Power System Protection
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Lecture 07
Recursive and Half Cycle DFT and Cosine Filter

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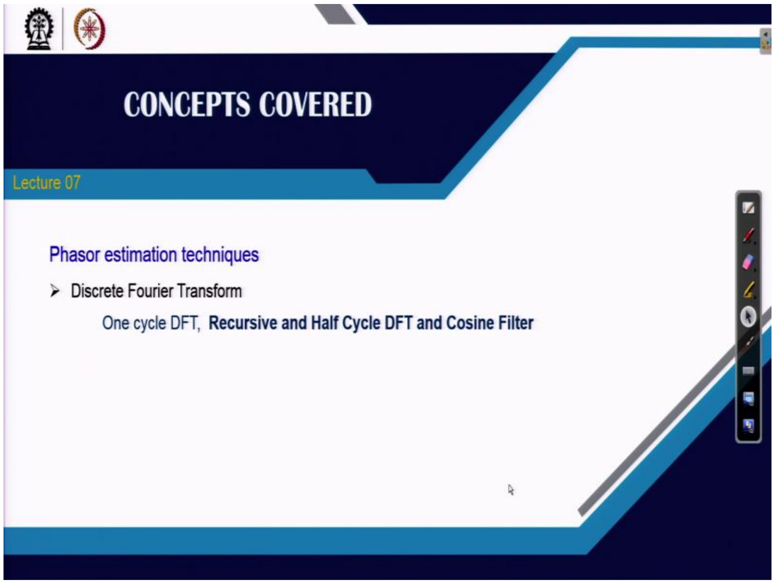

Welcome to the lecture 7 on module 2 and we are continuing with phasor estimation, we are learning different techniques in this lecture we will extend this idea of phasor estimation technique using one cycle DFT, where you will see more efficient techniques using recursive DFT, half cycle DFT and then will emphasize on cosine filtering approach.

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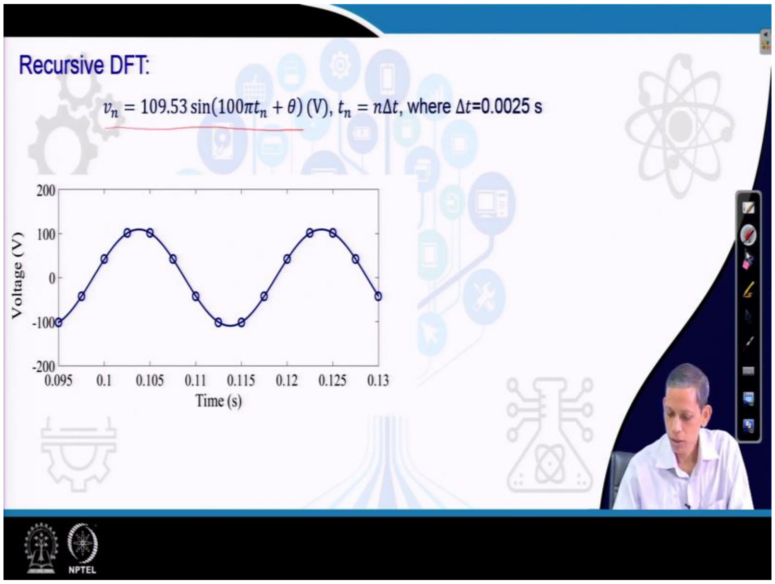

So, let us continue with the same signal, last time we discussed on the

$$v_n = 109.53\sin(100\pi t_n + \theta)$$

We are continuing the same sampling rate 400 Hz for the 50 Hz signal and therefore the corresponding Δt times remains to be 0.0025 s where this t_n is expressed as

$$t_n = n\Delta t$$

Where, n corresponds to the sampling number. So, these are the samples that are being acquired by the A to D processing part in the relay and then you can say that in last lecture we mentioned about how the one cycle DFT calculations can be carried out with a with progressing window, v_1, v_2, v_3, v_4 and so on.

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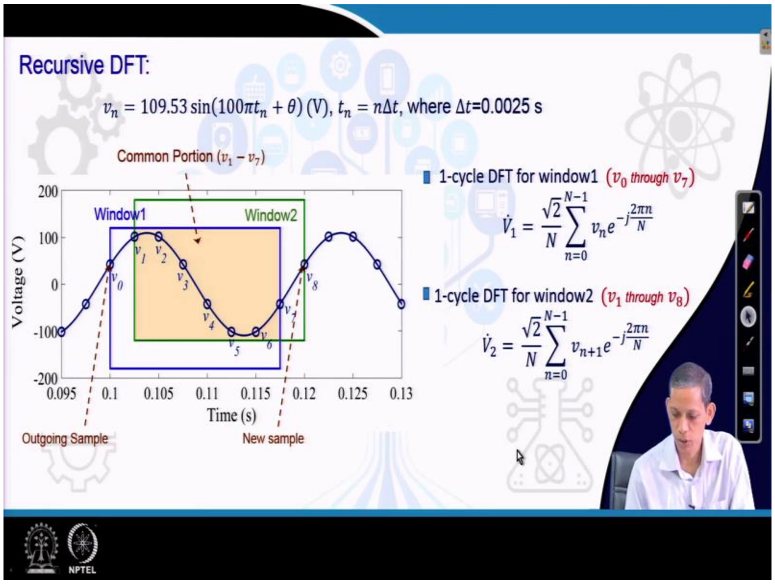

So, we proceed with this one cycle DFT the corresponding first window phasor, this window where the corresponding window samples are v_0 through v_7 and then the corresponding equation becomes

$$\dot{V}_1 = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n e^{-j\frac{2\pi n}{N}}$$

So, we see for the second window when the first sample is acquired by the processor, now the fresh sample is v_8 then you can see that the corresponding second window becomes v_1 through v_8. So, we see here that the corresponding expression becomes in this case

$$\dot{V}_2 = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_{n+1} e^{-j\frac{2\pi n}{N}}$$

So, if you put n equals to 0 to N-1 and again same window but then you can see that the sample sequence there becomes v_{n+1}, and the Fourier coefficient remaining same as that in the first window also. So, Fourier coefficient points remain same only the samples value is shifted by this, so what you see from this one that a new sample is acquired in the second window and the window is now updated and it discards the last sample of the first window at the old resting point, in this these way the outgoing sample and we consider new sample required by the second window. So, for this perspective we see that the common points between these two windows and these common points are nothing but v_1 through v_7, so 1 through 7, the seven points are common to window one and window two, so this set of samples are common to window one and window two, therefore it is being expected that there is a relation between $\dot{V}_2$ and $\dot{V}_1$ that is corresponding phasor for the second window and the first window, we would like to explore what is that this commonness you can say that provides us the corresponding relations between this and we like to explore how that can be used for the further computation process.

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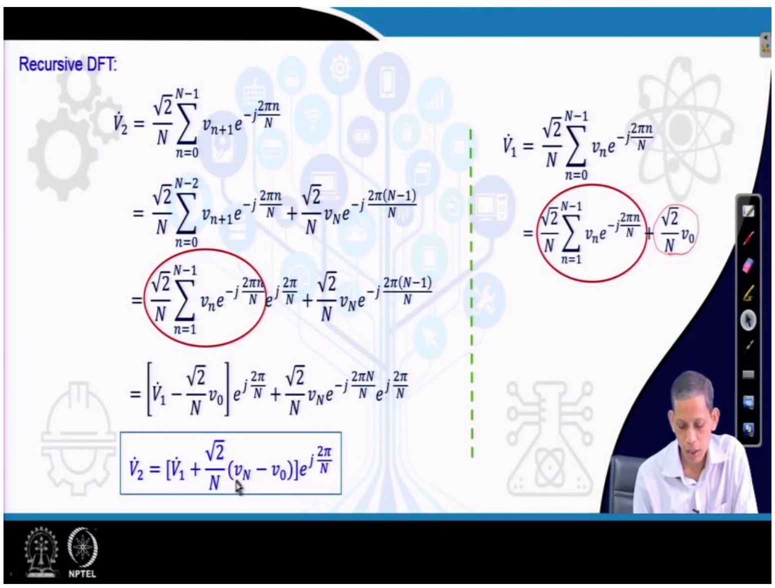

Now, see here in these expressions for $\dot{V}_2$ is

$$\dot{V}_2 = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_{n+1} e^{-j\frac{2\pi n}{N}}$$

It can be simplified by

$$\dot{V}_2 = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-2} v_{n+1} e^{-j\frac{2\pi n}{N}} + \frac{\sqrt{2}}{N} v_N e^{-j\frac{2\pi(N-1)}{N}}$$

$$= \frac{\sqrt{2}}{N}\sum_{n=1}^{N-1} v_n e^{-j\frac{2\pi n}{N}} e^{j\frac{2\pi}{N}} + \frac{\sqrt{2}}{N} v_N e^{-j\frac{2\pi(N-1)}{N}} \quad \text{.... (1)}$$

From the phasor of window 1

$$\dot{V}_1 = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n e^{-j\frac{2\pi n}{N}} = \frac{\sqrt{2}}{N}\sum_{n=1}^{N-1} v_n e^{-j\frac{2\pi n}{N}} + \frac{\sqrt{2}}{N} v_0 \;\ldots\ldots\; (2)$$

From (1) and (2)

$$\dot{V}_2 = (\dot{V}_1 - \frac{\sqrt{2}}{N} v_0) e^{j\frac{2\pi}{N}} + \frac{\sqrt{2}}{N} v_N e^{-j\frac{2\pi N}{N}} e^{j\frac{2\pi}{N}}$$

$$= [\dot{V}_1 + \frac{\sqrt{2}}{N}(v_N - v_0)] e^{j\frac{2\pi}{N}}$$

It shows the relation between $\dot{V}_2$ and $\dot{V}_2$ and that we are trying to explore, this gives us platform for efficient computation of the phasor where we do not require all the computations what we did earlier in the last lesson of DFT calculations using this multiplication with the cosine and sine weight and adding them to find the phasor.

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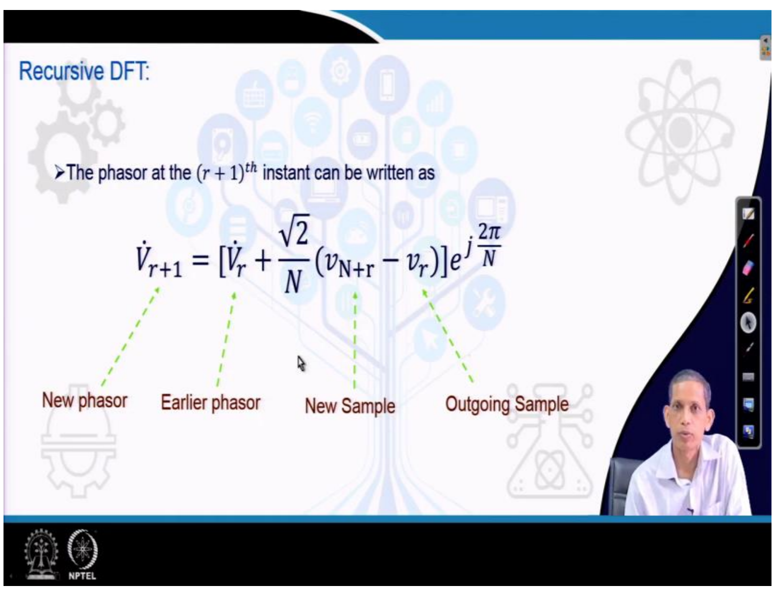

So, in general we can conclude that at any instant $(r+1)^{th}$ instant that any phasor can be obtained from the just earlier phasor computed at the r^{th} instance written as

$$V_{r+1} = [\dot{V_r} + \frac{\sqrt{2}}{N}(v_{N+r} - v_r)] e^{j\frac{2\pi}{N}}$$

So, therefore this is nothing but a phase shift by $\frac{2\pi}{N}$ for each sample and this part is nothing but in the last lesson we calculated it for N = 8, this corresponds to 45^0, so the phasor shifting by 45^0 in anticlockwise direction is nothing but the part of the computation of the things. So, we will see how the corresponding the calculation process in a further calculation can be obtained in a recursive way from one after the other from the phasors computed with the acquisition of the new and new samples.

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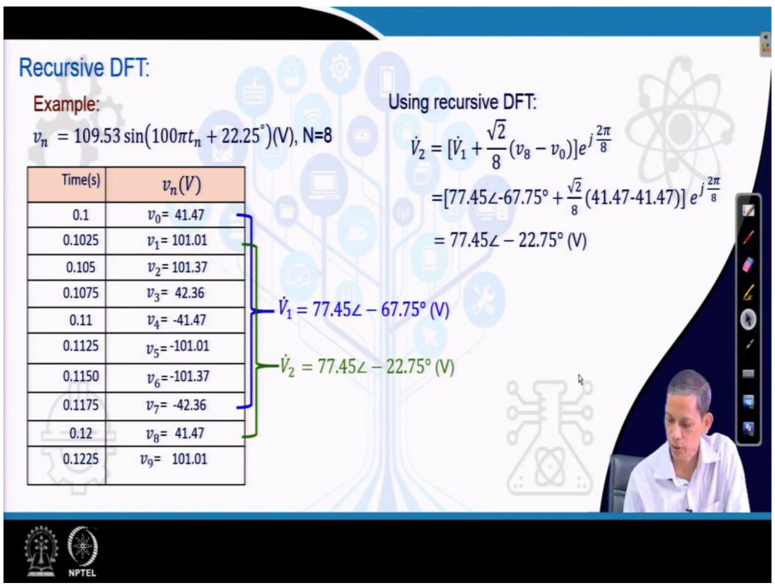

Let us see same example which we did in the last lesson

$$v_n = 109.53\sin(100\pi t_n + 22.25^0)\ \text{V}$$

And number of points per cycle is 8 and this is a 50 Hz signal. So, these about the time index and these are the samples number acquired by the relay. So, let us consider the first window from the v_0 through to v_7 and then gives us

$$\dot{V}_1 = 77.45\angle - 67.75^0\ \text{(V)}$$

Now, we like to calculate the $\dot{V}_2$ using the recursive DFT, $\dot{V}_2$ in earlier example using 1 cycle DFT similar to calculating for $\dot{V}_1$ equals to

$$\dot{V}_2 = 77.45\angle - 22.75^0\ \text{(V)}$$

That clearly shows a 45^0 shifting in the anti-clockwise direction with this angle, the magnitude remaining same. So, what you see now that using the recursive DFT we can express this V_2 equals

$$\dot{V}_2 = [\dot{V}_1 + \frac{\sqrt{2}}{8}(v_8 - v_0)]e^{j\frac{2\pi}{8}}$$

So, this case substitute the corresponding values $\dot{V}_2$ is calculated as

$$\dot{V}_2 = [77.45\angle - 67.75^0 + (41.47 - 41.47)]e^{j\frac{2\pi}{8}}$$

$$= 77.45\angle - 22.75^0$$

This is exactly matching with what you calculated for the one cycle DFT perspective. See here these values of v_8 and v_0 are same here because of the only the signal contains only sinusoidal perfect sinusoidal or 50 Hz systems but in practice the corresponding signal may be contaminated by different harmonics, noise and also decaying dc, transient during the fault situation the corresponding signal will go through lot of distortion also so we may not find same value in this case, however this one cycle DFT we saw that how it can reject different harmonics components also through an example also, so we will see how this filtering process is being good to capture the corresponding fundamental component in more details in further lessons also.

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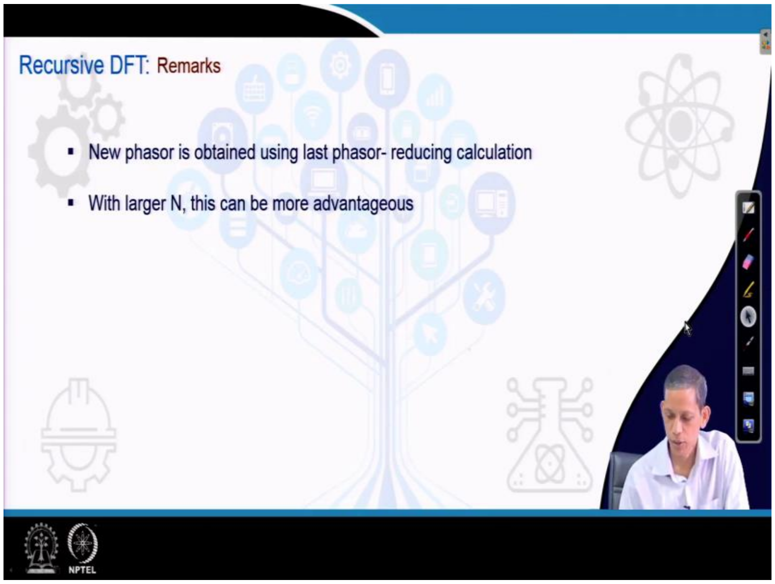

So, in conclusion we say that that recursive DFT is very useful in obtaining new phasors obtained from the last phasors and this reduces the calculation, like you can say that if the corresponding N becomes more and more it means number of samples per cycle becomes more and more higher sampling rate not 1 kHz you can go to the 2 kHz, 50 kHz, 80 kHz then the computational burdens becomes more for a one cycle DFT and that can be used substantially by using the recursive DFT algorithm approach. So, this is on recursive DFT.

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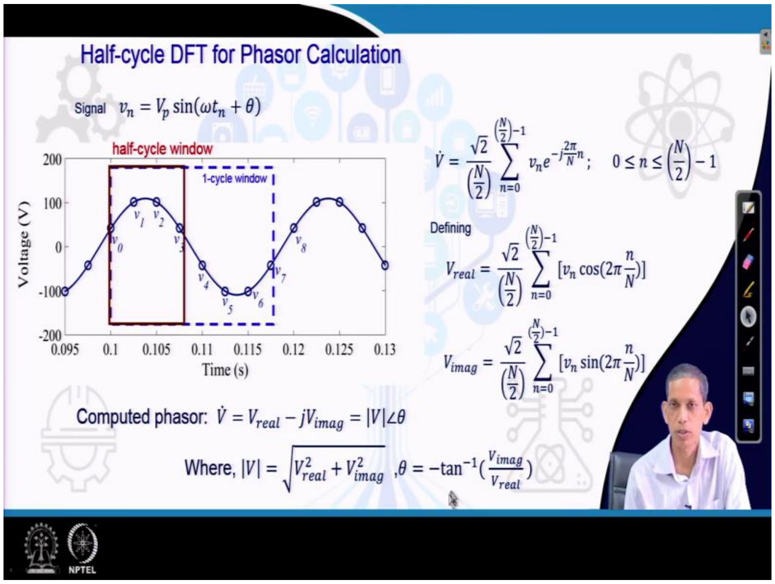

Now I will go to more, further more efficient way of doing things for the Phasor calculation using the concept of discrete Fourier transform, the next one is on half cycle DFT, so we see here same signal let us say

$$v_n = V_p \sin(\omega t_n + \theta)$$

A signal that we are considering and let us say similar to that signal which it earlier already mentioned in our discussion signals, in this window we have a set of eight data points are there in this one cycle of window and then will see.

Now, for this consider the half cycle, half cycle means that the corresponding window becomes half, so instead of eight points in the one cycle, one cycle window, one cycle DFT we are considering, now consider for the half cycle window v_0, v_1, v_2, v_3 these are the four points, so that constitute the half cycle of the 50Hz and sinusoidal signal so using that if you now apply the half cycle DFT concept then the corresponding voltage phasor $\dot{V}$ can be written as

$$\dot{V} = \frac{2\sqrt{2}}{N} \sum_{n=0}^{\frac{N}{2}-1} v_n e^{-j\frac{2\pi n}{N}}$$

So, here the Fourier coefficient remaining same, the same pool consider the relations to this summation remaining same only that the corresponding number of samples we are considering here is half and that is why this becomes also this weight factor becomes half and then as you see the samples varying from 0 to $\frac{N}{2} - 1$, so in this case capital N becomes 8, so therefore $\frac{N}{2} - 1$ is 3, from 0 to 3 it becomes 4 points only. Using this concept of half cycle DFT we say again consider this part becomes a complex number so we can segregate into real and imaginary part, so real part is nothing but the cos weights with cos weight and the imaginary part with having the sin weights just like we did for one cycle DFT, only the summation part the number of samples we have considering half, so the corresponding summation over n equals to 0 to $\frac{N}{2} - 1$ in this case. So, proceeding we can compute the phasors using the

$$\dot{V} = V_{real} - jV_{imag} = |V|\angle\theta$$

That becomes in polar from. So once you can compute the corresponding real part and imaginary part you can get the corresponding phasor like this as you did for the one cycle DFT and the magnitude and θ becomes

$$|V| = \sqrt{V_{real}^2 + V_{imag}^2} \;\; ; \; \theta = -\tan^{-1}\frac{V_{imag}}{V_{real}}$$

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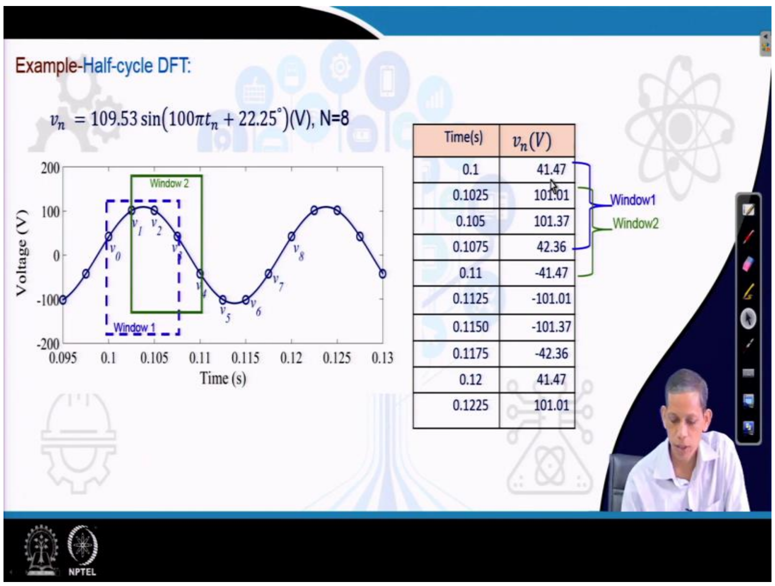

So, let us see an example how good is the corresponding half cycle DFT in computing phasor, so for the same signal now we will take half cycle DFT, for the window one, so in this window we require four points now instead of the eight points in one cycle DFT and then that we considered first point, second point, third point and fourth point, this four points are there for the window one and for the window two, it shifts by one point so it acquires a new samples and discards the oldest sample and by this becomes window two, so we will calculate for these two windows how the corresponding half cycle DFT can be computed.

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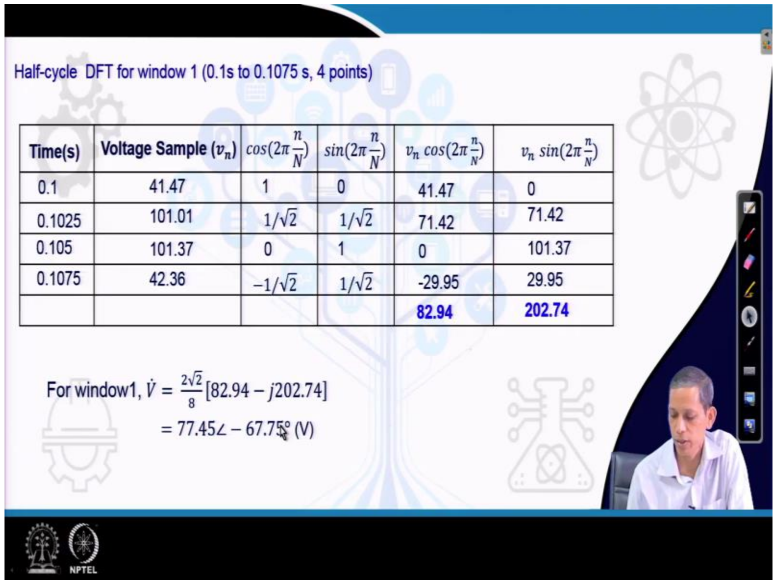

So, see here you can consider that for the first four samples window number one, 0.1 seconds to point 0.1075 second, these are the set of samples the cos weights are these and the sin weights are these for the Fourier coefficient for the four samples and then you multiply the corresponding cos and sin weights with these samples, this samples with this weight and this weight by the cos weights and you get a set of multiplying things $v_n \cos(\frac{2\pi n}{N})$. Similarly, you get the corresponding set of $v_n \sin(\frac{2\pi n}{N})$. Then we make the corresponding summation of this series and then the summation of this series and just like the one cycle DFT, so then we can convert, get the corresponding phasor value

$$\dot{V} = \frac{2\sqrt{2}}{8}[82.94 - j202.74] = 77.45\angle - 67.75^0 \text{ (V)}$$

So therefore, we say that in one cycle DFT what you got you are also getting half cycle DFT same value of the phasors,

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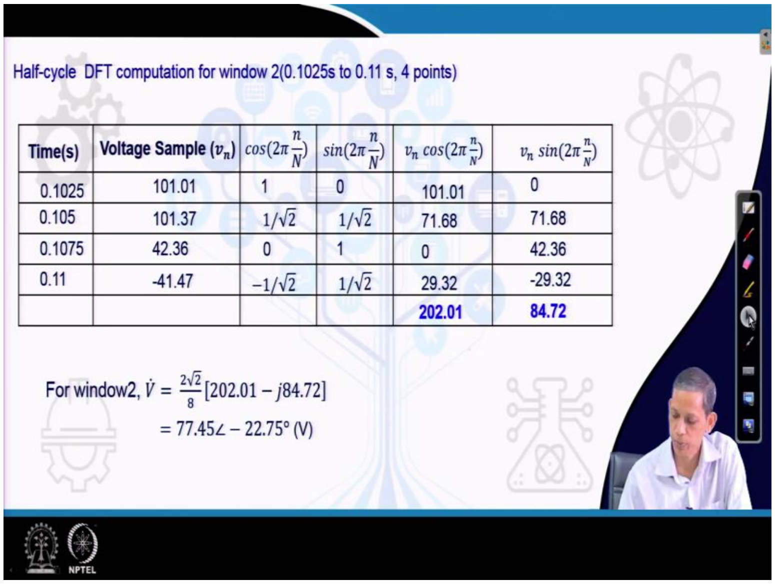

Now, going to the second window in a similar field now the window acquires the fresh samples of this and so it discards the other one and the window is being updated with the new samples the weights, cos weights and sin weights remaining same, we might get the multiplier to the samples to the cos and sin weights and then you take the summation over this and use this corresponding things to calculate the phasor value and then this corresponding to

$$\dot{V} = \frac{2\sqrt{2}}{8}[202.01 - j84.72] = 77.45\angle - 22.75^0 \text{ (V)}$$

So, what you obtain is that this window two phasor is being shifted by 45^0 anticlockwise, to the first phasor what you got from the window one, so that also we see that observation also find in case of one cycle DFT and so also here and in window two also we got the same phasor value what you obtained for the one cycle DFT with the window, for the second window also.

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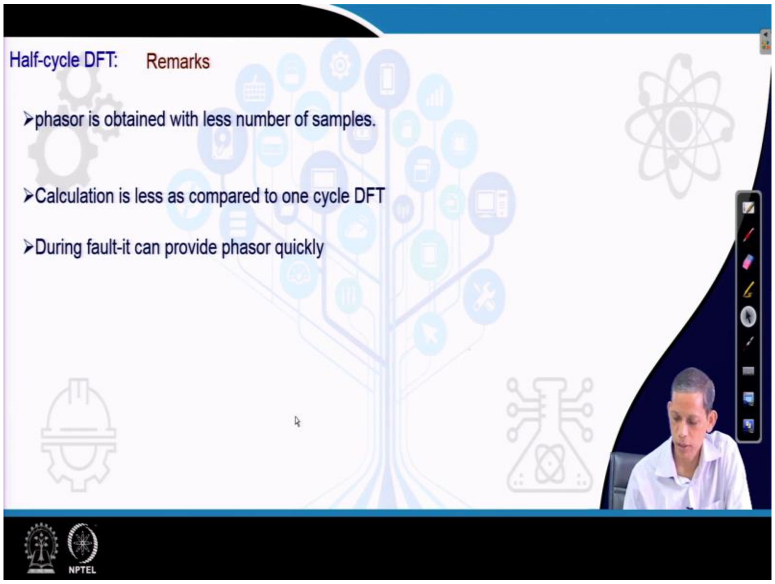

Now, our general remarks for the half cycle DFT, that the phasors can be computed with less number of samples, it means that the corresponding multiplication and the corresponding addition, in overall the corresponding computation burden will be substantially reduced in this case and so therefore you can say it becomes a more efficient in terms of utilization of the system computation perspective and if we go into the fault and the corresponding phasor computation during fault the number of points that are in that window of calculations we require eight points for the fault region for the accurate you can say that the calculations, for accurate calculation of the one cycle DFT here require only four points to calculate the corresponding phasor in the faulted region of the signal. That means that the phasors can be computed very quickly but subsequently we will see that how in terms of performance wise during fault how efficient, how correct, how accurate this half cycle DFT as compared to the one cycle DFT in our further discussion.

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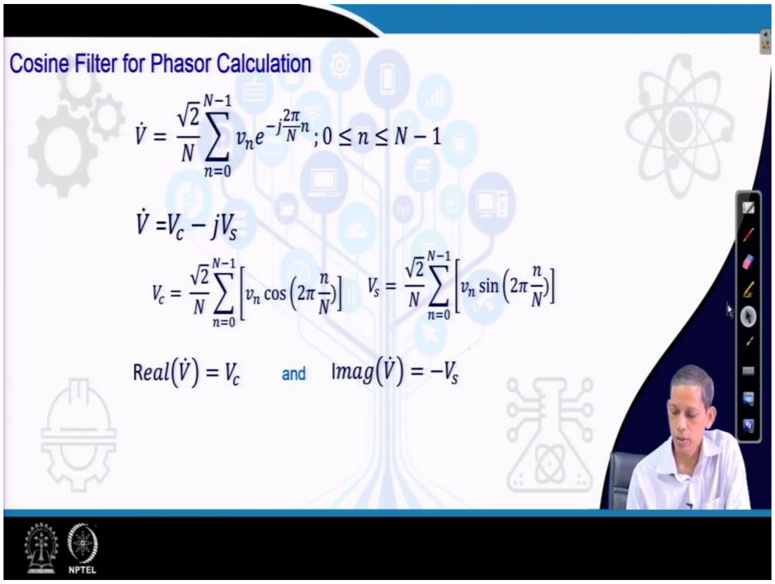

Now I will go to another category of filter extension of the one cycle DFT with term mention is that cosine filter and then again for the phasor calculation aspect. So, let us see that how this cosine filter uses for phasor calculation and then we will see through example how we can obtain the corresponding phasors for different windows of a signal.

$$\dot{V} = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n e^{-j\frac{2\pi n}{N}}$$

This is what one cycle DFT computations equation we have seen, so if you segregate the corresponding real, imaginary part

$$\dot{V} = V_c - jV_s$$

The c stands for here cosine part and the s stands for sinusoidal part, so we see here this V_c, V_s are represented by

$$V_c = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n \cos(\frac{2\pi n}{N}); \quad V_s = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n \sin(\frac{2\pi n}{N})$$

So, we represent them in terms of Vc minus jVs here you can see there is a negative sign here and we do not consider negative sign here so this relation becomes equals to V_c-jV_s, c for cos and s for sin part. So real value of V is V_c and imaginary value of V is -jVs where Vs equals to this summation over this you can say that part.

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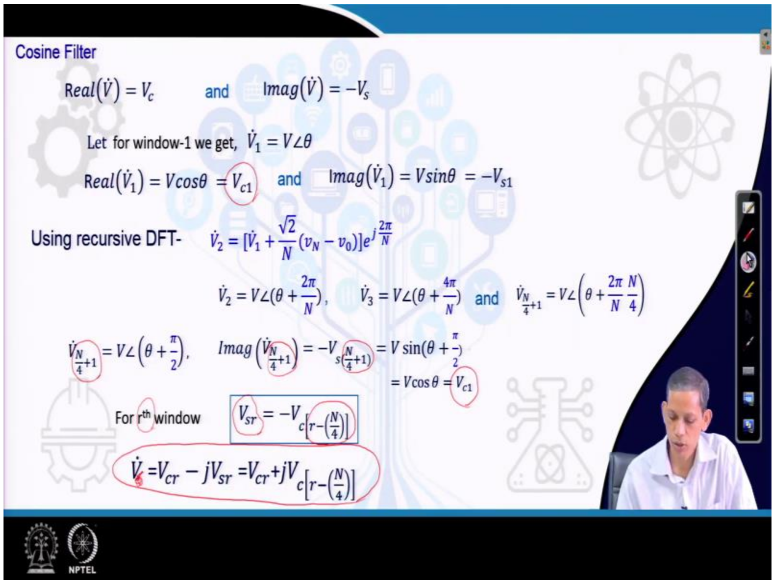

Now with this if you proceed further then, now the real part of V is Vc and we have already mentioned imaginary part is -jVs. Let us consider for the window one and we got a voltage phasor as

$$\dot{V}_1 = |V|\angle\theta$$

So,

$$Real(\dot{V}_1) = Vcos\theta\ ;\ \ Imag(\dot{V}_1) = Vsin\theta$$

Using recursive DFT, if we go for the second window and you get the $\dot{V}_2$ becomes equals to

$$\dot{V}_2 = [\dot{V}_1 + \frac{\sqrt{2}}{N}(v_N - v_0)]e^{j\frac{2\pi}{N}}$$

So, now this $\dot{V_2}$ can be expressed in terms of if you substitute the corresponding $\dot{V_1}$ =|V|∠θ

$$\dot{V_2} = |V|\angle(\theta + \frac{2\pi n}{N})$$

Because, we know here the v_n - v_0 as you have seen earlier also they become 0. Therefore this part becomes null and therefore $e^{j\frac{2\pi}{N}}$ will be multiplied with $\dot{V_1}$ and that gives us the corresponding $\dot{V_2}$. Similarly, if you go to the next window with third window V_3 that becomes

$$\dot{V_3} = |V|\angle(\theta + \frac{4\pi n}{N})$$

So, if you proceed like this then for the same phase,

$$V_{\frac{N}{4}+1}^{\cdot} = |V|\angle\left(\theta + \frac{2\pi}{N}\frac{N}{4}\right) = |V|\angle\left(\theta + \frac{\pi}{2}\right)$$

This results in the imaginary part of this, this becomes equals to

$$Imag(V_{\frac{N}{4}+1}^{\cdot}) = -V_{s(\frac{N}{4}-1)} = Vsin\left(\theta + \frac{\pi}{2}\right) = Vcos\theta = V_{c1}$$

so in general for r th window,

$$V_{sr} = -V_{c|r-\frac{N}{4}}$$

Any window the sinusoidal part of that window, the sinusoidal part of that window because equals to minus of cosine part of the window and the value corresponds to r – N/ 4, So, therefore in general we can summarize that the corresponding things that the

$$\dot{V} = V_{cr} - jV_{sr} = V_{cr} + jV_{c|r-\frac{N}{4}}$$

So, this gives us the complete phasor which you can compute only using the cosine term there is no need of sin term computation considered here, so that is the equals the cosine filter computation for the phasor so any phasor instance can be the corresponding voltage phasor or the corresponding current phasor can be computed frequency that from the cosine term at that instant plus the cosine term N / 4 instance earlier what is being computed for the cos part. So, if you consider substituting for the imaginary part this one then we are getting the corresponding phasor to be this one and that gives a platform for efficient use of this one, we will see in the next lesson also that how this part becomes advantageous, such cosine filter is advantageous to one cycle DFT and why many relates prefer for this such calculation.

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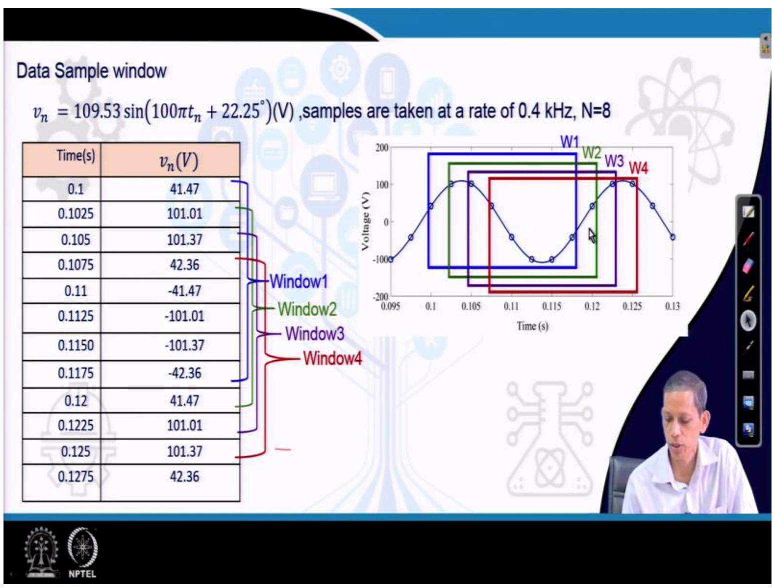

So, we will see how this being there, let us validate what you mentioned in the earlier derivation whether that is true or not for the same signal which we have earlier discussed, so we have taken four windows, window two, three, four as the corresponding fresh phasor and fresh samples are available to the relay so it progresses like this. So, this is the window we are mentioning like this.

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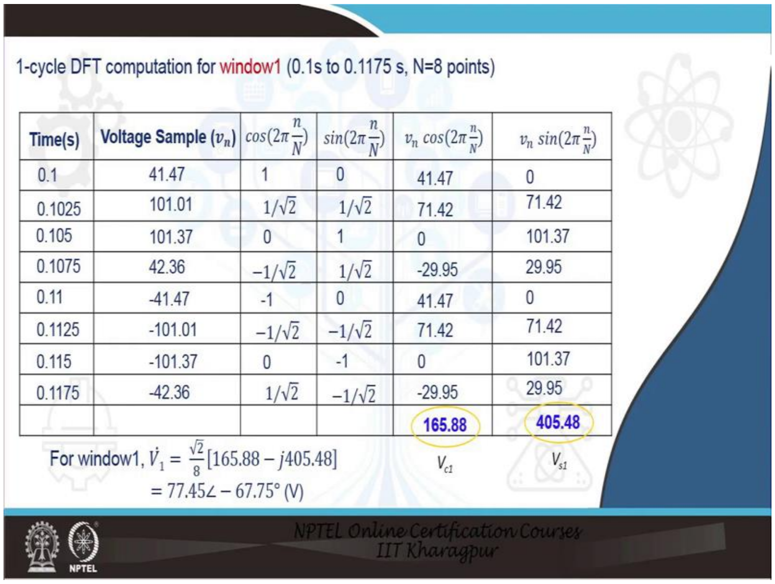

So, with this if we go like this then for the first window one cycle DFT we have computed as in the last lesson also same you can say things then we are getting the corresponding cos part here, V_{c1} here and the corresponding V_{s1} here and then we are substituting this to get the corresponding things to here and then you get the corresponding $\dot{V}_1$ to be like this, so these are the V_{c1} and this considered nothing but the our V_{c2}, V_{s1} part, sinusoidal part and the corresponding cosine part, this is for window one.

Now, next we will see for the window two, one cycle so it takes a fresh samples and discard the older one then you compute the similar way then you are getting V_{c2} to be 404 and V_{s2} the sinusoidal part with this and then the corresponding phasors we go at 45^0 shifted and magnitude remaining same to be like this, this we have computed in the last lesson also.

Similarly, if we proceed for the third window so we are getting V_{c3} and V_{s3} for 404 or 405.48 and -165.88 and then we go for the computation of the V_3 which happens to be $77.45\angle 22.25^0$, again 45^0 shifting with respect to this V_2. Now, I will go to the fourth window and in the fourth window also we get the corresponding V_{c4} and V_{s4} like this and then we get the V_4 phasor to be $77.45\angle 67.25^0$ and for this perspective again 45^0 anticlockwise rotation with respect to V_3 phasor.

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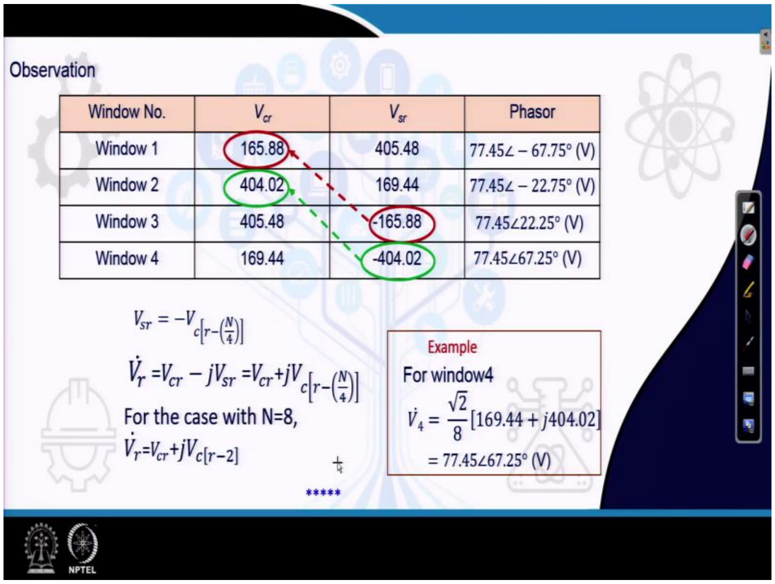

So, in overall we can conclude that window 1, window 2, window 3, window 4, four windows we have computed and we have computed the corresponding V_{cr} and V_{sr} the cosine part and the sin part for the corresponding window, we got the value to be what we obtained these values are noted and the corresponding phasor values are noted down here. So, what we derived in the cosine filter perspective is that

$$V_{sr} = -V_{c|r-\frac{N}{4}}$$

Now, you see here for this case, from the observations let us say third one this is V_{s3}, this V_{s3} -165.88 and these V_{cr} you can say 165 only with a negative sign, if you go to the fourth V_{s4}, this V_s is - 404 and V_{c2} is having 404. So, this is the relation that we are talking about, here we see here this for this example capital N = 8, 8/ 4 = 2, so whatever samples we are talking about at the present moment this is the real part, that is the V_{cr} part this remains and the other part, the sin part in place of the sin part we can replace by the cosine part, how much? By factor N/4 earlier values that becomes here 8/ 4 it is two 2. So, two samples earlier, so whatever values you are getting and if you say negative sign you consider that gives you the sin part of that one. So, negative of this is nothing but the sin part of this one and therefore the corresponding V_r phasor at that instant is nothing but

$$\dot{V}_r = V_{cr} - jV_{sr} = V_{cr} + jV_{c|r-\frac{N}{4}}$$

For N = 8 this becomes equals to

$$\dot{V}_r = V_{cr} + jV_{c|r-2}$$

two points earlier value whatever cosine value we are getting we just add that and you can say that the corresponding point is positive sign instead of the negative sign what we are using earlier in one cycle DFT because of that negative sign we are getting here. So, if we see that one example here from this data calculations, so for the fourth window if you like to compute the phasor using the cosine filter so

$$\dot{V}_4 = \frac{\sqrt{2}}{8}[169.44 + j404.02] = 77.45\angle - 67.25 \text{ (V)}$$

Exactly what you have got you can see that in our one cycle DFT calculation which we have done in our earlier slides we have demonstrated. So, this conclude that instead of going for sinusoidal computation part computations addition, subtraction and so we can say that addition, multiplication and so for the sinusoidal part we do not need to consider that one if you have already stored the corresponding cosine part of the calculation in this one, so from that real part of the corresponding Fourier coefficient we can directly obtain using the relation generally $V_{cr} + jV_{c|r-\frac{N}{4}}$ this is what more efficient usage of the processor and also we will see subsequently that this filtering, cosine filtering is better than one cycle DFT in filtering out unwanted components in the power system signal. So, in the next class we will discuss about more on this phasor compression perspective and we will see more techniques how they can be useful in relaying application, thank you.