**Advanced Microwave Guided-Structures and Analysis**
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**Indian Institute of Technology, Kharagpur**
**Lecture 13**
**Instantaneous Form of Maxwell's Equations Tutorials**

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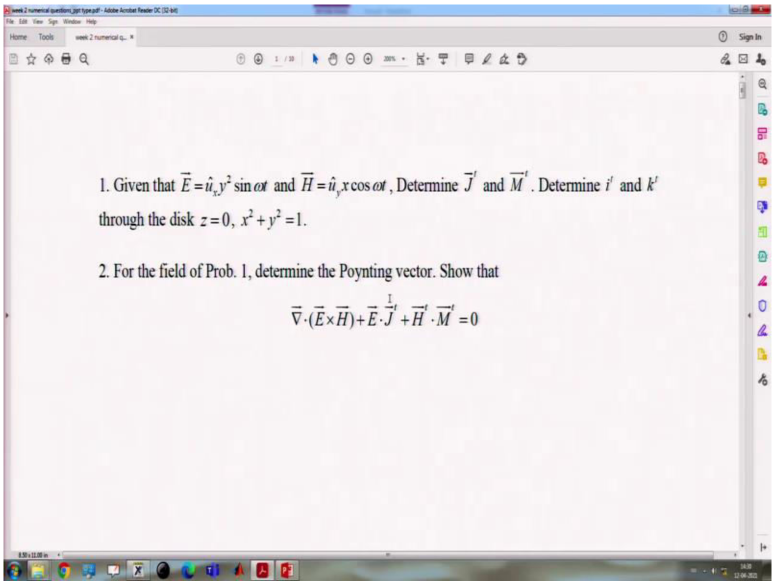


Welcome to the next tutorial class. Today we will do some numerical problems on Poynting vectors. So, this is the first problem. So, given that

$$\vec{E} = \hat{u}_x y^2 \sin \omega t, \quad \vec{H} = \hat{u}_y x \cdot \cos \omega t$$

Determine $\mathbf{J}^t$ and $\mathbf{M}^t$. So, here $\mathbf{J}^t$ is the total electric current density and $\mathbf{M}^t$ is the total magnetic current density. And in the next part determine $i^t$ and $k^t$ through the disk z is equal to 0 and $x^2 + y^2 = 1$. So, first we will do this problem.

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$$\vec{E} = \hat{u}_x y^2 \sin\omega t, \quad \vec{H} = \hat{u}_y x \cdot \cos\omega t$$

$$\vec{J}^i = \vec{\nabla} \times \vec{H} = \left(\frac{\partial}{\partial x}\hat{u}_x + \frac{\partial}{\partial y}\hat{u}_y + \frac{\partial}{\partial z}\hat{u}_z\right) \times \hat{u}_y x \cdot \cos\omega t$$

$x \; y \; z$

$$= \hat{u}_z \cos\omega t$$

$$\vec{M}^i = -\vec{\nabla} \times \vec{E}$$

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$$= -\left(\frac{\partial}{\partial x}\hat{u}_x + \frac{\partial}{\partial y}\hat{u}_y + \frac{\partial}{\partial z}\hat{u}_z\right) \times \hat{u}_x y^2 \sin\omega t$$

$$= -(-\hat{u}_z)\, 2y \sin\omega t = \hat{u}_z\, 2y \sin\omega t$$

$$i^i = \iint \vec{J}^i \cdot \vec{dS} = \int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z \cos\omega t \cdot dx\, dy\, \hat{u}_z$$

$$= \cos\omega t \int_0^1 \sqrt{1-x^2}\cdot dx = \frac{\pi}{4}\cos\omega t$$

$$K^t = \iint \vec{M}^t \cdot \vec{dS} = \int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z\, 2y \sin\omega t \cdot dx\, dy\, \hat{u}_z$$

$$= \sin\omega t \int_0^1 (1-x^2)\, dx = \frac{2}{3}\sin\omega t$$

So, here vector electric field and vector magnetic field is given

$$\vec{E} = \hat{u}_x y^2 \sin\omega t, \quad \vec{H} = \hat{u}_y x \cdot \cos\omega t$$

.

So, $\mathbf{J}^t$ can be found out by

$$\vec{J}^t = \vec{\nabla} \times \vec{H} = \left(\frac{\partial}{\partial x}\hat{u}_x + \frac{\partial}{\partial y}\hat{u}_y + \frac{\partial}{\partial z}\hat{u}_z\right) \times \hat{u}_y x \cdot \cos\omega t$$

$$= \hat{u}_z \cos\omega t$$

Similarly, we can find out the Mt, Mt will be minus of del cross E, we can write

$$\vec{M}^t = -\vec{\nabla} \times \vec{E}$$

$$= -\left(\frac{\partial}{\partial x}\hat{u}_x + \frac{\partial}{\partial y}\hat{u}_y + \frac{\partial}{\partial z}\hat{u}_z\right) \times \hat{u}_x y^2 \sin\omega t$$

$$= -(-\hat{u}_z)\, 2y \sin\omega t = \hat{u}_z\, 2y \sin\omega t$$

Similarly, in the question it is written determine $i^t$ and $k^t$ through that disk z equal to 0 and $x^2 + y^2 = 1$. So,

$$i^t = \iint \vec{J}^t \cdot \vec{dS} = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z \cos \omega t \cdot dx\, dy\, \hat{u}_z$$

$$= \cos \omega t \int_{0}^{1} \sqrt{1-x^2} \cdot dx = \frac{\pi}{4} \cos \omega t$$

Similarly, we can find out the magnetic current kt,

$$k^t = \iint \vec{M}^t \cdot \vec{dS} = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \hat{u}_z\, 2y \sin \omega t \cdot dx\, dy\, \hat{u}_z$$

$$= \sin \omega t \int_{0}^{1} (1-x^2)\, dx = \frac{2}{3} \sin \omega t$$

So, this will come and now we can go to the next problem.

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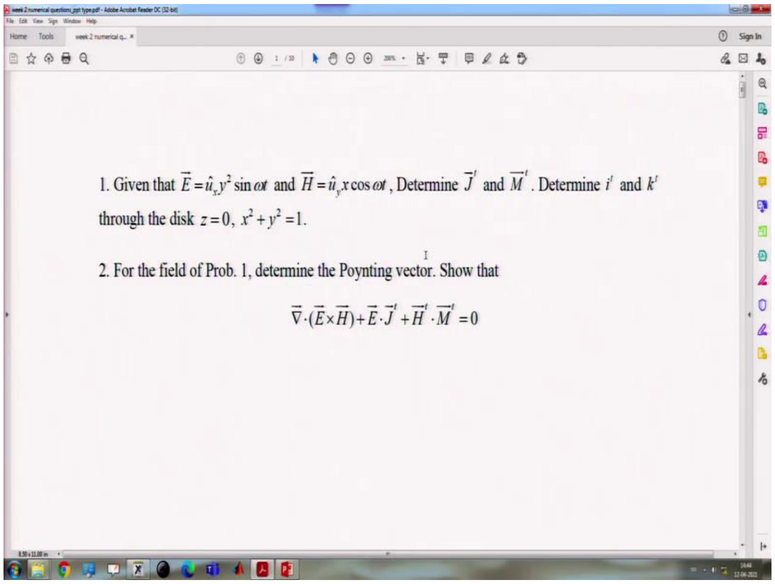


Next problem is it for the field of problem 1, determined the Poynting vector and show :

$$\overrightarrow{\nabla}\cdot(\overrightarrow{E}\times\overrightarrow{H})+\overrightarrow{E}\cdot\overrightarrow{J}^i+\overrightarrow{H}^i\cdot\overrightarrow{M}^i=0$$

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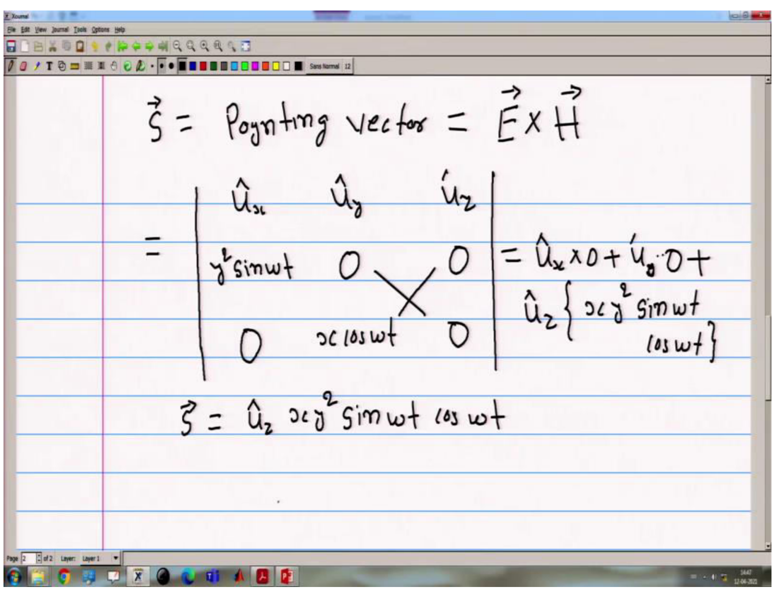


So, Poynting vector is defined as

$$\vec{S} = \text{Poynting vector} = \vec{E} \times \vec{H}$$

$$= \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ y^2 \sin\omega t & 0 & 0 \\ 0 & x\cos\omega t & 0 \end{vmatrix} = \hat{u}_x \times 0 + \hat{u}_y \cdot 0 + \hat{u}_z \left\{ x y^2 \sin\omega t \cos\omega t \right\}$$

$$\vec{S} = \hat{u}_z \, x y^2 \sin\omega t \cos\omega t$$

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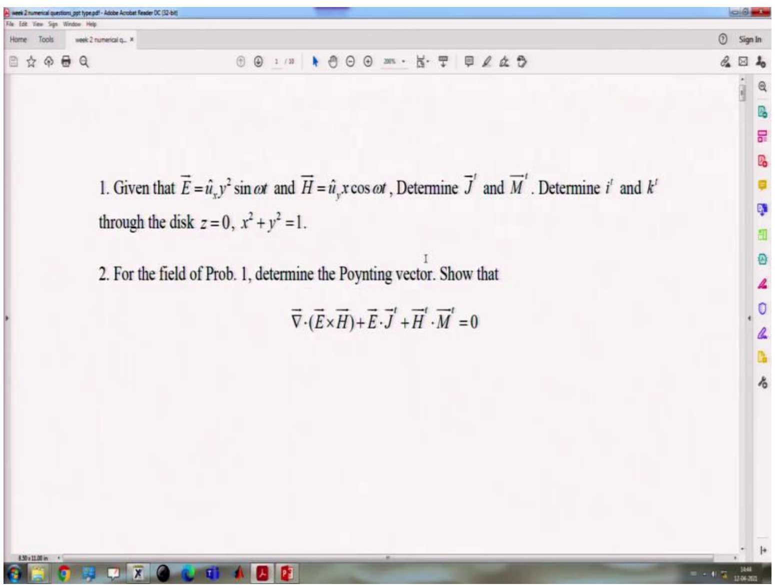
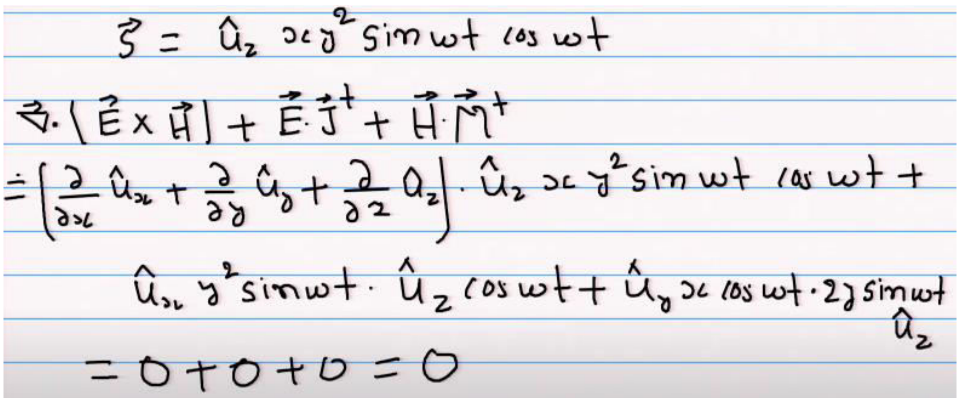



In the next part we have to prove that

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) + \vec{E}\cdot\vec{J}^t + \vec{H}^t\cdot\vec{M}^t = 0$$
.

So, we can derive that as follows:

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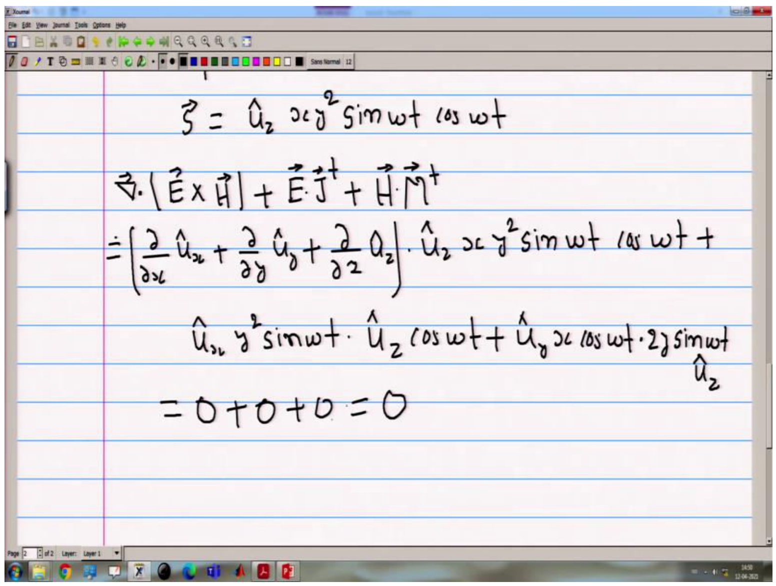


Now, we can go to the next problem.

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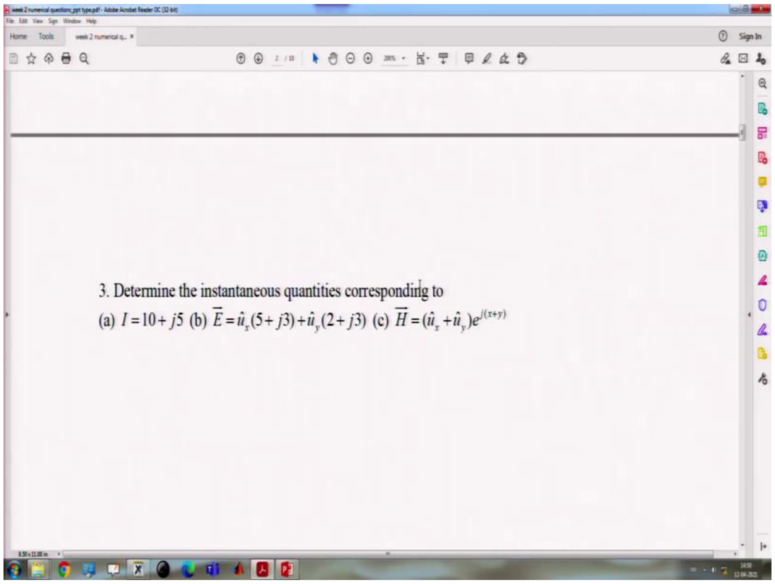


Determine the instantaneous quantities corresponding to

$$I = 10 + j5$$

So, in the (a) part, I is equal to 10 plus j5. For this term we have to find out the instantaneous quantities.

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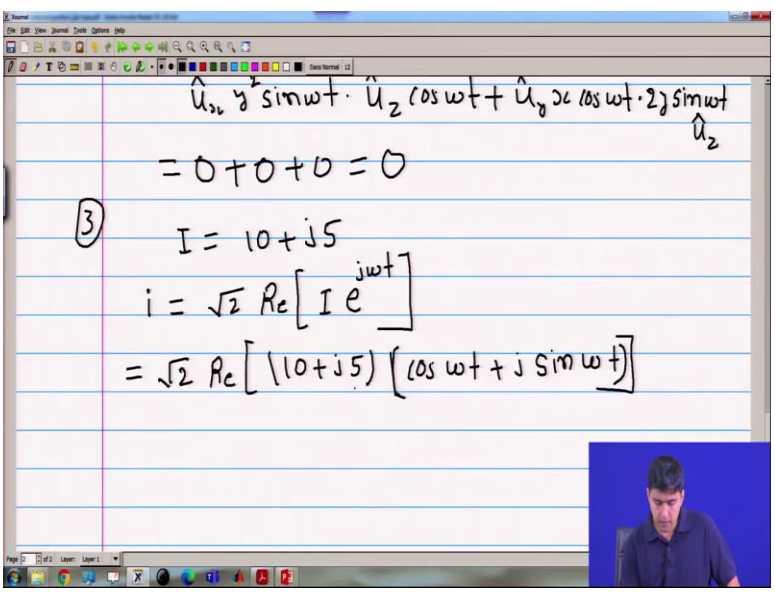

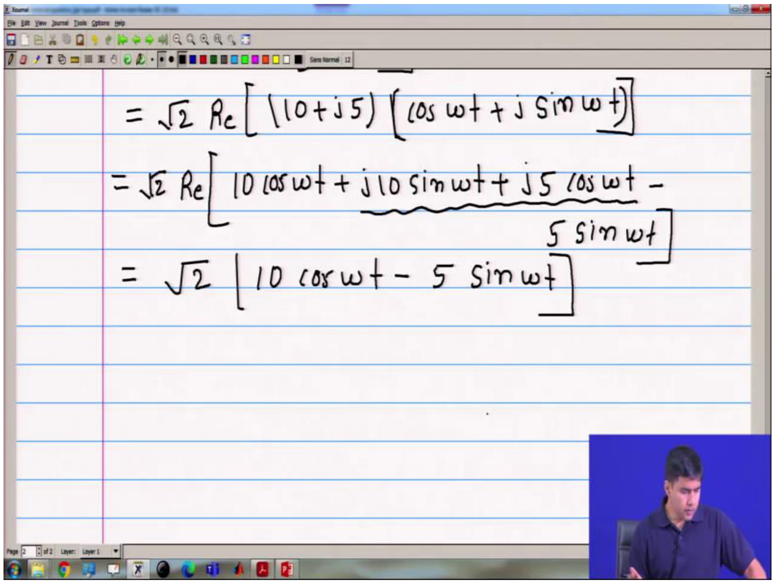


$$I = 10 + j5$$

$$i = \sqrt{2}\,\mathrm{Re}\left[ I e^{j\omega t} \right]$$

$$= \sqrt{2}\,\mathrm{Re}\left[ (10 + j5)(\cos \omega t + j \sin \omega t) \right]$$

$$= \sqrt{2}\,\mathrm{Re}\left[ 10 \cos \omega t + \underbrace{j10 \sin \omega t + j5 \cos \omega t}_{} - 5 \sin \omega t \right]$$

$$= \sqrt{2}\left[ 10 \cos \omega t - 5 \sin \omega t \right]$$

Similarly, we can do for the next part.

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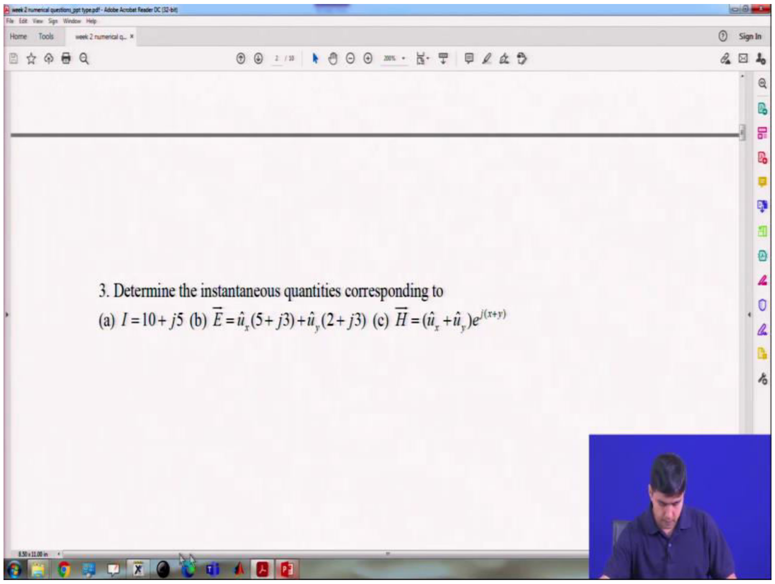


In the next part :

$$\vec{E} = \hat{u}_x (5 + j3) + \hat{u}_y (2 + j3)$$

$$\vec{\mathcal{E}} = \sqrt{2}\, Re\left[ \vec{E}\, e^{j\omega t} \right]$$

$$= \sqrt{2}\, Re\left[ u_x (5 + j3)\, e^{j\omega t} + \hat{u}_y (2 + j3)\, e^{j\omega t} \right]$$

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$$= \sqrt{2}\, Re\left[(10 + j5)(\cos\omega t + j\sin\omega t)\right]$$

$$= \sqrt{2}\, Re\left[10\cos\omega t + \underbrace{j10\sin\omega t + j5\cos\omega t}_{} - 5\sin\omega t\right]$$

$$= \sqrt{2}\left[10\cos\omega t - 5\sin\omega t\right]$$

(b) $\vec{E} = \hat{u}_x (5 + j3) + \hat{u}_y (2 + j3)$

$$\vec{\mathcal{E}} = \sqrt{2}\, Re\left[\vec{E} e^{j\omega t}\right]$$

$$= \sqrt{2}\, Re\left[u_x (5 + j3) e^{j\omega t} + \hat{u}_y (2 + j3) e^{j\omega t}\right]$$

$$= \sqrt{2}\, Re\left[\hat{u}_x (5 + j3) e^{j\omega t} + \hat{u}_y (2 + j3) e^{j\omega t}\right]$$

$$= \sqrt{2}\, Re\left[u_x (5 + j3)(\cos\omega t + j\sin\omega t) + u_y (2 + j3)(\cos\omega t + j\sin\omega t)\right]$$

$$\sqrt{2}\left[\hat{u}_x (5\cos\omega t - 3\sin\omega t) + \hat{u}_y (2\cos\omega t - -3\sin\omega t)\right]$$

On expanding we get:

$$= \sqrt{2}\, Re\left[u_x (5 + j3)(\cos\omega t + j\sin\omega t) + u_y (2 + j3)(\cos\omega t + j\sin\omega t)\right]$$

$$\sqrt{2}\left[\hat{u}_x (5\cos\omega t - 3\sin\omega t) + \hat{u}_y (2\cos\omega t - -3\sin\omega t)\right]$$

So this will be the final answer.

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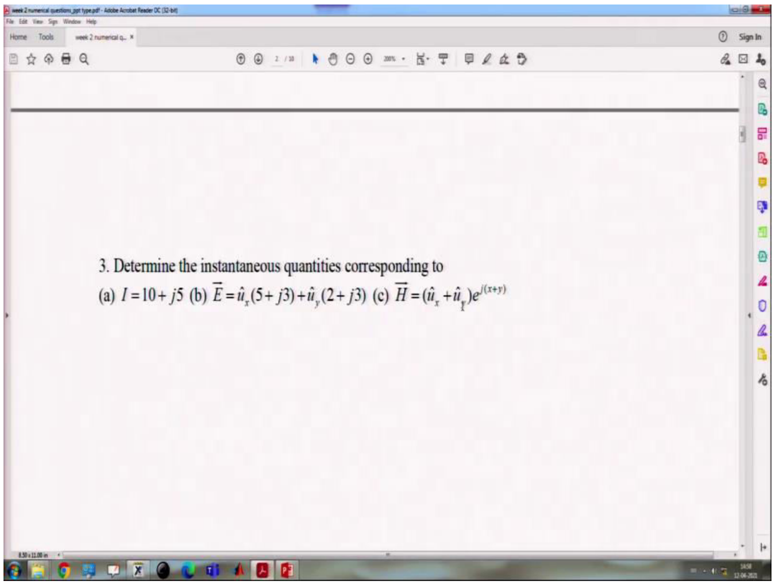


For the next part H is given

(c) $\vec{H} = (\hat{u}_x + \hat{u}_y)\, e^{j(x+y)}$

$$\vec{\mathcal{H}} = \sqrt{2}\,Re\left[\vec{H}\, e^{j\omega t}\right] = \sqrt{2}\,Re\left[(\hat{u}_x + \hat{u}_y)\, e^{j(x+y)} \cdot e^{j\omega t}\right]$$

$$= \sqrt{2}\,Re\left[(\hat{u}_x + \hat{u}_y)\left(\cos(x+y) + j\sin(x+y)\right)\left(\cos\omega t + j\sin\omega t\right)\right]$$

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$$- j\sin\omega t)]$$

(c) $\vec{H} = (\hat{u}_x + \hat{u}_y)\, e^{j(x+y)}$

$$\vec{\mathcal{H}} = \sqrt{2}\, Re\left[\vec{H} e^{j\omega t}\right] = \sqrt{2}\, Re\left[(\hat{u}_x + \hat{u}_y)\, e^{j(x+y)} \cdot e^{j\omega t}\right]$$

$$= \sqrt{2}\, Re\left[(\hat{u}_x + \hat{u}_y)\left(\cos(x+y) + j\sin(x+y)\right)\left(\cos\omega t + j\sin\omega t\right)\right]$$

$$(\cos\omega t + j\sin\omega t)]$$

$$= \sqrt{2}\, Re\left[(\hat{u}_x + \hat{u}_y)\left(\cos(x+y)\cos\omega t - \sin(x+y)\cdot\sin\omega t + j\cos(x+y)\cdot\sin\omega t + j\sin(x+y)\cos\omega t\right)\right]$$

$$= \sqrt{2}\,(\hat{u}_x + \hat{u}_y)\left(\cos(x+y)\cos\omega t - \sin(x+y)\sin\omega t\right)$$

So, we can multiply these two and we can extract the real part. So, after multiplying we will get


$$= \sqrt{2}\, Re\left[(\hat{u}_x + \hat{u}_y)\left(\cos(x+y)\cos\omega t - \sin(x+y)\cdot\sin\omega t + j\cos(x+y)\cdot\sin\omega t + j\sin(x+y)\cos\omega t\right)\right]$$

$$= \sqrt{2}\,(\hat{u}_x + \hat{u}_y)\left(\cos(x+y)\cos\omega t - \sin(x+y)\sin\omega t\right)$$


So this is the answer for the third part. Thank you.