Advanced Microwave Guided Structures and Analysis
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Lecture No. 11
Instantaneous Form of Maxwell's Equations (Contd.)

Welcome to the next session on Instantaneous form of Maxwell's equations.

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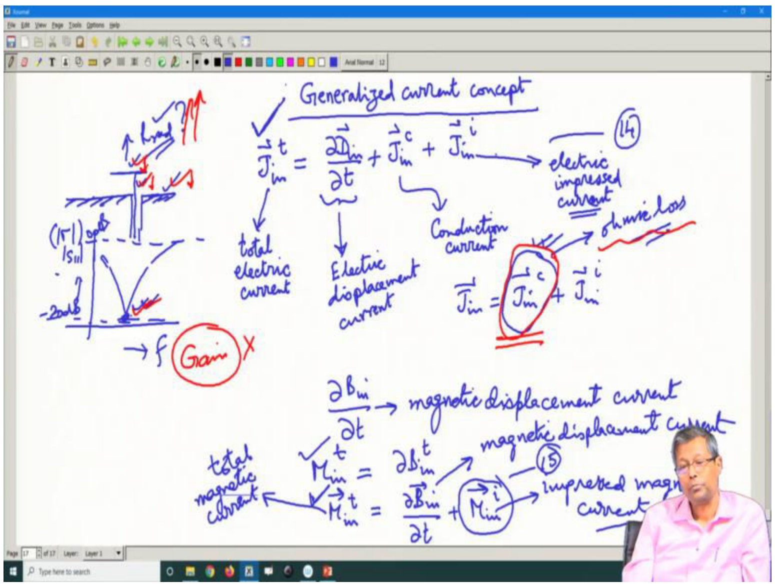

We next focus our attention on what we call the generalized current concept. We saw that we deal with the right hand side of the equations; we have these terms $\frac{\partial \vec{B}}{\partial t}$, or $\frac{\partial \vec{D}}{\partial t}$ and impressed current; that is what we saw in Maxwell's equations. So, what we want to do is that we do not want to keep those terms $\frac{\partial \vec{D}}{\partial t}$ and impressed current to be separate. So, we would like to bring all those terms close together. We want to club those terms in a total current. We do not want to distinguish between impressed current and $\frac{\partial \vec{D}}{\partial t}$.

That way it is easier to manipulate Maxwell's equations. So, we will say in this new form of Maxwell's equations that actually the $\frac{\partial \vec{D}}{\partial t}$ term is one particular kind of current that generates the field and J is another type of term which generates the field. I mean the magnetic field in this case. And similarly for the other equation we have the term $-\frac{\partial \vec{B}}{\partial t}$. So, that is the term which is responsible for the electric field; so that way it is easier to manipulate the the terms.

So, therefore we say $\vec{J}_{in}^{t}$ is $\frac{\partial D}{\partial t} + \vec{J}_{in}^{c} + \vec{J}_{in}^{i}$. So, this is the total electric current; this term is called the electric displacement current, this is the conduction current; and this is the electric impressed current. So, you will call this equation, equation number-14. So, if we compare this with equation-1; we will see that the term in equation-1, which is $\vec{J}_{in}$. It is given by $\vec{J}_{in}^{c} + \vec{J}_{in}^{i}$; so it is the impressed current plus the conduction current. In this case it might be useful to understand when we were referring to the process of radiation previously.

We said that the radiated power was contributed by the real part of the complex power. Now, this real part of the complex power is basically composed of two parts; we will come to that. But, before we do that we have to first of all get a little bit prior idea, as to the physics of what is happening. And that is the real part of the power consists of the power, which is lost as ohmic loss in the conductor and the power that is radiated. Both of this power contribute to the real part of the power, and contribute to the matching problem.

But, the part of the power which is radiated, which is the real part of the power is the desired power for the antenna. The other part, which is lost in the ohmic loss, which is lost as ohmic loss; which is the power due to $\vec{J}_{in}^{c}$; the conductor power, the power due to the current in the conductor. This contributes or this generates ohmic loss; or this is the source of ohmic loss. So, I will again repeat that this ohmic loss this will heat up the conductors; but they will contribute to the real part of the power.

But, we want to maximize the radiated power for the antenna; we want to minimize this power loss. But, both of them will be contributing to the real part of the complex impedance; which is the resistive part of the complex impedance. So, the resistive part of the complex impedance will be contributed by both $\vec{J}_{in}^{c}$ and the radiated power. We will come to that, but this is a very important concept. Why it is an important concept? Because you might feed power to an antenna; and you might see that the antenna is giving me very acceptable reflection coefficient. No reflection, no reflected power; it does not mean that the antenna is radiating. For example, if I have a structure like this again, I feed power. Particularly this can happen in a miniaturized antenna scenario, where the efficiency of such an antenna degrades. You might see that you are getting an S_{11} versus frequency, or a reflection coefficient; S_{11} is reflection coefficient mod gamma versus frequency to be like that. You might get a dip of minus 20 dB here at this point; so let it let this be 0 dB. So, it cannot be above 0 dB, so let me just modify; so this is 0 dB.

So, at this point you might imagine that we are getting a very good match to the antenna that is fine. You are indeed getting a very good match to the antenna; but it does not mean that this antenna is radiating. This does not mean that the antenna is radiating, why? Because it means that somebody is absorbing power from you. So, the power is not reflected back at you; but who is that somebody?

There can be two candidates. One the power is taken by completely by the ohmic loss; so this is the component which is causing heating of the patch; heating of all these metallic surface, heating of the patch, heating of the ground plate or it can be contributed by a mixture of radiated power, and this ohmic loss powers power loss due to ohmic loss. Or, it can be like mostly radiated power; so, this third situation is what I would like.

I want mostly to maximize this, I want to maximize this and minimize these quantities. That is I want to minimize this ohmic loss; and maximize my radiated power. And therefore in order to see that this is indeed taking place that we are getting a decent amount of radiated power; we have to measure the antenna gain. If we measure the antenna gain, or if we simulate the antenna gain and if you get a decent gain that means we are radiating a decent amount of power; and most of the power probably is radiated, and we need not worry.

But, if we get a good match and we do not get a good gain; it means that mostly the power is getting to hit the conductors. And therefore the power is due to the conductor loss term or the ohmic loss terms, which is undesired. It is not desired, this power loss is not desired. But, whether you like it or not, this is going to take place; and this is the role of this conduction current.

So, because of the symmetricity of Maxwell's equation, we are going to consider this term $\frac{\partial B}{\partial t}$ as the magnetic displacement current; we call this the magnetic displacement current. And therefore, $\vec{M}_{in}^{t} = \dfrac{\partial \vec{B}}{\partial t} + \vec{M}_{in}^{i}$. So, this is the total magnetic current, this is the magnetic displacement current; and this is the impressed magnetic current. It must be remembered here that magnetic sources do not exist in nature; so, this is actually a fictitious quantity. It does not exist in nature, but yet we invoke magnetic current; we invoke the concept of magnetic current.

Why the concept of magnetic current is invoked? We will come to a later lecture, which will be devoted on the concept of magnetic current, and its utility in electromagnetics. So, let us preserve that for later. Now, accept the the existence of the magnetic current; so, we call this equation, equation number-15. And therefore using these equations 14 and 15, we can write the Maxwell's equations using this generalized current $\vec{M}_{in}^{t}$ and $\vec{J}_{in}^{t}$. What will be those forms of Maxwell's equations? Very simple to write.

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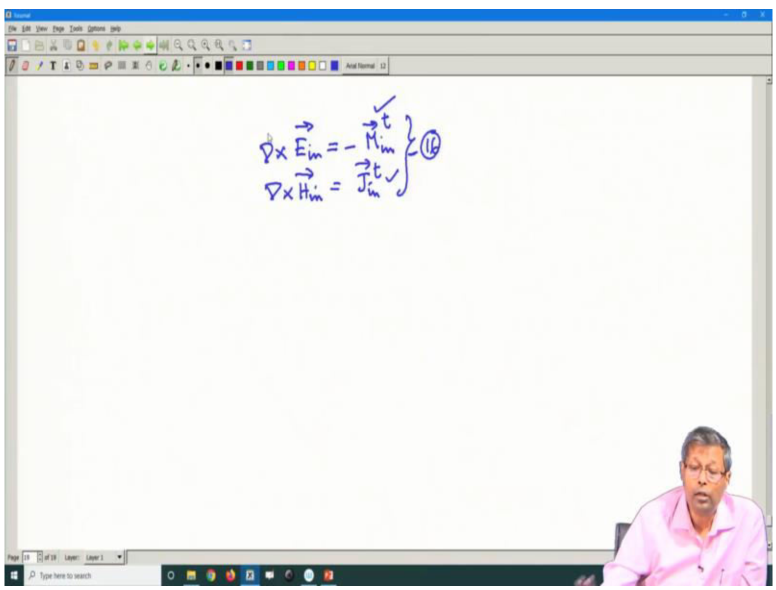

Just from our first set of equations, so let us put a vector here. So, this will be called equation, let us call this equation-16. So, this means that we have now expressed Maxwell's equation in terms of the generalized current $\vec{M}_{in}^{t}$ and $\vec{J}_{in}^{t}$. We have completely expressed, we have completely forgotten that or completely compressed the idea $\vec{M}_{in}^{t}$ contains the compressed term, composed of the displacement magnetic current and the impressed magnetic current.

So, that is contained in $\vec{M}_{in}^{t}$, similarly, $\vec{J}_{in}^{t}$ is a term which contains the electric conduction current; the electric impressed current and the electric displacement current. So, now we armed with this knowledge, we going to find out the power in an electromagnetic wave.

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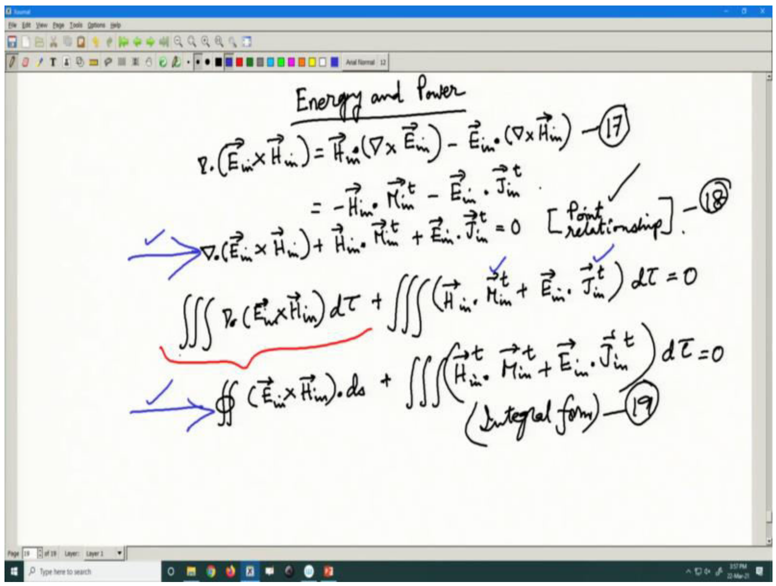

We are going to energy and power. So, what constitutes power flow in electromagnetic wave? How do we formulate? How much power is carried by an electromagnetic wave? How do we define that from where the definition comes? The journey will start from a vector identity, which will lead to the conservation of power. And then we will use our knowledge of low frequency in order to isolate the different terms in that vector identity. And then we will distinguish the terms leading to stored energy, leading to power flow, and leading to radiated and leading to power loss, or the ohmic power loss.

So, start with the vector identity which takes the shape of the conservation of power. And that vector identities. So, this is the vector identity: $\nabla \cdot (\vec{E}_{in} + \vec{H}_{in}) = \vec{H}_{in} \cdot (\nabla \times \vec{E}_{in}) - \vec{E}_{in} \cdot (\nabla \times \vec{H}_{in})$. So, this is an identity which is a general identity for any two vectors $\vec{a}$ and $\vec{b}$. It is not restricted to electric and magnetic fields; we apply that identity to the vectors, electric field and magnetic field. $\vec{E}_{in}$ and $\vec{H}_{in}$, as before denote the instantaneous electric and instantaneous magnetic fields respectively.

So, now from here if we call this equation-17; so we can write this as using the generalized current concept. In equation-16, - $\vec{H}_{in}$ dot $\vec{M}^{t}_{in}$, minus $\vec{E}_{in}$ dot $\vec{J}^{t}_{in}$; and then we bring all the terms to the left hand side. So we have brought all the terms to the left hand side; and this is a point relationship which is valid at a particular point. So, this differential relationship is valid at a particular point; we can construct an equivalent volume relationship.

So, we can integrate in a region and establish or derive an equivalent relationship, which is valid in a region of space as before. So, we take the triple integral and then we apply divergence theorem to this term. And therefore, we get closed integral $\vec{E}_{in}$ cross $\vec{H}_{in}$ dot ds, plus triple integral $\vec{H}_{in}$; the rest of the term remains the same.

We call this equation, equation number-18, which is the point relationship; and we call this equation, equation number-19, which is valid in a region of space. This is the integral form in contrast to the point form there. So, we have two, we have two forms of relationship; and these two forms eventually will take the shape of the conservation of energy. That can be easily seen by plugging in these terms $\vec{J}^{t}_{in}$, $\vec{M}^{t}_{in}$ here. If I write down the equations of $\vec{J}^{t}_{in}$, $\vec{M}^{t}_{in}$, which we derived before; we will evaluate this and find out that each of these terms turn out to be different terms, which fit in together in the jigsaw puzzle. And in totality these two equations read as the conservation of power equations. In order to do that let us find out happens when we equate when we put in substitute the values.

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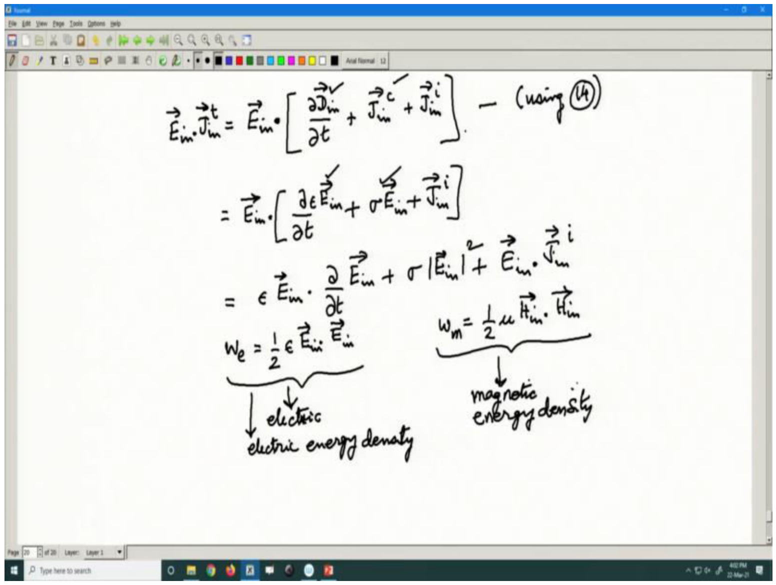

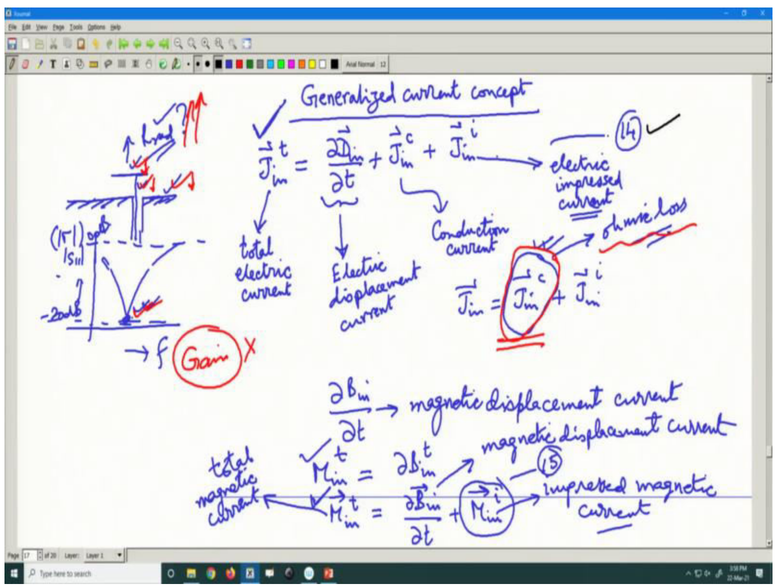

So, $\vec{E}_{in}$ dot $\vec{J}^{t}_{in}$; we will just substitute $\vec{J}^{t}_{in}$; the displacement current, the conduction current and the impressed current that is using equation-14, this relationship. So, now I can write this as using the constitutive relationship, I can write this as epsilon $\vec{E}_{in}$. This I can write as $\sigma\vec{E}_{in}$ and this I can write as $\vec{J}^{i}_{in}$; because d is epsilon $\vec{E}_{in}$, and $\vec{J}_{in}$ is $\sigma\vec{E}_{in}$. And then I can write this as; sorry, I should have put a vector here. So, epsilon $\vec{E}_{in}$ dot del del t of $\vec{E}_{in}$, plus sigma mod $\vec{E}_{in}$ square, plus $\vec{E}_{in}$ dot $\vec{J}^{i}_{in}$.

Now, the terms We as half epsilon $\vec{E}_{in}$ dot $\vec{E}_{in}$ this term; and Wm as half mu $\vec{H}_{in}$ dot $\vec{H}_{in}$. These two terms, this term can be identified as the electric stored energy; the electric stored energy of static fields. So, electric and so this is the electric energy density; and this is the magnetic energy density.

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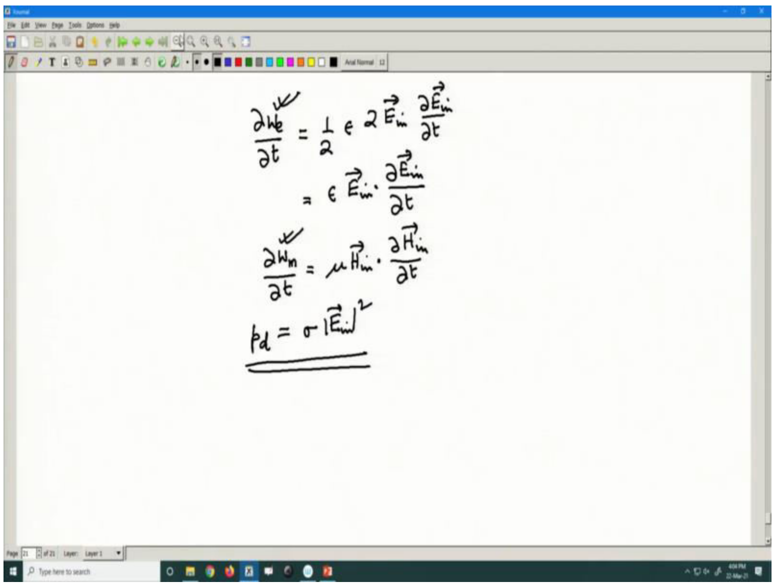

So, now if we perform $\frac{\partial W_e}{\partial t}$; it is a simple matter; we will get half epsilon $2\vec{E}_{in}, \frac{\partial \vec{E}_{in}}{\partial t}$. That is epsilon $\vec{E}_{in}$ dot $\frac{\partial \vec{E}_{in}}{\partial t}$; and similarly $\frac{\partial W_m}{\partial t}$ will be mu $\vec{H}_{in}$ times $\frac{\partial \vec{H}_{in}}{\partial t}$. So, they refer to the rate of change of stored electric for $\frac{\partial W_e}{\partial t}$; and the rate of change of stored magnetic energy densities $\frac{\partial W_m}{\partial t}$. The term Pd defined as sigma mod E in square is the density of power converted to heat energy. It is the density of power converted to heat energy or the ohmic loss term.

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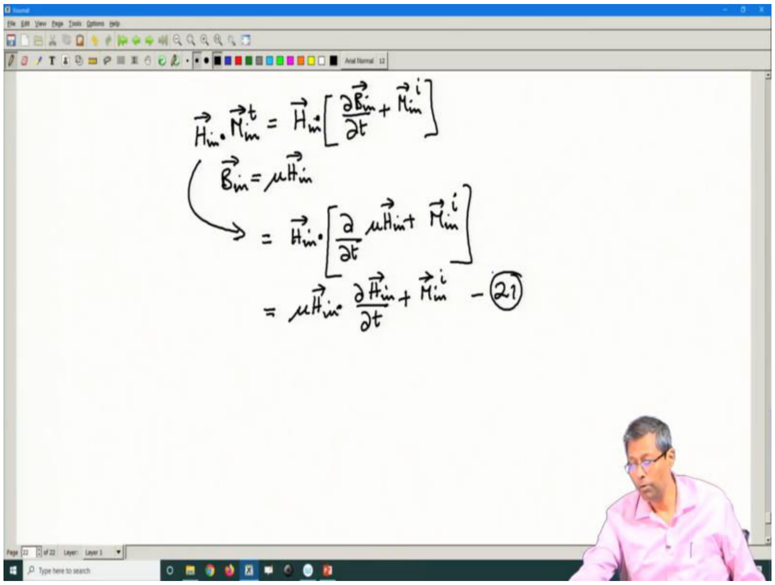

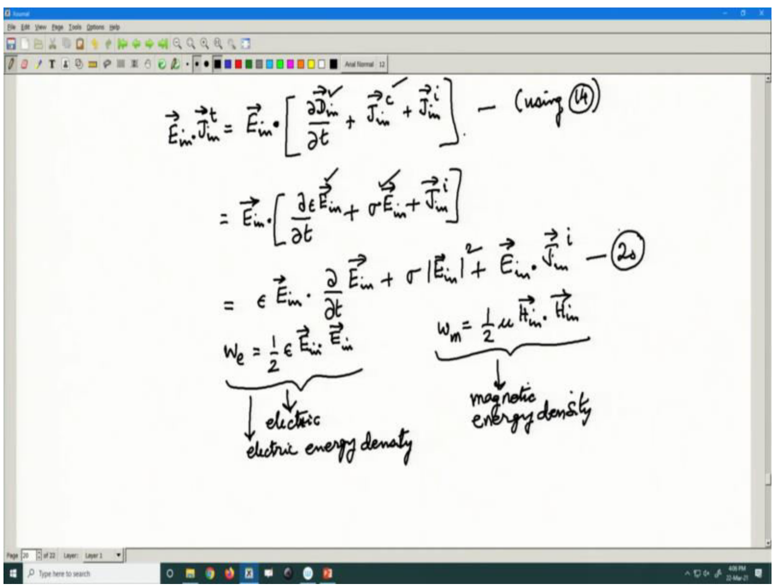

So, if we look at the term the other term $\vec{H}_{in}$ dot $\vec{M}^t_{in}$; it is going to give me if I break up $\vec{M}^t_{in}$ dot, plus $\vec{M}^i_{in}$. So, next using B equal to mu $\vec{H}_{in}$; I can write this term as equal to plus $\vec{M}^i_{in}$ simple matter. So, that is mu $\vec{H}_{in}$ dot $\frac{\partial \vec{H}_{in}}{\partial t}$, plus $\vec{M}^i_{in}$. So, let us call let us call this equation as 20; and let us call this equation as 21.

So, now we have all the ingredients, how to write the original vector identity in terms of terms; or in terms of quantities, which relate to the stored energy density. The ohmic loss term and the impressed power and the power flow terms. So, we will interpret those terms and we will find that what the vector identity we originally wrote, is basically a manifestation or basically an expression of the conservation of energy. So, let us come to that in the next class. So, we stop here from now; we will continue here. Thank you.