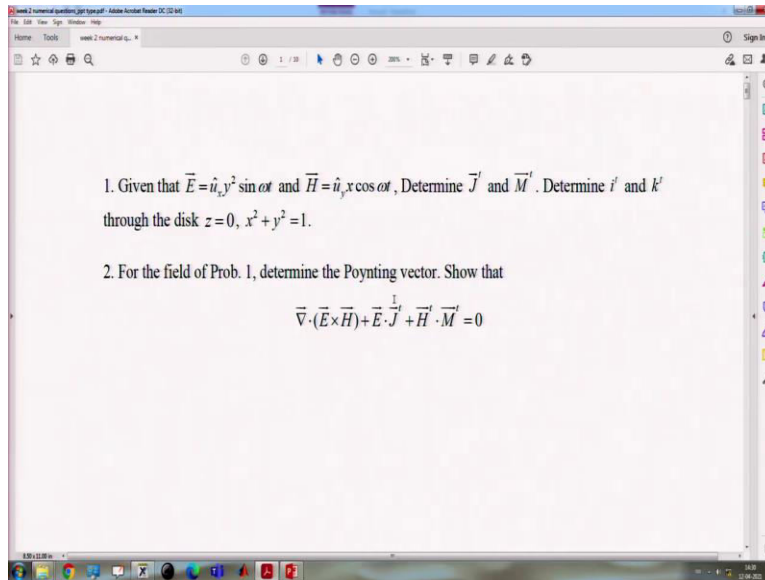


Advanced Microwave Guided-Structures and Analysis
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Lecture 13
Instantaneous Form of Maxwell's Equations Tutorials

(Refer Slide Time: 00:18)



Welcome to the next tutorial class. Today we will do some numerical problems on Poynting vectors. So, this is the first problem. So, given that

$$\vec{E} = \hat{u}_y y^2 \sin \omega t, \quad \vec{H} = \hat{u}_y x \cos \omega t$$

Determine \vec{J}^t and \vec{M}^t . So, here \vec{J}^t is the total electric current density and \vec{M}^t is the total magnetic current density. And in the next part determine i^t and k^t through the disk z is equal to 0 and $x^2 + y^2 = 1$. So, first we will do this problem.

(Refer Slide Time: 00:55)

$$\vec{E} = \hat{u}_x y^2 \sin \omega t, \quad \vec{H} = \hat{u}_y x \cdot \cos \omega t$$
$$\vec{J} = \vec{\nabla} \times \vec{H} = \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \times \hat{u}_y x \cdot \cos \omega t$$
$$\vec{J} = \hat{u}_z \cos \omega t$$
$$\vec{M}^+ = -\vec{\nabla} \times \vec{E}$$

$$\vec{M}^+ = -\vec{\nabla} \times \vec{E}$$
$$= - \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \times \hat{u}_x y^2 \sin \omega t$$
$$= - (-\hat{u}_z) 2y \sin \omega t = \hat{u}_z 2y \sin \omega t$$
$$i^+ = \iint \vec{J}^+ \cdot \vec{\delta S} = \int \int_{x=0, y=0}^{\sqrt{1-x^2}} \hat{u}_z \cos \omega t \cdot dx dy \hat{u}_z$$

$$\begin{aligned}
 &= \cos \omega t \int_0^1 \sqrt{1-x^2} \cdot dx = \frac{\pi}{4} \cos \omega t \\
 \vec{K}^t &= \iint \vec{M}^t \cdot d\vec{S} = \int_{x=0}^1 \int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z 2y \sin \omega t \cdot dx dy \hat{u}_z \\
 &= \sin \omega t \int_0^1 (1-x^2) dx = \frac{2}{3} \sin \omega t
 \end{aligned}$$

So, here vector electric field and vector magnetic field is given

$$\vec{E} = \hat{u}_x y^2 \sin \omega t, \quad \vec{H} = \hat{u}_y x \cdot \cos \omega t$$

So, \vec{J}^t can be found out by

$$\begin{aligned}
 \vec{J}^t &= \vec{\nabla} \times \vec{H} = \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \times \hat{u}_y x \cdot \cos \omega t \\
 &= \hat{u}_z \cos \omega t
 \end{aligned}$$

Similarly, we can find out the \vec{M}^t , \vec{M}^t will be minus of del cross E, we can write

$$\begin{aligned}
 \vec{M}^t &= -\vec{\nabla} \times \vec{E} \\
 &= - \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \times \hat{u}_x y^2 \sin \omega t \\
 &= - (-\hat{u}_z) 2y \sin \omega t = \hat{u}_z 2y \sin \omega t
 \end{aligned}$$

Similarly, in the question it is written determine i^t and k^t through that disk z equal to 0 and $x^2 + y^2 = 1$. So,

$$i^t = \int \int \vec{j}^t \cdot \vec{dS} = \int_{x=0}^1 \int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z \cos \omega t \cdot dx dy \hat{u}_z$$

$$= \cos \omega t \int_0^1 \sqrt{1-x^2} \cdot dx = \frac{\pi}{4} \cos \omega t$$

Similarly, we can find out the magnetic current k^t ,

$$k^t = \int \int \vec{M}^t \cdot \vec{dS} = \int_{x=0}^1 \int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z 2y \sin \omega t \cdot dx dy \hat{u}_z$$

$$= \sin \omega t \int_0^1 (1-x^2) dx = \frac{2}{3} \sin \omega t$$

So, this will come and now we can go to the next problem.

(Refer Slide Time: 10:12)

1. Given that $\vec{E} = \hat{u}_y y^2 \sin \omega t$ and $\vec{H} = \hat{u}_x x \cos \omega t$, Determine \vec{J}' and \vec{M}' . Determine i' and k' through the disk $z=0, x^2 + y^2 = 1$.

2. For the field of Prob. 1, determine the Poynting vector. Show that

$$\nabla \cdot (\vec{E} \times \vec{H}) + \vec{E} \cdot \vec{J}' + \vec{H}' \cdot \vec{M}' = 0$$

Next problem is it for the field of problem 1, determined the Poynting vector and show :

$$\nabla \cdot (\vec{E} \times \vec{H}) + \vec{E} \cdot \vec{J}' + \vec{H}' \cdot \vec{M}' = 0$$

(Refer Slide Time: 10:59)

$\vec{S} = \text{Poynting vector} = \vec{E} \times \vec{H}$

$$= \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ y^2 \sin \omega t & 0 & 0 \\ 0 & x \cos \omega t & 0 \end{vmatrix} = \hat{u}_x \times 0 + \hat{u}_y \cdot 0 + \hat{u}_z \{ x y^2 \sin \omega t \cos \omega t \}$$
$$\vec{S} = \hat{u}_z x y^2 \sin \omega t \cos \omega t$$

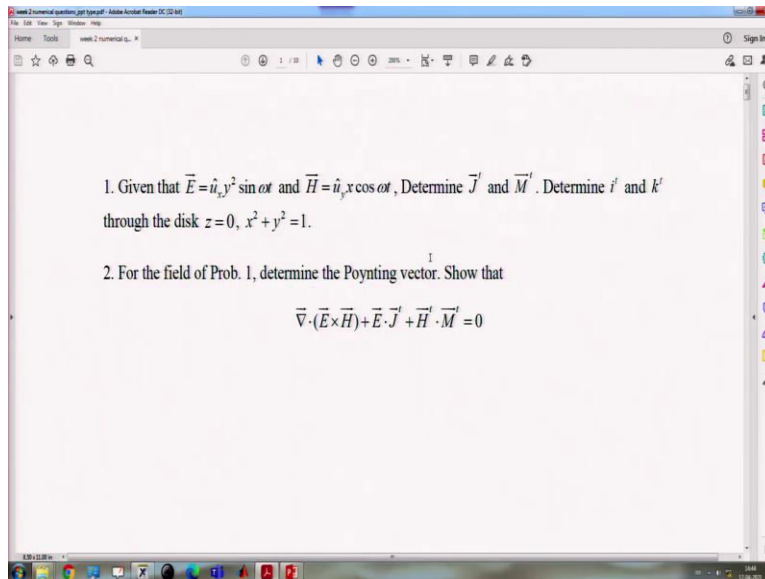
So, Poynting vector is defined as

$$\vec{S} = \text{Poynting vector} = \vec{E} \times \vec{H}$$

$$= \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ y^2 \sin \omega t & 0 & 0 \\ 0 & \omega \cos \omega t & 0 \end{vmatrix} = \hat{u}_x \times 0 + \hat{u}_y \times 0 + \hat{u}_z \{ \omega y^2 \sin \omega t \cos \omega t \}$$

$$\vec{S} = \hat{u}_z \omega y^2 \sin \omega t \cos \omega t$$

(Refer Slide Time: 13:43)



In the next part we have to prove that

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) + \vec{E} \cdot \vec{J}^t + \vec{H}^t \cdot \vec{M}^t = 0$$

So, we can derive that as follows:

$$\vec{S} = \hat{u}_z \propto \gamma^2 \sin \omega t \cos \omega t$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) + \vec{E} \cdot \vec{J} + \vec{H} \cdot \vec{M}$$

$$= \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \cdot \hat{u}_z \propto \gamma^2 \sin \omega t \cos \omega t +$$

$$\hat{u}_x \gamma^2 \sin \omega t \cdot \hat{u}_z \cos \omega t + \hat{u}_y \propto \cos \omega t \cdot 2 \gamma \sin \omega t \hat{u}_z$$

$$= 0 + 0 + 0 = 0$$

(Refer Slide Time: 13:58)

The image shows a digital whiteboard with the same mathematical derivation as the first image. The equations are:
$$\vec{S} = \hat{u}_z \propto \gamma^2 \sin \omega t \cos \omega t$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) + \vec{E} \cdot \vec{J} + \vec{H} \cdot \vec{M}$$

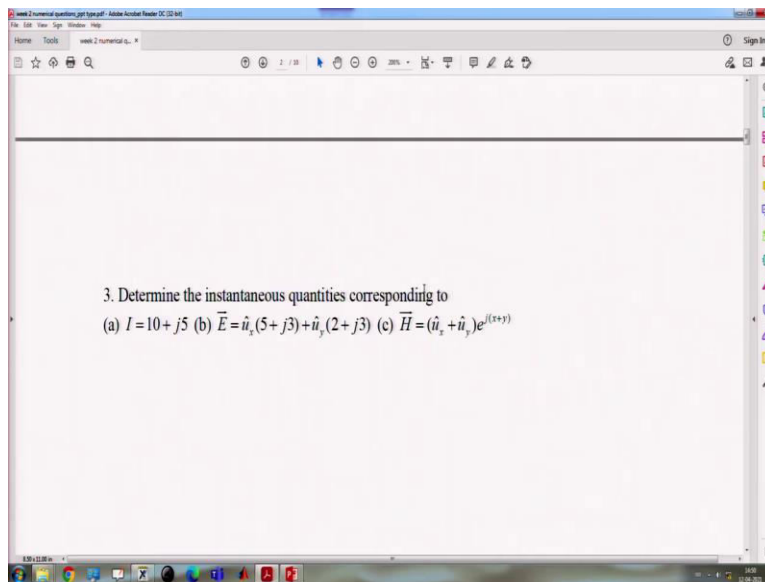
$$= \left(\frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \cdot \hat{u}_z \propto \gamma^2 \sin \omega t \cos \omega t +$$

$$\hat{u}_x \gamma^2 \sin \omega t \cdot \hat{u}_z \cos \omega t + \hat{u}_y \propto \cos \omega t \cdot 2 \gamma \sin \omega t \hat{u}_z$$

$$= 0 + 0 + 0 = 0$$

Now, we can go to the next problem.

(Refer Slide Time: 17:06)



Determine the instantaneous quantities corresponding to

$$I = 10 + j5$$

So, in the (a) part, I is equal to 10 plus $j5$. For this term we have to find out the instantaneous quantities.

(Refer Slide Time: 17:31)

$\hat{u}_x \cdot \hat{u}_x \sin \omega t \cdot \hat{u}_z (\cos \omega t + \hat{u}_y \cos \omega t) \cdot 2j \sin \omega t$
 $= 0 + 0 + 0 = 0$

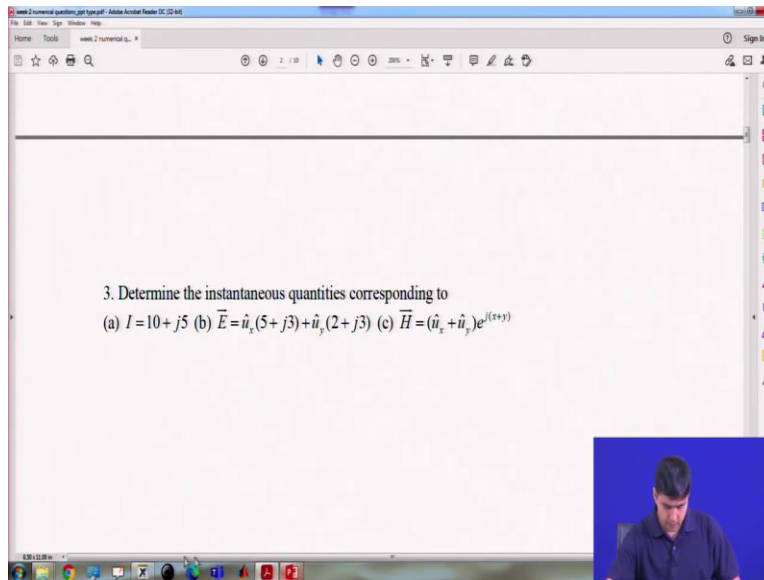
③ $I = 10 + j5$
 $i = \sqrt{2} \operatorname{Re} [I e^{j\omega t}]$
 $= \sqrt{2} \operatorname{Re} [(10 + j5) (\cos \omega t + j \sin \omega t)]$

$$\begin{aligned}
 &= \sqrt{2} \operatorname{Re} [(10+j5)(\cos \omega t + j \sin \omega t)] \\
 &= \sqrt{2} \operatorname{Re} [10 \cos \omega t + j 10 \sin \omega t + j 5 \cos \omega t - 5 \sin \omega t] \\
 &= \sqrt{2} [10 \cos \omega t - 5 \sin \omega t]
 \end{aligned}$$

$$\begin{aligned}
 I &= 10 + j5 \\
 i &= \sqrt{2} \operatorname{Re} [I e^{j\omega t}] \\
 &= \sqrt{2} \operatorname{Re} [(10+j5)(\cos \omega t + j \sin \omega t)] \\
 &= \sqrt{2} \operatorname{Re} [10 \cos \omega t + j 10 \sin \omega t + j 5 \cos \omega t - 5 \sin \omega t] \\
 &= \sqrt{2} [10 \cos \omega t - 5 \sin \omega t]
 \end{aligned}$$

Similarly, we can do for the next part.

(Refer Slide Time: 20:27)



3. Determine the instantaneous quantities corresponding to
(a) $I = 10 + j5$ (b) $\vec{E} = \hat{u}_x(5 + j3) + \hat{u}_y(2 + j3)$ (c) $\vec{H} = (\hat{u}_x + \hat{u}_y)e^{j(100\pi t)}$

The screenshot shows a presentation slide with the above text. In the bottom right corner, there is a small video inset of a person with dark hair, wearing a dark blue shirt, looking down at a desk.

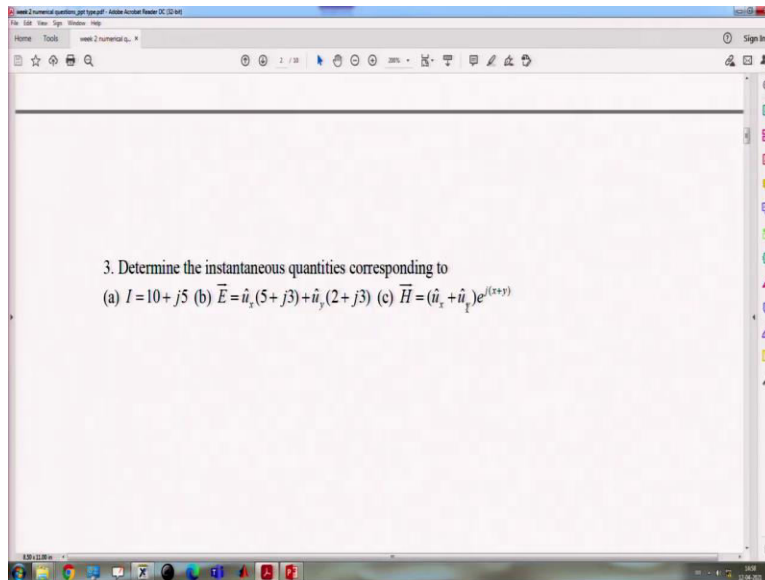
In the next part :

$$\begin{aligned}\vec{E} &= \hat{u}_x (5 + j3) + \hat{u}_y (2 + j3) \\ \vec{E} &= \sqrt{2} \operatorname{Re} \left[\vec{E} e^{j\omega t} \right] \\ &= \sqrt{2} \operatorname{Re} \left[u_x (5 + j3) e^{j\omega t} + u_y (2 + j3) e^{j\omega t} \right]\end{aligned}$$

(Refer Slide Time: 20:33)

So this will be the final answer.

(Refer Slide Time: 24:09)



For the next part H is given

$$\begin{aligned} \text{(c)} \quad \vec{H} &= (\hat{u}_x + \hat{u}_y) e^{j(\omega t + \theta)} \\ \vec{h} &= \sqrt{2} \operatorname{Re} \left[\vec{H} e^{j\omega t} \right] = \sqrt{2} \operatorname{Re} \left[(\hat{u}_x + \hat{u}_y) e^{j(\omega t + \theta)} \cdot e^{j\omega t} \right] \\ &= \sqrt{2} \operatorname{Re} \left[(\hat{u}_x + \hat{u}_y) (\cos(\omega t + \theta) + j \sin(\omega t + \theta)) (\cos \omega t + j \sin \omega t) \right] \end{aligned}$$

(Refer Slide Time: 24:22)

$$\begin{aligned}
 \vec{H} &= (\hat{u}_x + \hat{u}_y) e^{j(\alpha t + \beta)} \\
 \tilde{H} &= \sqrt{2} \operatorname{Re} [\vec{H} e^{j\omega t}] = \sqrt{2} \operatorname{Re} [(\hat{u}_x + \hat{u}_y) e^{j(\alpha t + \beta)} \cdot e^{j\omega t}] \\
 &= \sqrt{2} \operatorname{Re} [(\hat{u}_x + \hat{u}_y) (\cos(\alpha t + \beta) + j \sin(\alpha t + \beta)) (\cos \omega t + j \sin \omega t)]
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{2} \operatorname{Re} [(\hat{u}_x + \hat{u}_y) (\cos(\alpha t + \beta) \cos \omega t - \sin(\alpha t + \beta) \sin \omega t \\
 &\quad + j \cos(\alpha t + \beta) \sin \omega t + j \sin(\alpha t + \beta) \cos \omega t)] \\
 &= \sqrt{2} (\hat{u}_x + \hat{u}_y) (\cos(\alpha t + \beta) \cos \omega t - \sin(\alpha t + \beta) \sin \omega t)
 \end{aligned}$$

So, we can multiply these two and we can extract the real part. So, after multiplying we will get

$$\begin{aligned}
 &= \sqrt{2} \operatorname{Re} [(\hat{u}_x + \hat{u}_y) (\cos(\alpha t + \beta) \cos \omega t - \sin(\alpha t + \beta) \sin \omega t \\
 &\quad + j \cos(\alpha t + \beta) \sin \omega t + j \sin(\alpha t + \beta) \cos \omega t)] \\
 &= \sqrt{2} (\hat{u}_x + \hat{u}_y) (\cos(\alpha t + \beta) \cos \omega t - \sin(\alpha t + \beta) \sin \omega t)
 \end{aligned}$$

So this is the answer for the third part. Thank you.