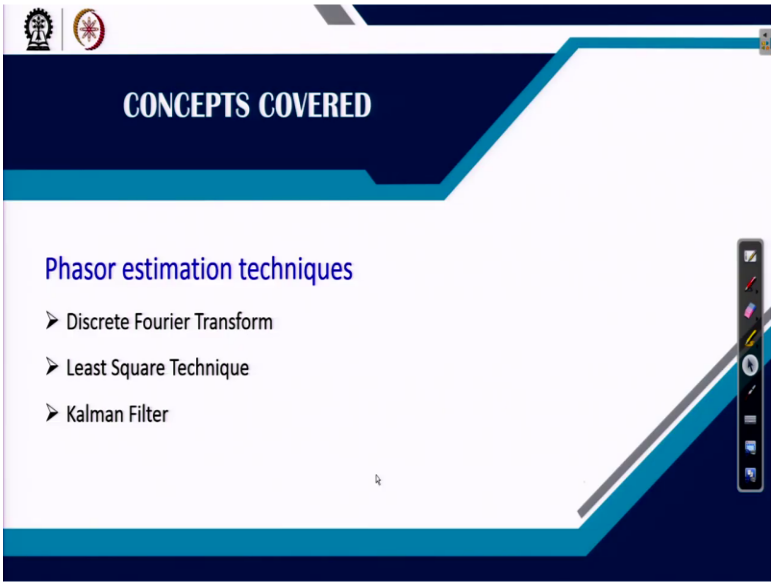

So, in this lecture we will learn a phasor estimation technique particularly on Discrete Fourier Transform and this module includes different other Phasor estimation techniques and growing Least square technique, and Kalman filter technique, this is an input into considered topic in numerical algorithm perspective.

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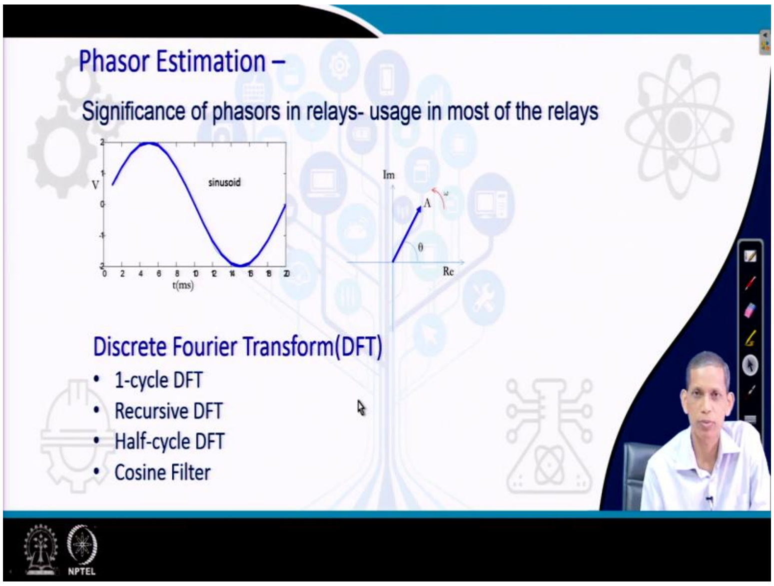

Now, we know that if a phasor represents for a sinusoidal quantity like you see here, this is a voltage signal of sinusoidal quantity and then these can be represented in terms of a phasor, an anti clockwise rotating vector in the same speed, considered as ω that of the sinusoidal quantity. So, these you can say that this vector represented by a magnitude of A and its projection on Y-axis represents for this sinusoidal quantity.

Now, this is what we remember that to use the sequence components you are in need of the V_A, V_B, V_C and I_A, I_B, I_C to get the corresponding positive negative and 0 sequence components which are being widely used in different relaying principle algorithms.

So, essentially from a sinusoidal quantity, we require the corresponding phases to be computed and these phasor considered are very much required for the different computation process like one example, you can say that, when we are talking about impedance in a distance relay, this impedance is a function of a voltage and current phasors. These voltage and current phasors can be represented in a complex form either in polar or rectangular quantity and then you can find corresponding impedance, a complex number in this one and in impedance relay, we use the R-X plane and there the corresponding decision is being carried out by the relay using that complex number.

So, you can say that most of the relays available numerical relays use the phasor for the decision making process. Now, for this perspective, there are different phasor estimation algorithms available and we will start with the very commonly used technique that is 1-cycle DFT and subsequently we will see also how the corresponding DFT can be computed in a better way using recursive DFT also its variance in the form of half-cycle DFT and the Cosine filters that an extension of this concept of the 1-cycle DFT will be considered applying those techniques.

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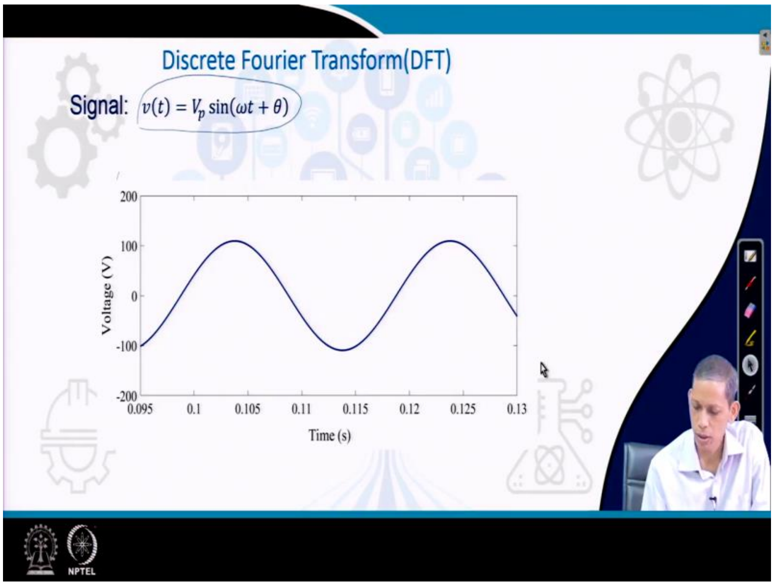

Now, let us you can say that this signal considered here is represented here by

$$v(t) = V_p \sin(\omega t + \theta)$$

Where, ω the angular speed of this voltage signal, with plotted on the time axis we get the corresponding signal like this. The objective is how to extract the corresponding fundamental component and that should be represented in the form of a phasor and that phasor is being used in relay algorithm. So, you can say that data acquisition system has to be accomplished to the A to D conversion process and those samples you can say will be used for the subsequent phasor estimation technique.

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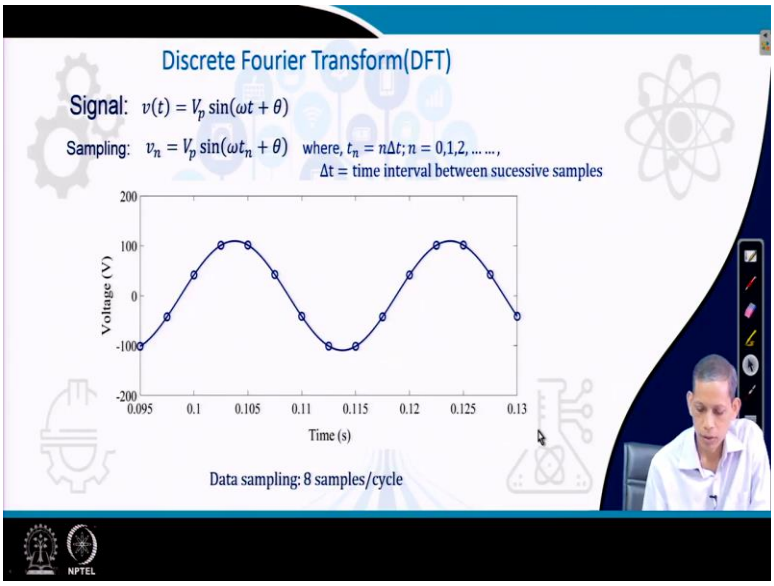

So, let us you can say that we sample the signal like this these are the different samples values in this process so, these sample is value represent V_n to the same signal

$$v_n = V_p \sin(\omega t_n + \theta)$$

This t_n is represented as

$$t_n = n\Delta t$$

Where, Δt is the time interval between two consecutive points and *n* is the number of samples. One by one the relay possess the newer and newer samples then the corresponding *n* also goes on increasing one by one in this order the corresponding time also on the time axis goes on increasing. For this continuous sinusoidal form, these are the samples which are being acquired by the relay to the A to D process. After obtaining these samples, we will like to use the samples for the phasor computation process. Here, in this sampling perspective we considered 8 sample per cycle for this 50 Hz signal. Therefore, you can say that

$$\Delta t = \frac{0.02}{8} = 2.5 \text{ ms}$$

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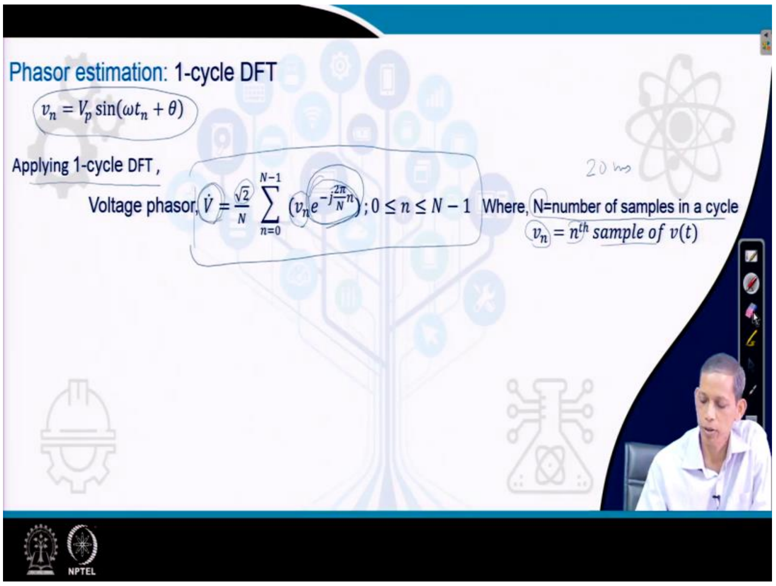

Now, see the you can say that how the 1-cycle Discrete Fourier Transform algorithm is being applied for the phasor estimation process, this is you can say that the corresponding sample value being acquired by the relay as already mentioned and then, you can say that it will apply the corresponding one cycle DFT technique, the phasors which is being computed by this 1-cycle DFT process is represented by

$$\dot{V} = \frac{\sqrt{2}}{N} \sum_{n-0}^{N-1} v_n e^{-j\frac{2\pi n}{N}} \; ; \; 0 \le n \le N-1$$

Where, N is the number of samples in a cycle, 1-cycle period in the 50 Hz system is 20 ms and v_n is the nth sample of $v(t)$. So, there is a complex coefficient individual to v_n , if we multiply and then take the summation for all N number of points multiplied by $\frac{\sqrt{2}}{N}$ the corresponding fundamental component is estimated. Typically you see in some literature you will find that $\frac{2}{N}$ is considered because, in power systems the phasors are mostly represented by RMS instead of the peak value. So, therefore you can say that peak divided by √2 gives you 1/√2 so that root √2 factor is coming considered because, of the RMS value is being used in this perspective.

Therefore, you can compute the corresponding phasor using this mathematical relations. So, once fresh sample is being obtained by the A to D conversion process relay gets it and uses a set of N number of points available to that and then the each value is multiplied with corresponding Fourier coefficients to find out the corresponding phasor.

So, these being consider the complex number the corresponding $\dot{V}$ comes out to be a complex number in terms of that, so what you can say that from this relation is that to the each sample value the corresponding Fourier weight being multiplied and then the relay computes the corresponding phasor value.

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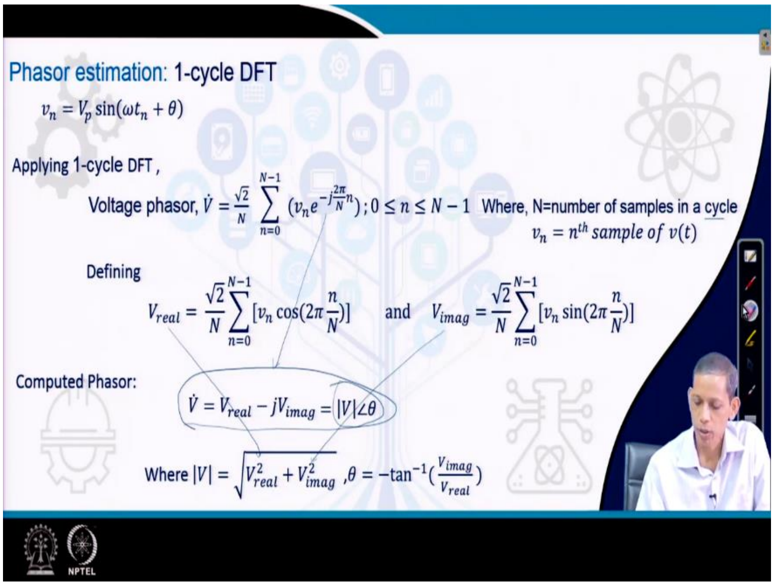

So, now you can say that for these complex number for the computational process we can make it easier also in the algorithm process the corresponding processing element may not be able to execute the complex number, if we segregate the corresponding real and imaginary part, the real part of these you can say that phasor becomes,

$$V_{real} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} v_n \cos(\frac{2\pi n}{N})$$

Corresponding imaginary part becomes,

$$V_{imag} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} v_n \sin(\frac{2\pi n}{N})$$

So, this you can say that we divide the corresponding complex number to the real and imaginary part you can call it the cosine and sine component part for the corresponding phasor $\dot{V}$, we can say that this $\dot{V}$that is the voltage Phasor for the sinusoidal quantity which we are representing can be expressed in terms of

$$\dot{V} = V_{real} - jV_{imag} = |V|\angle\theta$$

So, we see clearly that from the voltage samples, we can multiply the corresponding Fourier coefficients and then, we can get the corresponding $\dot{V}$phasor you can say that in terms of these and which, we can express in terms of a polar form in terms of $|V|\angle\theta$ and where the corresponding magnitude and are angle can be obtained from

$$|V| = \sqrt{V_{real}^2 + V_{imag}^2}; \quad \theta = -\tan^{-1}\frac{V_{imag}}{V_{real}}$$

So, this in our all you can say that gives us the corresponding phasors computation process you can say that for this facility in overall with the samples value for the phasor for the signal being available for 1-cycle, we multiply with the corresponding Fourier coefficients and find the summation of that one multiply by $\frac{\sqrt{2}}{N}$ provides the corresponding voltage phasor for that particular window of the signal.

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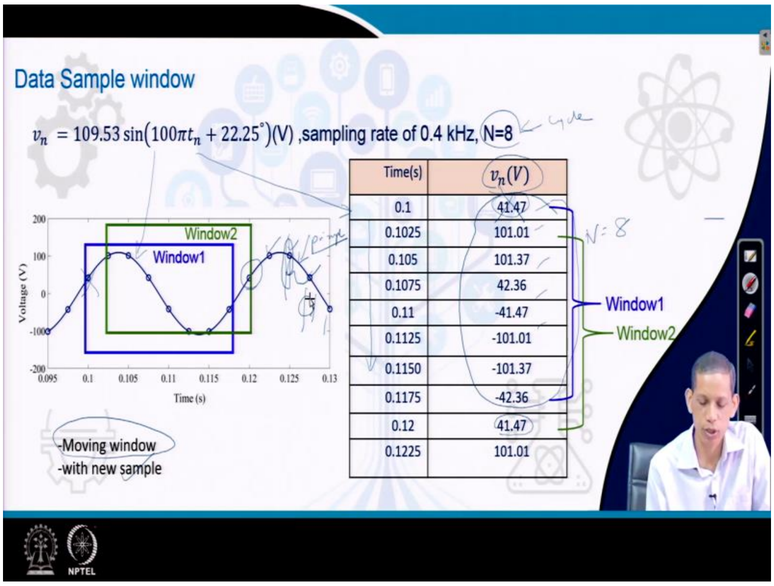

Now, we will see how we can say this computational process is being accomplished through an example let us you can say that a signal a sinusoidal signal given by

$$v_n = 109.53\sin(100\pi t_n + 22.5^0)$$

The sampling rate we have taken here is 0.4 kHz, 400 Hz that is and because, this is a 50Hz signal so the number of points per cycle that is N=8. So, this is a simple calculation process however, a typical relay today can be taken, N= 64 or more for more accurate and efficient calculation process.

So, here you see this is the voltage signal which you can see that these are the samples in this process and these are the time index of that corresponding t_n and these are the corresponding v_n value corresponding to sample value of the voltage of this one.

So, we have a record from point 1 here, the corresponding value is 41.47 like that as we progress the relay acquire data for the different time index and store the corresponding voltage points one by one in terms of that.

Now, we see here in this example, this require the 1-cycle data you can say for this one so, this window you can say that from a source of 1-cycle, this is from point 1 to the 1-cycle data, you can say that the corresponding 1-cycle data here are being from 41.47 to -42.37 you can say that the next sample again comes out to be 41.47. So, we you can say that one set of data you that is nothing but corresponding to N=8 in this case are to be now processed by the 1-cycle DFT for computation of the phasor. Similarly, when we are going for another window you can say that as the next window progress in this case a fresh sample is being acquired by the relay and the corresponding windows see that 1 sample, it discards this sample and then progresses ahead.

Therefore, again you can say that in the second window we acquires the new samples and discard this sample that is being the new fresh window comes out to be there in this way this is a progressive window with a newer and newer samples are being acquired by the relay the corresponding window shifts and 1-cycle DFT computes the phasor using the formula which we have already shown in the earlier slide. This you can say the moving window approach you can compute the phasor with every new samples, the corresponding window is being updated and relay computes the corresponding phasor for the further calculations, one point again I am telling here, that once this simple is being acquired the corresponding phasor is being computed the relay then starts computing for further things like currents, voltage phasors, impedance and so, and there you can say that the relay the uses its principle in this, interval and using that principle and or the method the corresponding relay decides that whether to trip or not trip. So, when you can say that the new fresh sample comes the relay by that time computes the phasor and again you can say that goes for the processing of its own algorithm inside this one. So, that is why in this interval of time you can say that the relay has to accomplished data acquisition and as well as to compute the phasor or so, you can say that the corresponding decisions to be there and that is you can say is the typical relay performance security is achieved. Therefore, if the relay or algorithm uses higher and higher number of samples then, the interval you can say that which is available to the relay will be smaller and smaller but, the relay has to take a decision within that smaller interval or time and that becomes challenging; therefore, it has associated cost and that is you can say limits the design process and they are based on all this perspective and all these things, the relay has to take the decision for successful operation of the protection scheme and so. So, this you can say that the corresponding things

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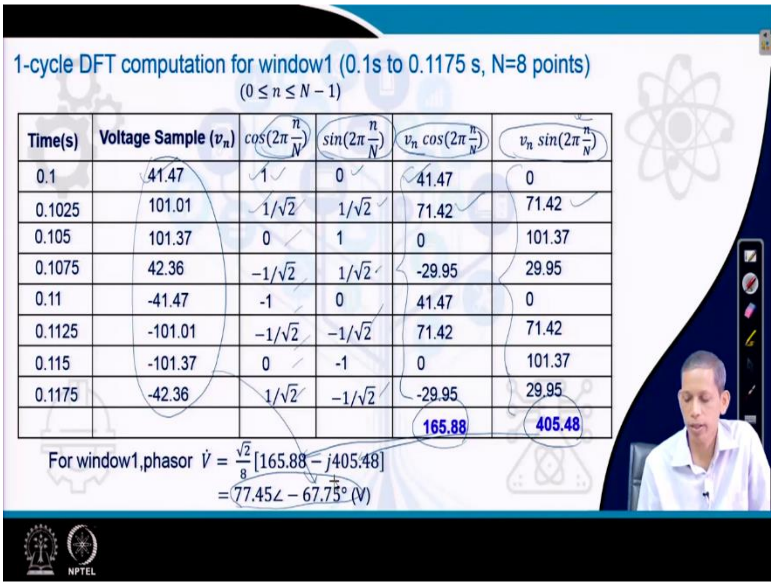

Now, let us see how we can compute the phasor using the using that 1-cycle DFT see here, you can say that, these are the data sets which we have already mentioned in the earlier slide and this sequence corresponding time index for the perspective, so the corresponding Fourier coefficient cos part and sin part you can say that are being $\cos(\frac{2\pi\, n}{N})$ and $\sin(\frac{2\pi\, n}{N})$ part and this corresponds to cos weights and this correspond to your sin weights the *n* you can say that in corresponds to the sample number and N is number of samples per cycle that is nothing but 8 in this case. So, if you substitute value of n equals to 0 to (N-1) n equals to 0 means cos becomes 1 and sin becomes 0 and then you can say that for n = 1, that gives $\frac{2\pi}{8}$, provides 45^0 therefore, this becomes equals to $\frac{1}{\sqrt{2}}$, also for the sin weight and like this you can say that you substitute the n equals to 0, 1, 2, 3, 4, 5, 6, 7 and then, you can say that we are getting the all that corresponding 8 weights for the purpose; similarly, you get the corresponding sin weights in this perspective. Then, in this case you can say that, after multiply the corresponding weights to the sample values and these, you can say that sample values and the weights are being multiplied individually and like for example,

$$41.47 \times 1 = 41.47;\ 41.47 \times 0 = 0\ ; 101.04 \times \frac{1}{\sqrt{2}} = 71.42$$

Like this, you can say that the corresponding sample value and the corresponding cos and sin weights are being multiplied and you are getting a set of values for the cos part and set of values for the sin part that are being used in the next case.

Now what we say that in the 1-cycle DFT you get the summation of these cosine parts gives us 165.88 and summation of this sine gives us 405.48. Now, using the corresponding DFT algorithm, voltage phasor can be represented as

$$\dot{V} = \frac{\sqrt{2}}{8}[165.88 - j405.48] = 77.45\angle - 67.75^0$$

Note that, this is for the particular window of data which you have taken for these 8 sample value and the corresponding phasor you can say that gives you this one with a fresh sample in this window will be again updated as already mentioned and then you can say that we can get corresponding phasor value to be different you can say that newer and newer samples are acquired and accordingly you can say that the corresponding phasor is being computed and that phasor is being used by the relay for decision making process.

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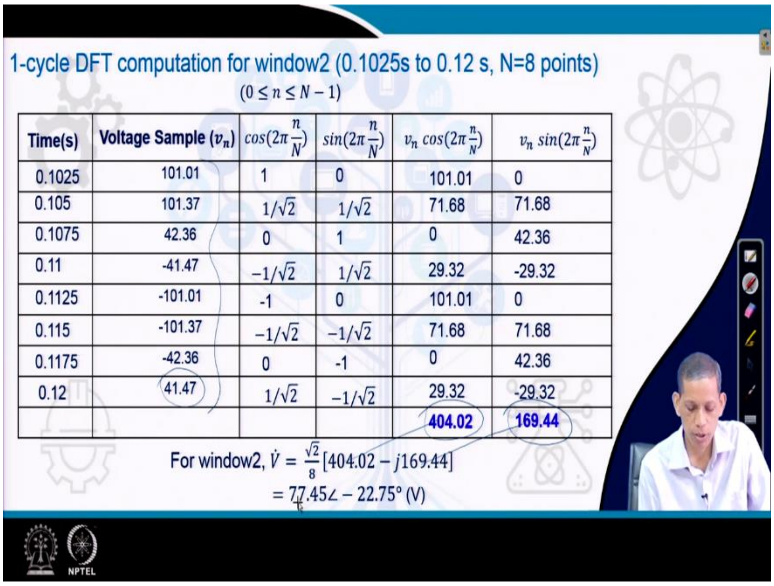

So, now you can say that, in the second window, you can say that, what you have already mentioned that the fresh sample is being acquired and this fresh sample is being used by the relay while discarding the older sample and this a new set of data you can say that for the second window. Using this window again and multiply the corresponding sample with the respective cos and sine weights after adding these values the real part or the cos part in this case to be 404.02 and sin part summation gives you 169.44. Therefore, the corresponding phasor value in this case becomes,

$$\dot{V} = \frac{\sqrt{2}}{8}[404.02 - j169.44] = 77.45\angle - 22.75^0$$

So, this what we see can say that, with the second set of window, in a similar way we can computed the different weights and corresponding multiplication of the different weights and the summation will provide the corresponding phasor you can say that to be computed for the second window.

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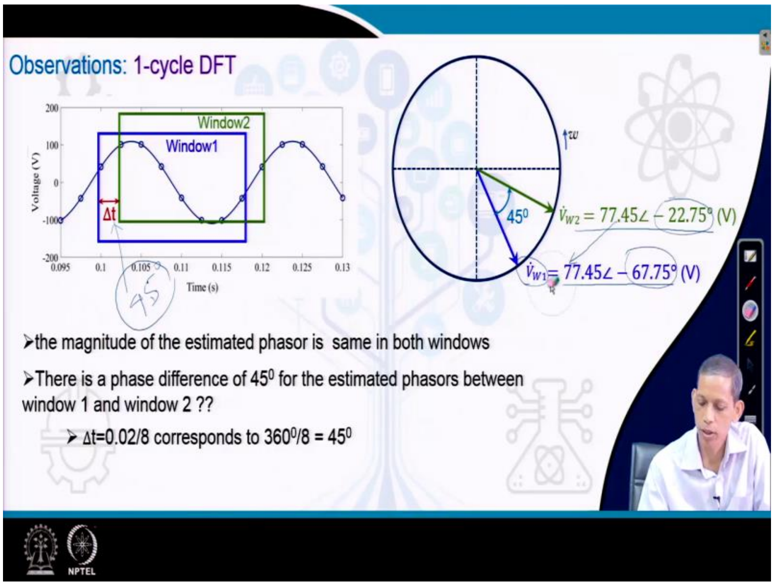

So, what we see from these two windows that the corresponding things for first window the blue one and then you can say that the second window the green one so with a new sample the second window is being updated accordingly now if you see this computations and get the polar plot of the corresponding phasor, you can say that it will rotating in anti clockwise direction with the same speed as ω without the sinusoidal quantity. So, the fast window gives us the blue phasors magnitude and angle you have already mentioned, and the second window gives us the corresponding magnitude and angle like this, what we see from these two that that the corresponding magnitude remains same but the angles in both the cases are been different.

Now, what is the difference you will notice here, you see here that, the magnitude of both the phasors are same but, the angle difference becomes 45^0 between window 1 and window 2, if you see here for this case the corresponding Δt between these two windows the window two shifts by a time of Δt, what is the Δt talks about? It is nothing but $\frac{0.02}{8}$s samples that is 8 corresponds to that a capital N value and this you can say that, is nothing but corresponds to

$$\frac{360^0}{8} = 45^0$$

Therefore, that 45^0 shifting of the window 2 phasor in the time domain is being clearly reflected in the phasors domain and that is you can say that, the reason we are getting this shifting angle of 45^0 by this one.

Note that, you can say that, the magnitude of the signal remains stationary, therefore you can the corresponding magnitude part remains to be same in the both the cases. So, that clearly shows this 1-cycle DFT is able to capture the phase and the magnitude information note that, the 77.45 is nothing but the corresponding signal you can say that, it talk about signal divide by $\sqrt{2}$ so, that is your result you can say that, 77.45 magnitude, you can say that. Now, what we see from this perspective is that with a further and further shifts in windows the corresponding phasors can be computed in the proper way.

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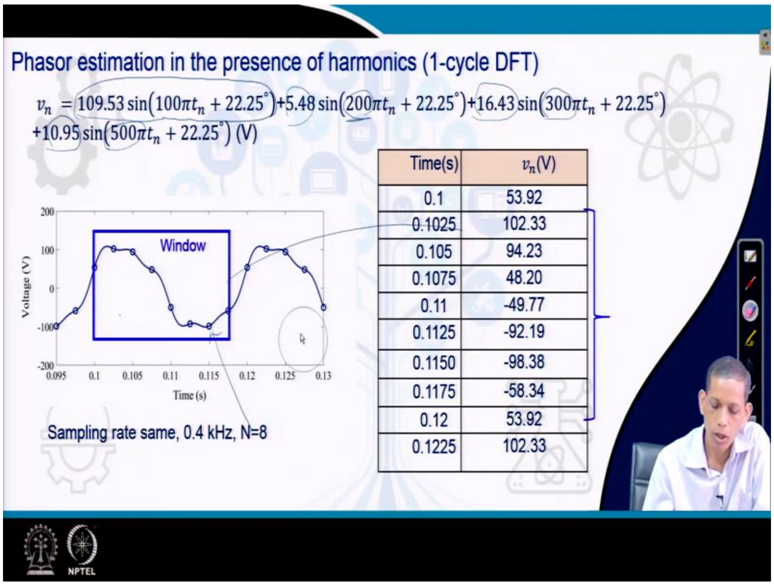

Now, let us you can say that, see how good you can say that, is this corresponding 1-cycle DFT in filtering or the different unwanted component like in this example, we have taken you can say that, a signal you can say that, these signals you can say that, it is contaminated by different harmonic components and so, like you can say that, same sinusoidal signal we consider you can say that, again you can say that $109.53\sin(100\pi t_n + 22.5^0)$ the same signal we have computed the phasors with a magnitude of 77.45, and with certain angle depending upon the different windows and we add this second harmonic component 200 now, instead of 100, this is 300 now, third harmonic component and then, we have also fifth harmonic component you can say that the magnitude of fifth harmonic component is 10.95 then 16.43 and then you can say 5.43 different percentage of second, third and fifth harmonic are being added to this one, this signal is now highly distorted you can say that from the fundamental which is clearly visible from this plot of the signal.

Now, you see you can say that, this signal you can say that, the data acquisition to be accomplished by the A to D process and then again we are maintaining the same 400 Hz sampling rate with the capital N=8 for the 50 Hz signal. Now, consider this window we have 8 number of samples, this 8 number of samples corresponds to window becomes like this and are to be processed to compute the 1-cycle DFT. So, how the 1-cycle DFT finds the corresponding signal you can say for this case we will try to evaluate using the same calculation process as we have already mentioned earlier also.

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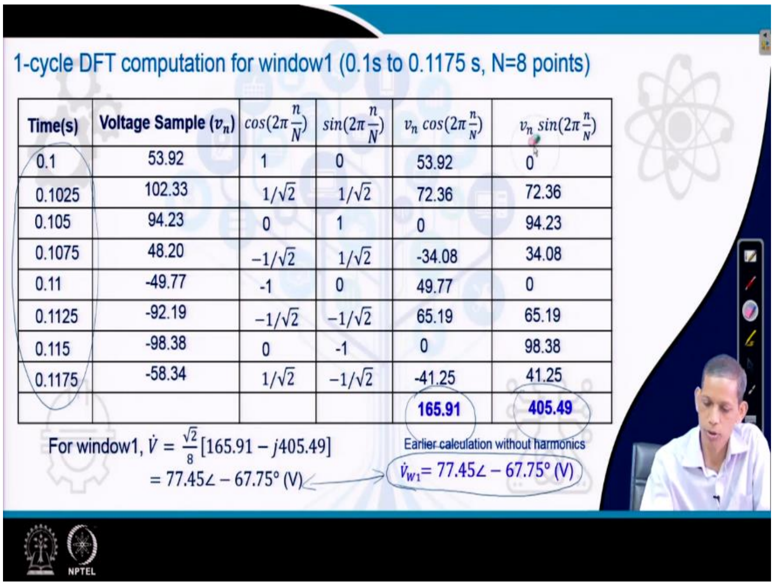

So, the calculation reveals that if we applied the corresponding cosine and sine weight to the 1-cycle data window which we have mentioned for this period the corresponding value you real part becomes 165.91, and the imaginary part comes out to be 405.49. If you see this you can say that, two components, it is we have already seen these two component earlier so, and corresponding phasor

$$\dot{V} = 77.44\angle - 67.75^0$$

So, what we see here, that the corresponding phasor is same as earlier calculation of that without harmonics in the fast window. So, these two you can say that, values are you can say that, exactly same this clearly shows that the different harmonic components second, third and fifth are completely being rejected by the 1-cycle DFT and that is what, we want in relay also, because if the phasor is to be used by the relay the phasor should be accurately as should be estimated from the signal for successful operation you can say that accurate operation, accurate decision by the corresponding relay in the system.

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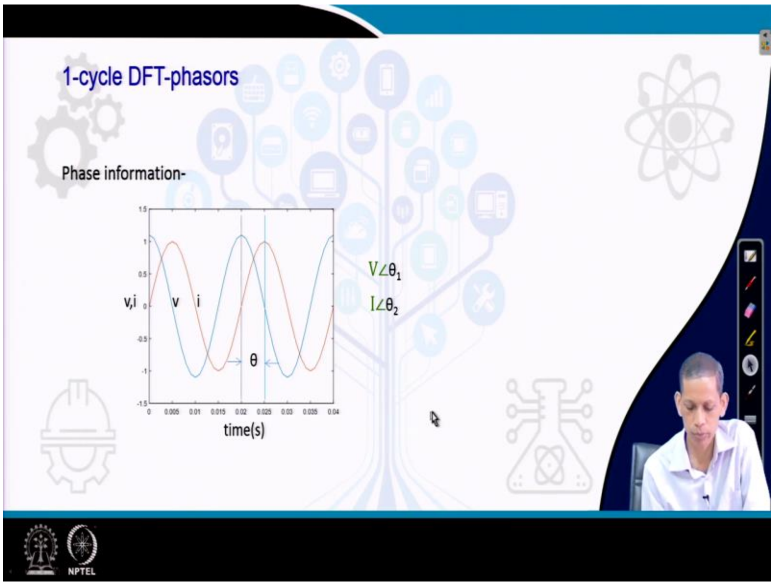

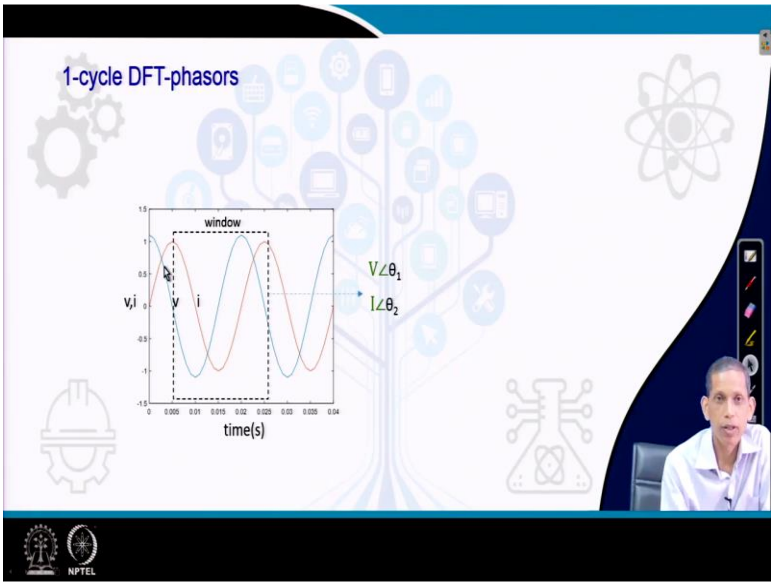

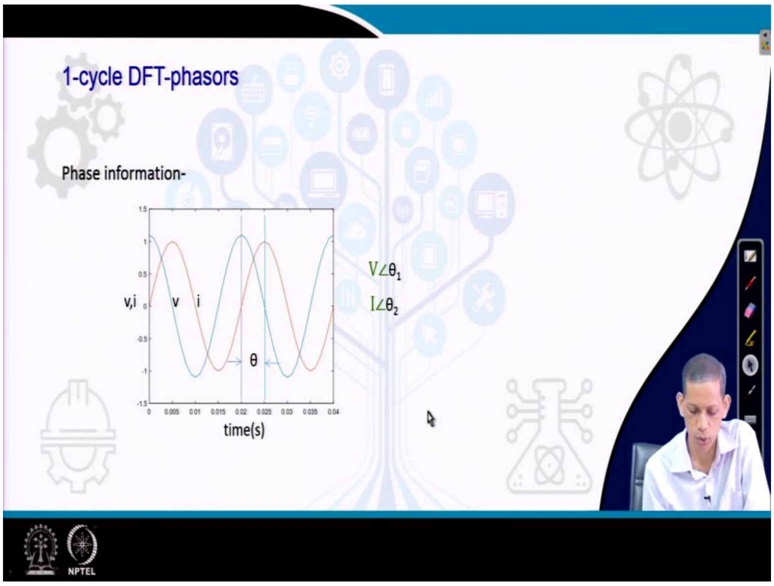

Now, we see here that, as you have already mentioned you can say that are being useful in relay decision process to have that picture on that you can say take sinusoidal signals, voltage and current. So, blue one is voltage and the red one is current perspective in this case, for example, now you select an window and in this window you compute the corresponding phasors for both voltage and current so you can get the corresponding voltage phasor and the corresponding current phasor in polar form or in rectangular form at particular instance $V\angle\theta_1$ and the corresponding current for this case, let us calculate by the 1-cycle DFT is $I\angle\theta_2$. So, clearly you can say for this case you can find the corresponding angle between, voltage and current phasors ($\theta_1 - \theta_2$) you can say that is indicative of what is the corresponding theta value so, this phase information is very important in relay applications like, directional relay and in many other you can say that, application perspective including the magnitude but, also you can say use both the voltage and current magnitude and angle are being used in distance relay application and so.

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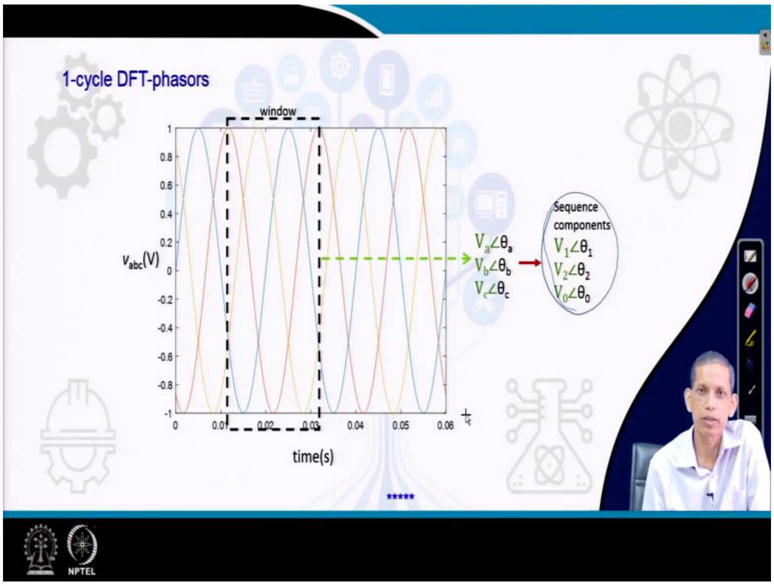

Now, come to you can say that, another one this you can say that take how, 1-cycle DFT can be useful you can say that, in this sequence component perspective, let us say here, you can say that, this three plots are for voltages you can say that, in a system A B C. So, these voltage signals like to be used to compute the corresponding phasors using the 1-cycle DFT.

Let us say at one window you can say that, we will take this window of length containing N number of samples and then, you can say that, for each phasor for each signal you can say that, will compute the corresponding phasors $V_a\angle\theta_a$, $V_b\angle\theta_b$, $V_c\angle\theta_c$ using the 1-cycle DFT for this window. Then, using the transformation some as you have already learned that we can compute the $V_1\angle\theta_1$, $V_2\angle\theta_2$, $V_0\angle\theta_0$ corresponding to these sequence components, are being widely used for different numerical relays you can say that, for decision making process. So, now this you can say that, with you can say that, window progressing the corresponding V_a V_b V_c will be also estimated using the 1-cycle DFT and the corresponding sequence component will also be updated accordingly.

So, what you learn from this is that different current and voltage signals using the 1-cycle DFT algorithm which is just the different samples value and using the corresponding Fourier coefficients to it you can estimate the corresponding fundamental component using that phasor estimation principle very efficiently and which can be used for the relay decision making process. So, in the next class, we will see how variants of the 1-cycle DFT algorithms can also be used for phasor estimation for better relay decision making process. Thank you.