Power System Protection
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Lecture No. 09
Frequency Response of Estimation Techniques and Decaying DC

Welcome to the NPTEL power system protection course. So, we have already seen the phasor estimation techniques, now I will like to evaluate these techniques who are good in a relative basis, so for that we have a lecture on frequency response of phasor estimation technique.

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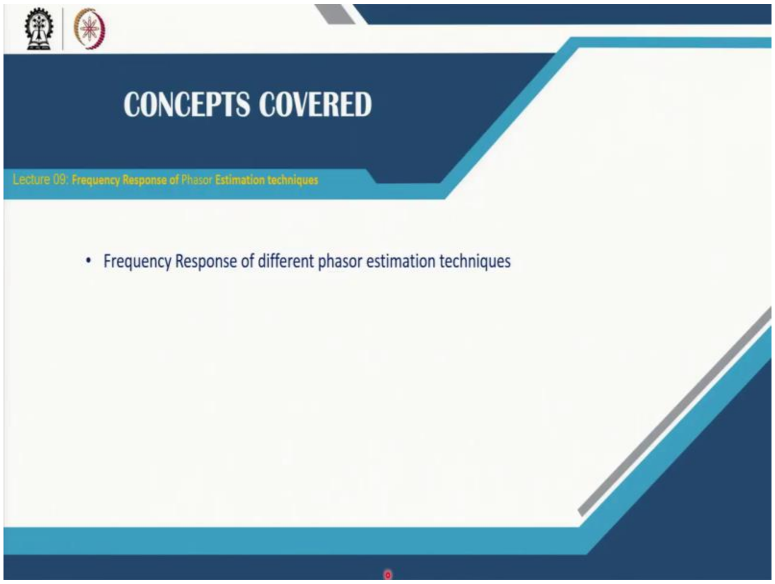

Here in this one we will apply the frequency response perspective to different phasor estimation techniques and evaluate them.

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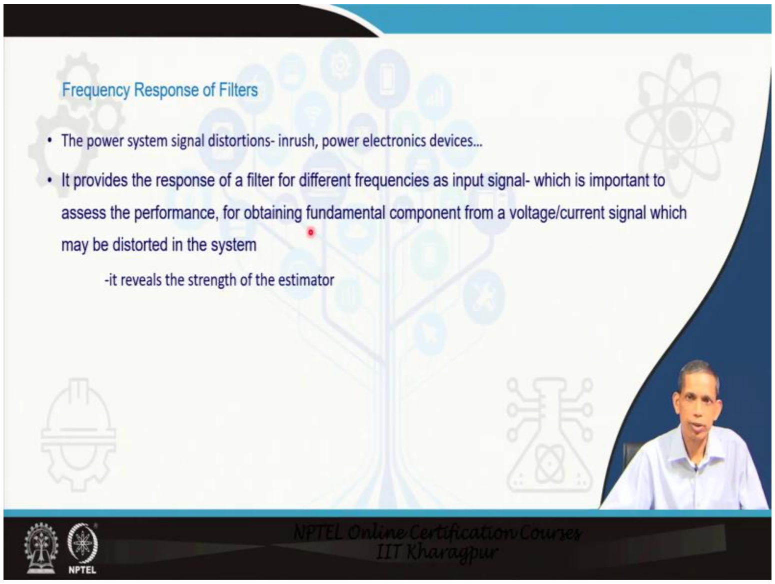

So, what this frequency response of filters, you must have seen in while studying filters, z transform domain, digital filtering techniques and so. Our phasor estimation techniques are filtering techniques, we are working in numerical relaying, and these are numerical algorithms in digital techniques.

So what it say, that the power system signals are distorted. Why? Events like inrush, events like transient things, momentary transient, power electronics devices introduces harmonics. The power system elements are interconnected, so disturbances may be happening in this element or that, so that propagates also. So, these modulates the voltage current signals as seen by the relay at times and furthermore during fault the current and voltage go through transient.

The frequency response provides the response of filters for different frequency as input signal which is important to assess the performance of the filter or the estimation techniques which we are of interest. Our interest is here to obtain the fundamental component faithfully from the, the voltage and current signals, even though they may be distorted. So, this frequency response of filter reveals the strength of the phasor estimation techniques.

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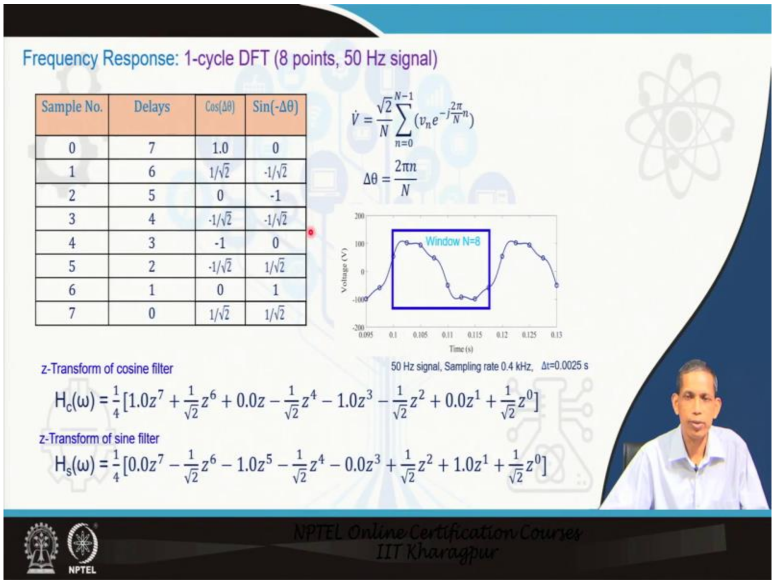

We will see that how we can evaluate the frequency response of different phasor estimation techniques which we have discussed. First let us consider for 1 cycle DFT widely used in relaying principles. So, same 50 Hz signal, 400 Hz sampling means 1 cycle corresponds to 8 points, we will be discussing again for understanding of the concept.

So this is that signal, distorted signal and we are now with a window and the window is having N equals to 8, 8 samples for the 400 Hz sampling rate, 50 Hz signal. The interval between two consecutive points in time axis is this much of time. Now these are the set of signals, so what we say that we will be applying the 1 cycle DFT, govern by

$$V = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} v_n e^{j\frac{2\pi n}{N}}$$

where the Δθ corresponds to

$$\Delta\theta = \frac{2\pi n}{N}$$

Now, if you see this window, this is the current sample and this is the oldest sample in this window. So let us, we number the samples, so this is 0 1 2 3 4 5 6 7 samples, so 0 1 2 3 4 5 6 7 samples, this is the current sample. So, if we consider the current sample, the associated delay is 0 from here, to this 6th sample and the associated delay from the current sample is Δt.

So, we have 1Δt, this third sample from this side or fifth one from this side is having 2 delays; 1Δt + 1Δt for 2 delays and like that. So, when you reach to this one, so we have 1 2 3 4 5 6 7 delays, 7 delays. So, these 7 delays means 7Δt as compared to the current sample, this sample. Now as already we have done for the DFT computation, so for this corresponding samples we have cosine weights and the sine weights; cosΔθ and sinΔθ with a negative sign, where Δθ corresponds to this. So, these are the sets of weights, we have already use for DFT calculation earlier.

These weights, so when you see these corresponding points, these are the corresponding weights, so now what we will do for the cosine filters and sine filters that are being used in the 1 cycle DFT computation, we will have the z transform of the cosine filter and the z transform of the sine filters to compute the frequency response.

z-transform of cosine filter

$$H_c(\omega) = \frac{1}{4}[1.z^7 + \frac{1}{\sqrt{2}}z^6 + 0.z^6 - \frac{1}{\sqrt{2}}z^4 - 1z^3 - \frac{1}{\sqrt{2}}z^2 + 0.z^1 + \frac{1}{\sqrt{2}}z^0]$$

z-transform of sine filter

$$H_s(\omega) = \frac{1}{4}[0.z^7 - \frac{1}{\sqrt{2}}z^6 - 1.z^5 - \frac{1}{\sqrt{2}}z^4 - 0.z^3 + \frac{1}{\sqrt{2}}z^2 + 1.z^1 + \frac{1}{\sqrt{2}}z^0]$$

So for this one, what any frequency corresponds to ω, the cosine filter z transform will be considering that z^0 this corresponds to no delay that is for this sample the corresponding weight for the cosine filter is 1/√2, so 1/√2 z^1, 1delay, so that corresponds to this samples, the corresponding weight is 0 and like that so whenever the 7 delays this, the corresponding weight is 1, this is what for the 8 samples, we have corresponding cosine filters here. Similarly, the z transformation for the sine filter, if you see this is the current sample with no delay, weight is 1/√2, then 1, then 1/√2, like this and for the last sample so we have 0. That gives the z transform the cosine filter and sine filters.

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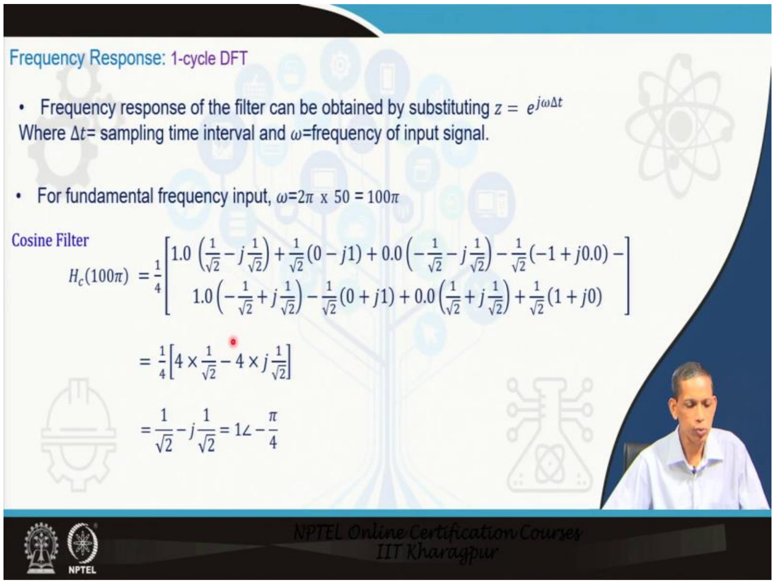

Moving forward, we see what you do with that, for the frequency response of the filters you bring that z transform the cosine and sine filters, if you substitute

$$z = e^{j\omega\Delta t}$$

Where, Δt = sampling interval, and ω is the frequency of the corresponding input signals, so we go on varying the input frequency accordingly ω also varies whereas and Δt the sampling time interval in our case we have 400 Hz sampling, so accordingly 1/400 gives you the Δt. For fundamental frequency input, let us bother about how would is the filter response for the 50 Hz or the 60 Hz component, in our case we have considered 50 Hz. So, this ω corresponds to

$$\omega = 2\pi \times 50 = 100\pi.$$

So, the cosine filter H_c for this frequency 50 Hz component, we substitute that in that z transform equation that all these values here as you have seen in the earlier slide, then you get this, for this case the

$$H_c(100\pi) = 1\angle -\frac{\pi}{4}$$

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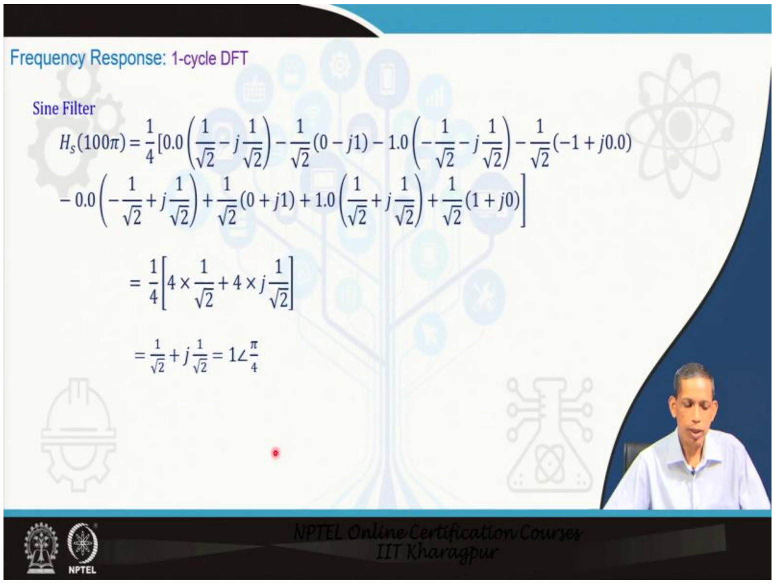

In a similar way, when you apply this same for the same 50 Hertz and to that z transform of this sine filter and substitute the values for all the 8 points, corresponding 8 points and the corresponding delays, then you get a

$$H_s(100\pi) = 1\angle\frac{\pi}{4}$$

For cosine you got $-\frac{\pi}{4}$, here you are getting $\frac{\pi}{4}$.

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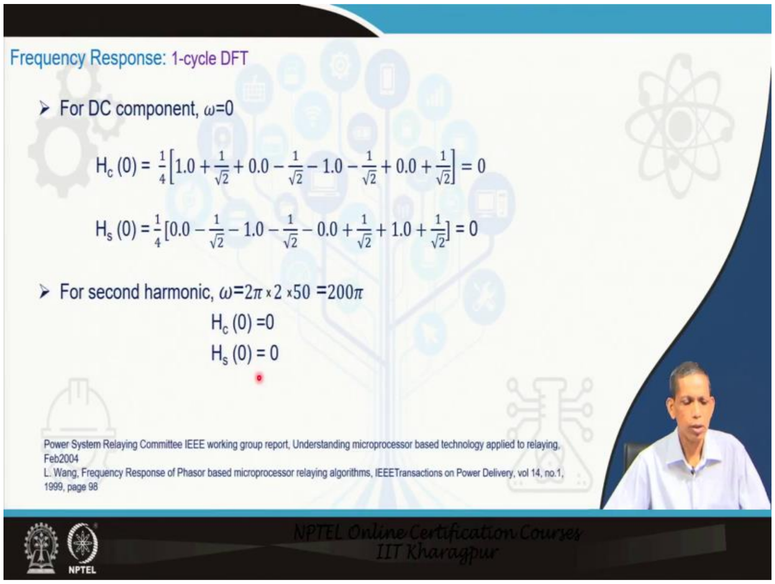

Now when you apply the same concept to the DC component that is $\omega = 0$, then the frequency response that from the z transform after substituting all the values here in that z transform equation, then we got

$$H_c(0) = 0$$

Similarly for the sine also after substituting you get the summation to be 0.

$$H_s(0) = 0$$

So, what does it mean? It means that the cosine filter and the sine filter both reject the DC component 100 percent. They eliminate the DC component, so if you pass any DC value through these filters the output will be 0. Now if we apply second harmonic signal to these filters which you have for the 1 cycle DFT the cosine or sine, so $\omega = 2\pi \times 100 = 200\pi$, substitute the value in this $H_c(\omega)$ and the $H_s(\omega)$,

$$H_c(200\pi) = 0\,; \;\; H_s(200\pi) = 0$$

Here also it comes out to be Hc equals to 0, Hs equals to 0. So, this also imply that the second harmonic component is eliminated by the two filters completely. Similarly, if we extend this to to third harmonic, fourth harmonic and so, we will see that they reject the corresponding harmonic components completely.

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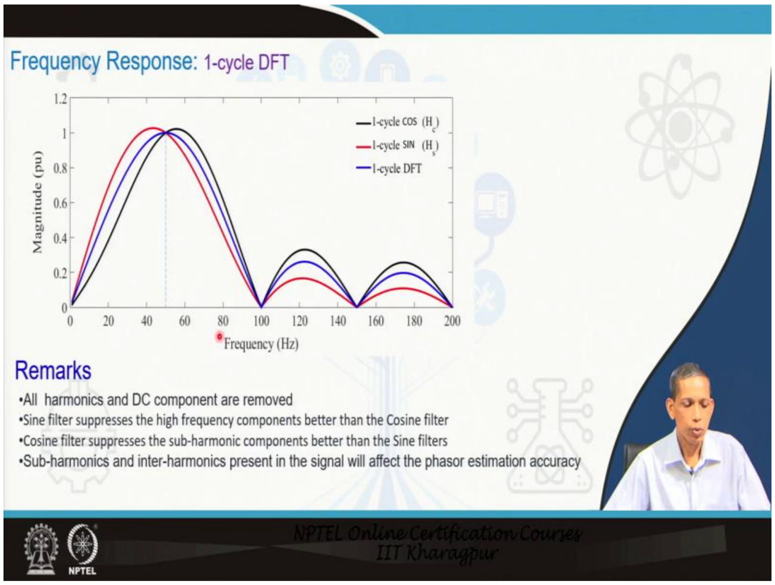

Finally, if you go to the different frequency components and find the corresponding magnitude response, then we plot that so like we will say that we talk about 50 Hz, we got magnitude 1 with inside an angle that one magnitude means whatever input signal, that will be faithfully estimated by the 1 cycle DFT or the sine or the cosine filters. The 100 percent input, 100 percent output but for other harmonic components like we talk about DC complete rejection, second harmonic complete rejection, third harmonic complete rejection, fourth complete rejection, note this is you can say that 400Hz sampling, so based on the Shanon's theorem we have shown here up to the 400/2 = 200 Hz.

Now these 3 plots of frequency versus the magnitude response, we have cosine the black one, red one is for the sine and the blue one is for the 1 cycle DFT. What we see here that these are the few remarks, that the all harmonic components including DC are being completely removed by all the 3 filters.

Sine filters suppresses high frequency component better, sine filters the red one suppresses high frequency component greater than the fundamental and better than the cosine filters, This is sine filter, this is cosine filters and in between we have the 1 cycle DFT but cosine filter suppresses the sub -harmonic, this is cosine, this is sine, so cosine filters suppresses the sub-harmonic components better than the sine filter.

However, you see here that the sub-harmonic component and the inter-harmonic components, not the harmonics, inter-harmonic components will affect the 1 cycle DFT that is what this frequency response plots reveal. Once again, that the sub-harmonic components and the inter-harmonic components will influence the estimation performance of all the 3 filters which we are describe here.

So with this, we see that which is better and what, note that because of the sub-harmonic components part, cosine filter is better than the sine, so that is found to be more suitable including that of the decaying DC elimination also which we will in the next lecture that has certain advantage over sine filters.

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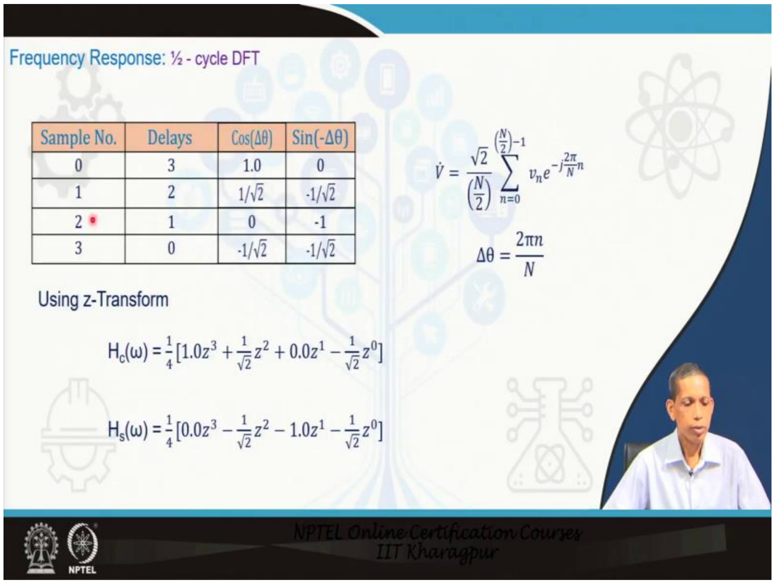

Now we will go forward to see how good is the half cycle DFT in terms of this frequency response. Therefore, we have 4 samples instead of 8 samples. So 0 1 2 3 delays, corresponding cosine weights and the sine weights, which we have already used in the half cycle DFT computation, N/2 here, and $\Delta\theta = \frac{2\pi n}{N}$

Now you go for the z transform for the cosine part and the sine part, so we have 4 points 1 2 3 4, no delay for this corresponding weight is -1/√2, then 0, then 1/√2 and then 1.Similarly, for the sine, -1/√2, - 1, 1/√2 and then the 0. So, by substituting this we got the z transform of this two filters.

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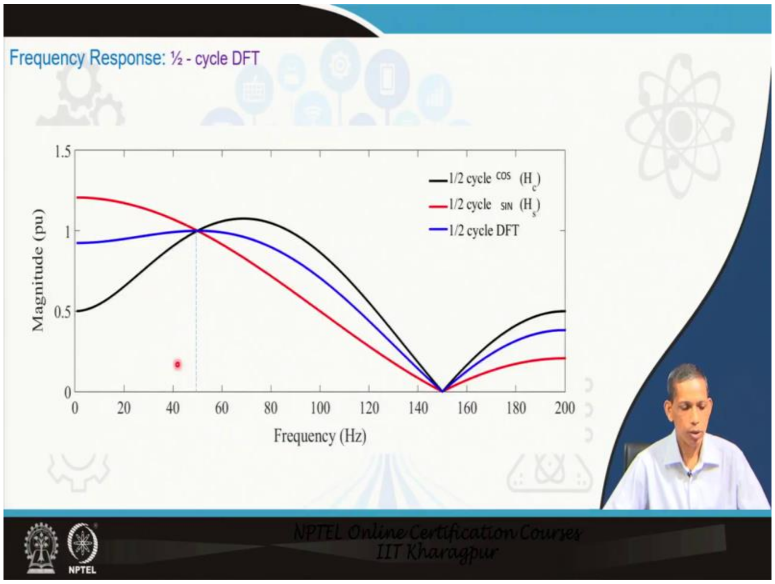

And now by similar calculations for different frequency component starting from DC, fundamental and different harmonic components, the frequency response plot that is frequency versus the magnitude plot for cosine, sine and the half cycle DFT blue one reveals that the DC values will now affect the half cycle DFT performance.

So it is unable to reject the DC component as you see for the 1 cycle DFT completely reject the DC values and which is not the case with half cycle DFT. And there is one important point, we now notice that half cycle DFT can do the business quicker, only with 4 sample as compared to 8 samples for full cycle in this example, but compromising the accuracy.

Now for 50 Hz, all are showing 1, so all 50 Hz at this signal only contains 50 Hz, it will be very good, that is also what we saw from the examples while computing the phasors, but if the signal contains other components including second harmonic, the corresponding half cycle DFT is not able to eliminate that as we found from the 1 cycle DFT but the third amount is being eliminated that is true but no other harmonics.

In addition, we saw that inter-harmonics and sub-harmonics part also we see that the rejection by this half cycle DFT is poor as compared to the 1 cycle DFT. So, when you are concerned about accuracy, then we will prefer 1 cycle DFT over half cycle DFT but if speed is a concern then you may go for half cycle DFT.

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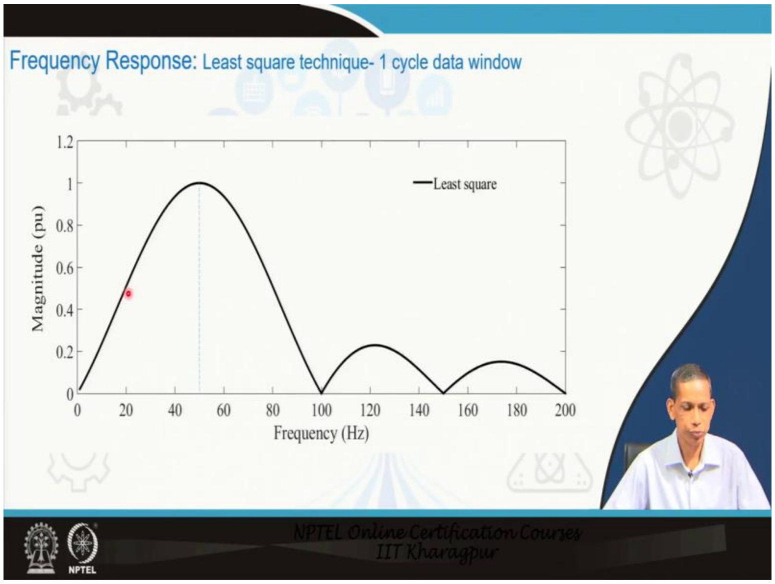

Now we will extend the same frequency response concept to least square technique and let us see here for 1 cycle data window. The similar you can go for the z transform approach and then apply

$$z = e^{j\omega\Delta t}$$

Then we will find the corresponding computations for different window because least square estimation can be formulated as you have seen from two samples to more and more collide samples also, and as you know, if you have more and more samples in the window, computation burden becomes more but accuracy becomes better.

Here also, we will see here this is 1 cycle least square estimation and the response as you can see it rejects the DC completely. For 50 Hz component perfect 1, so it is pretty good for fundamental. Harmonic components second, third and fourth are being eliminated. If you see this performance, in the frequency response performance curve, this is so similar to that of the 1 cycle DFT. So, they are comparable to each other but again they are having the one cycle data for both the cases.

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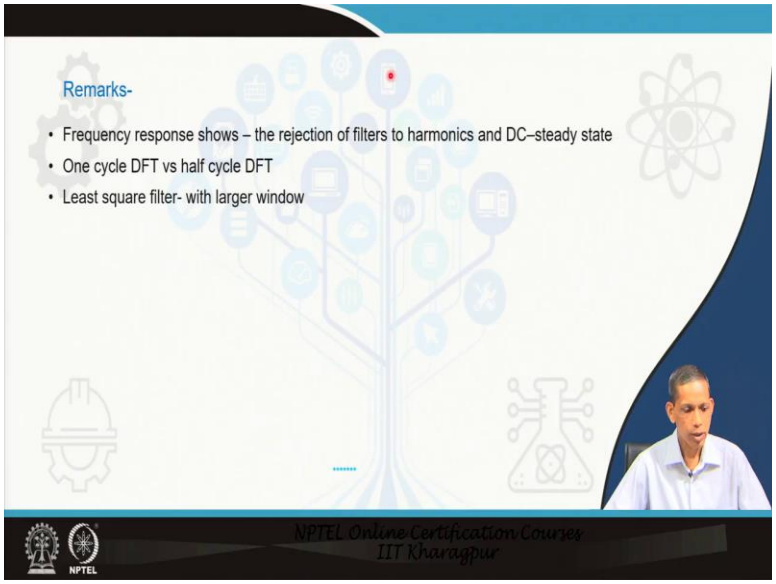

In overall, we see that the frequency response shows, how good is the filters for the estimation of the fundamental component, phasor estimation technique in rejecting harmonics and DC. Frequency response is a steady state perspective, but relay takes decision during transients. So, this study reveals that if the fault persists, then how would be the corresponding estimation process during that transient process.

One cycle DFT versus half cycle DFT we saw, that accuracy wise 1 cycle DFT will be preferred from the frequency response perspective and that is also during transient process also it is being observed. Least square filter with larger and larger window becomes better accuracy wise but we are compromising speed advantage. So, in overall we observed that frequency response approach gives which filters can be useful for protection decision in numerical relay. Thank you.