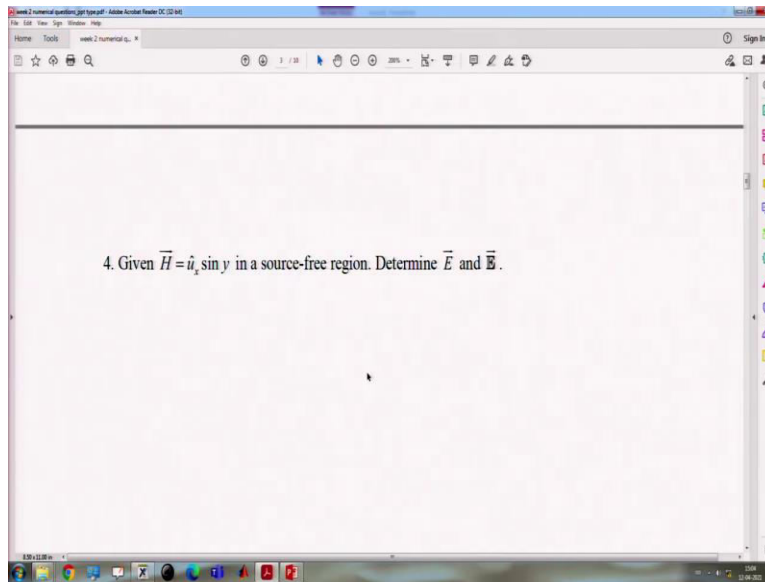


**Advanced Microwave Guided-Structures and Analysis**  
**Professor. Bratin Ghosh**  
**Department of Electronics and Electrical Communication Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture No. 14**  
**Instantaneous Form of Maxwell's Equations Tutorials (Contd.)**

Welcome back to the next tutorial class, so we are doing the problems on the Pointing vector.

(Refer Slide Time: 0:20)



So, next problem is,

4. Given  $\vec{H} = \hat{u}_x \sin y$  in a source-free region. Determine  $\vec{E}$  and  $\vec{E}$ .

So, first is the complex and second one is instantaneous electric field.

(Refer Slide Time: 0:47)

$$-\sqrt{2} (u_x + u_y) (\cos(x+y) \cos \omega t - \sin(x+y) \sin \omega t)$$

4)

$$\vec{H} = \hat{u}_x \sin y$$
$$\vec{H}_t = \sqrt{2} \operatorname{Re} [\vec{H} e^{j\omega t}] = \sqrt{2} \operatorname{Re} [\hat{u}_x \sin y e^{j\omega t}]$$
$$= \sqrt{2} \operatorname{Re} [\hat{u}_x \sin y (\cos \omega t + j \sin \omega t)]$$
$$= \sqrt{2} \hat{u}_x \sin y \cos \omega t$$

So, given is, so this is a forth problem.

$$\vec{H} = \hat{u}_x \sin y$$
$$\vec{H}_t = \sqrt{2} \operatorname{Re} [\vec{H} e^{j\omega t}] = \sqrt{2} \operatorname{Re} [\hat{u}_x \sin y e^{j\omega t}]$$
$$= \sqrt{2} \operatorname{Re} [\hat{u}_x \sin y (\cos \omega t + j \sin \omega t)]$$
$$= \sqrt{2} \hat{u}_x \sin y \cos \omega t$$

So, this is instantaneous forms of magnetic field and from the Maxwell's equation we can write.

(Refer Slide Time: 3:01)

The image shows a digital whiteboard with handwritten mathematical derivations. The top part shows the curl of the vector potential  $\vec{A}$  in terms of the time derivative of the electric field  $\vec{E}$ . The bottom part shows the general formula for the induced electric field  $\vec{E}$  as a line integral of the curl of  $\vec{A}$ .

$$= \sqrt{2} \hat{u}_x \sin y \cos \omega t$$
$$\vec{\nabla} \times \vec{A} = \frac{\partial \vec{B}}{\partial t} + \vec{j} = \frac{\partial (\epsilon \vec{E})}{\partial t} + 0$$
$$+$$
$$\vec{E} = \frac{1}{\epsilon_0 \epsilon_r} \int_0 \vec{\nabla} \times \vec{A} \cdot d\vec{l}$$

The image shows a digital whiteboard with handwritten mathematical derivations. It starts with the general formula for the induced electric field, then calculates the curl of the vector potential  $\vec{A}$  for a specific field. The final result is the induced electric field  $\vec{E}$  in terms of the time derivative of the vector potential.

$$\vec{E} = \frac{1}{\epsilon_0 \epsilon_r} \int_0 \vec{\nabla} \times \vec{A} \cdot d\vec{l}$$
$$\vec{\nabla} \times \vec{A} = \left( \frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right) \times \sqrt{2} \hat{u}_x \sin y \cos \omega t$$
$$= 0 + (-\hat{u}_z) \cos y \cos \omega t \sqrt{2} + 0$$
$$\vec{\nabla} \times \vec{A} = -\sqrt{2} \cos y \cos \omega t \hat{u}_z$$
$$\vec{E} = \frac{-\sqrt{2} \cos y \sin \omega t}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

$$\vec{\nabla} \times \vec{H} = -\sqrt{2} \cos \gamma \cos \omega t \hat{u}_z$$

$$\vec{\xi} = \frac{-\sqrt{2} \cos \gamma \sin \omega t}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

$$\vec{E} = \sqrt{2} \operatorname{Re} \left[ \vec{E} e^{j\omega t} \right]$$

$$\vec{E} = \frac{j \cos \gamma}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

So, because here it is source free, in the problem it is written it is source free region, so  $\mathbf{J}$  will be equal to 0.

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} = \frac{\partial (\epsilon \vec{E})}{\partial t} + 0$$

Because epsilon is constant it will come outside. And because there is differentiation on  $E$  it can be taken this side and it will be integration.

$$\vec{\xi} = \frac{1}{\epsilon_0 \epsilon_r} \int_0^t |\vec{\nabla} \times \vec{H}| \cdot dt$$

$$\vec{\nabla} \times \vec{H} = \left[ \frac{\partial}{\partial x} \hat{u}_x + \frac{\partial}{\partial y} \hat{u}_y + \frac{\partial}{\partial z} \hat{u}_z \right] \times \sqrt{2} \hat{u}_z \sin \gamma \cos \omega t$$

$$= 0 + (-\hat{u}_z) \cos \gamma \cos \omega t \sqrt{2} + 0$$

$$\vec{\nabla} \times \vec{H} = -\sqrt{2} \cos \gamma \cos \omega t \hat{u}_z$$

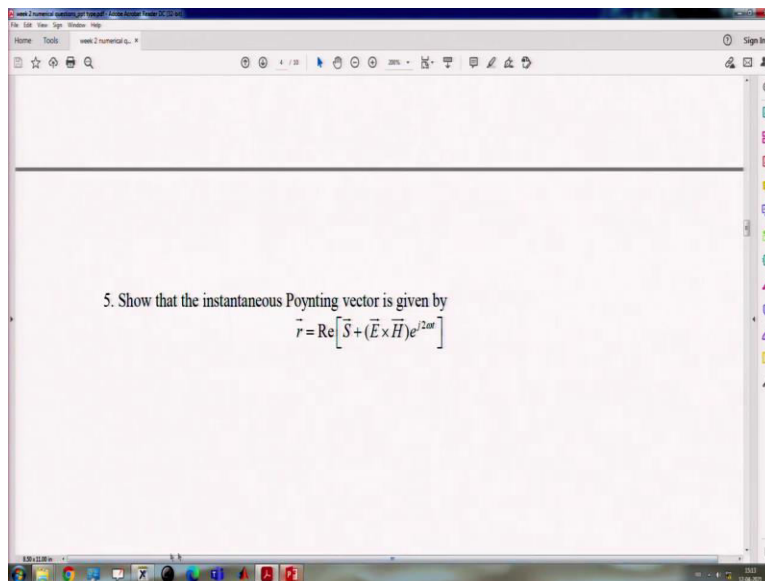
$$\vec{\xi} = \frac{-\sqrt{2} \cos \gamma \sin \omega t}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

now we have to compare and we have to find out, we know the left side, now will have to find out as:

$$\vec{E} = \sqrt{2} \operatorname{Re} \left[ \vec{E} e^{j\omega t} \right]$$
$$\vec{E} = \frac{j \cos \gamma}{\epsilon_0 \epsilon_r \omega} \hat{a}_z$$

So, we can go to the next problem. Next problem is.

(Refer Slide Time: 8:45)



Show, that the instantaneous Poynting vector is given by

5. Show that the instantaneous Poynting vector is given by

$$\vec{r} = \operatorname{Re} \left[ \vec{S} + (\vec{E} \times \vec{H}) e^{j2\omega t} \right]$$

(Refer Slide Time: 9:26)

$$E = \frac{j\omega U_2}{\epsilon_0 \epsilon_r \omega}$$
$$5) \quad \vec{S} = \text{Re} \left[ \vec{S} + \frac{|\vec{E} \times \vec{H}|}{e^{j2\omega t}} \right]$$
$$\vec{E} = \sqrt{2} \text{Re} \left[ \vec{E} e^{j\omega t} \right]$$
$$\vec{H} = \sqrt{2} \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$
$$\vec{S} = \vec{E} \times \vec{H} = 2 \text{Re} \left[ \vec{E} e^{j\omega t} \right] \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$

$$\vec{H} = \sqrt{2} \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$
$$\vec{S} = \vec{E} \times \vec{H} = 2 \text{Re} \left[ \vec{E} e^{j\omega t} \right] \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$
$$\vec{E} = \vec{E}_1 + j\vec{E}_2 \quad \vec{H} = \vec{H}_1 + j\vec{H}_2$$
$$\text{Re} \left[ \vec{E} e^{j\omega t} \right] = \text{Re} \left[ (\vec{E}_1 + j\vec{E}_2) (\cos \omega t + j \sin \omega t) \right]$$
$$= \vec{E}_1 \cos \omega t - \vec{E}_2 \sin \omega t$$

So, for the fifth problem here it is

$$\vec{S} = \text{Re} \left[ \vec{S} + \frac{|\vec{E} \times \vec{H}|}{e^{j2\omega t}} \right]$$
$$\vec{E} = \sqrt{2} \text{Re} \left[ \vec{E} e^{j\omega t} \right]$$
$$\vec{H} = \sqrt{2} \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$
$$\vec{S} = \vec{E} \times \vec{H} = 2 \text{Re} \left[ \vec{E} e^{j\omega t} \right] \text{Re} \left[ \vec{H} e^{j\omega t} \right]$$

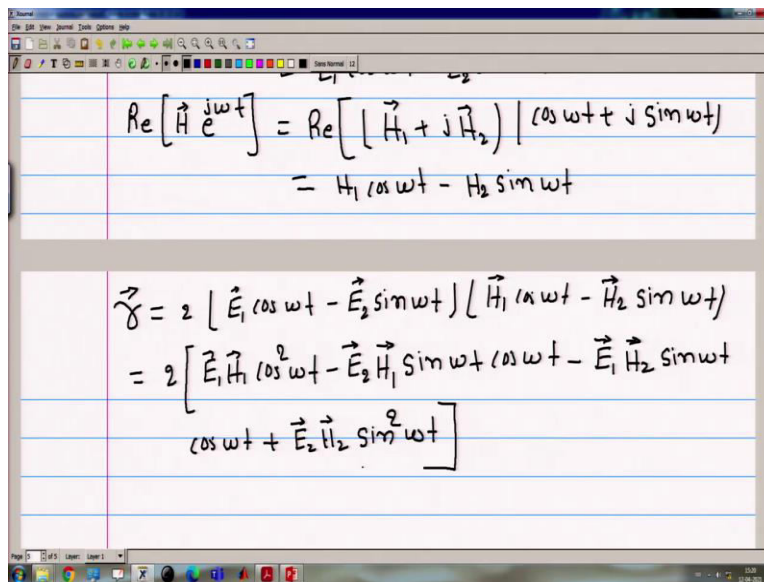
so we can define

$$\vec{E} = \vec{E}_1 + j\vec{E}_2 \quad \vec{H} = \vec{H}_1 + j\vec{H}_2$$

So we can write:

$$\begin{aligned} \operatorname{Re}[\vec{E} e^{j\omega t}] &= \operatorname{Re}[(\vec{E}_1 + j\vec{E}_2) (\cos \omega t + j \sin \omega t)] \\ &= \vec{E}_1 \cos \omega t - \vec{E}_2 \sin \omega t \\ \operatorname{Re}[\vec{H} e^{j\omega t}] &= \operatorname{Re}[(\vec{H}_1 + j\vec{H}_2) (\cos \omega t + j \sin \omega t)] \\ &= \vec{H}_1 \cos \omega t - \vec{H}_2 \sin \omega t \end{aligned}$$

(Refer Slide Time: 13:14)



The screenshot shows a presentation slide with two sections of handwritten mathematical derivations. The top section shows the real parts of the complex fields  $\vec{E}$  and  $\vec{H}$  multiplied by  $e^{j\omega t}$ . The bottom section shows the dot product of these real parts, resulting in a sum of terms involving  $\cos^2 \omega t$  and  $\sin^2 \omega t$ .

$$\begin{aligned} \operatorname{Re}[\vec{H} e^{j\omega t}] &= \operatorname{Re}[(\vec{H}_1 + j\vec{H}_2) (\cos \omega t + j \sin \omega t)] \\ &= \vec{H}_1 \cos \omega t - \vec{H}_2 \sin \omega t \end{aligned}$$
$$\begin{aligned} \vec{S} &= 2 [\vec{E}_1 \cos \omega t - \vec{E}_2 \sin \omega t] [\vec{H}_1 \cos \omega t - \vec{H}_2 \sin \omega t] \\ &= 2 [\vec{E}_1 \vec{H}_1 \cos^2 \omega t - \vec{E}_2 \vec{H}_1 \sin \omega t \cos \omega t - \vec{E}_1 \vec{H}_2 \sin \omega t \cos \omega t + \vec{E}_2 \vec{H}_2 \sin^2 \omega t] \end{aligned}$$

$$\begin{aligned}
 &= 2 \left[ \vec{E}_1 \vec{H}_1 \cos^2 \omega t - \vec{E}_2 \vec{H}_1 \sin \omega t \cos \omega t - \vec{E}_1 \vec{H}_2 \sin \omega t \right. \\
 &\quad \left. \cos \omega t + \vec{E}_2 \vec{H}_2 \sin^2 \omega t \right] \\
 &= \vec{E}_1 \vec{H}_1 (1 + \cos 2\omega t) - \vec{E}_2 \vec{H}_1 \sin 2\omega t - \vec{E}_1 \vec{H}_2 \sin 2\omega t \\
 &\quad + \vec{E}_2 \vec{H}_2 (1 - \cos 2\omega t) \\
 &= \vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2 + \cos 2\omega t (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) - \sin 2\omega t \\
 &\quad (\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2)
 \end{aligned}$$

$$\begin{aligned}
 &+ \vec{E}_2 \vec{H}_2 (1 - \cos 2\omega t) \\
 &= \vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2 + \cos 2\omega t (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) - \sin 2\omega t \\
 &\quad (\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2) \\
 \vec{S} &= \vec{E} \times \vec{H}^* = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 - j\vec{H}_2) \\
 &= \vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2 + j(\vec{E}_2 \vec{H}_1 - \vec{E}_1 \vec{H}_2)
 \end{aligned}$$

Now, to calculate the complex pointing vector

$$\begin{aligned}
 \vec{S} &= 2 \left[ \vec{E}_1 \cos \omega t - \vec{E}_2 \sin \omega t \right] \left[ \vec{H}_1 \cos \omega t - \vec{H}_2 \sin \omega t \right] \\
 &= 2 \left[ \vec{E}_1 \vec{H}_1 \cos^2 \omega t - \vec{E}_2 \vec{H}_1 \sin \omega t \cos \omega t - \vec{E}_1 \vec{H}_2 \sin \omega t \right. \\
 &\quad \left. \cos \omega t + \vec{E}_2 \vec{H}_2 \sin^2 \omega t \right] \\
 &= \vec{E}_1 \vec{H}_1 (1 + \cos 2\omega t) - \vec{E}_2 \vec{H}_1 \sin 2\omega t - \vec{E}_1 \vec{H}_2 \sin 2\omega t \\
 &\quad + \vec{E}_2 \vec{H}_2 (1 - \cos 2\omega t)
 \end{aligned}$$

On simplification we obtain:



$$\vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2 + \cos 2\omega t [\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2] - \sin 2\omega t [\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2]$$

so this will be the instantaneous Poynting vector and we can go now to the right side, so S will be

$$\begin{aligned} \vec{S} &= \vec{E} \times \vec{H}^* = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 - j\vec{H}_2) \\ &= \vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2 + j(\vec{E}_2 \vec{H}_1 - \vec{E}_1 \vec{H}_2) \\ \vec{E} \times \vec{H} &= (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 + j\vec{H}_2) \\ &= \vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2 + j(\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2) \end{aligned}$$

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$$\begin{aligned}\vec{E} \times \vec{H} &= (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 + j\vec{H}_2) \\ &= \vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2 + j(\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2) \\ (\vec{E} \times \vec{H}) e^{j2\omega t} &= \left[ (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) + j(\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2) \right] \\ &\quad (\cos 2\omega t + j \sin 2\omega t)\end{aligned}$$

So the other part,

$$\begin{aligned}(\vec{E} \times \vec{H}) e^{j2\omega t} &= \left[ (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) + j(\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2) \right] \\ &\quad (\cos 2\omega t + j \sin 2\omega t) \\ &= (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) \cos 2\omega t - [\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2] \sin 2\omega t \\ &\quad + j \sin 2\omega t (\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2) + j \cos 2\omega t [\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2]\end{aligned}$$

(Refer Slide Time: 24:30)

$$\begin{aligned}
 & (\cos 2\omega t + j \sin 2\omega t) \\
 & = (\vec{E}_1 \hat{H}_1 - \vec{E}_2 \hat{H}_2) \cos 2\omega t - [\vec{E}_2 \hat{H}_1 + \vec{E}_1 \hat{H}_2] \sin 2\omega t \\
 & \quad + j \sin 2\omega t (\vec{E}_1 \hat{H}_1 - \vec{E}_2 \hat{H}_2) + j \cos 2\omega t [\vec{E}_2 \hat{H}_1 + \vec{E}_1 \hat{H}_2] \\
 & \text{Re} \left[ \vec{S} + (\vec{E} \times \vec{H}) e^{j2\omega t} \right] \\
 & = (\vec{E}_1 \hat{H}_1 + \vec{E}_2 \hat{H}_2) + (\vec{E}_1 \hat{H}_1 - \vec{E}_2 \hat{H}_2) \cos 2\omega t - [\vec{E}_2 \hat{H}_1 + \vec{E}_1 \hat{H}_2] \sin 2\omega t
 \end{aligned}$$

$$\begin{aligned}
 & = \vec{E}_1 \hat{H}_1 (1 + \cos 2\omega t) - \vec{E}_2 \hat{H}_1 \sin 2\omega t - \vec{E}_1 \hat{H}_2 \sin 2\omega t \\
 & \quad + \vec{E}_2 \hat{H}_2 (1 - \cos 2\omega t) \\
 & = \vec{E}_1 \hat{H}_1 + \vec{E}_2 \hat{H}_2 + \cos 2\omega t (\vec{E}_1 \hat{H}_1 - \vec{E}_2 \hat{H}_2) - \sin 2\omega t [\vec{E}_2 \hat{H}_1 + \vec{E}_1 \hat{H}_2] \\
 & \vec{S} = \vec{E} \times \vec{H}^* = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 - j\vec{H}_2) \\
 & \quad = \vec{E}_1 \hat{H}_1 + \vec{E}_2 \hat{H}_2 + j(\vec{E}_2 \hat{H}_1 - \vec{E}_1 \hat{H}_2) \\
 & \vec{E} \times \vec{H} = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 + j\vec{H}_2) \\
 & \quad = \vec{E}_1 \hat{H}_1 - \vec{E}_2 \hat{H}_2 + j(\vec{E}_2 \hat{H}_1 + \vec{E}_1 \hat{H}_2)
 \end{aligned}$$

So, this is the final expression, after this we have to find out the real part of S plus E cross H into e to the power j omega t.

For that we have to find out the real part, real part of S plus E cross H into e to the power j 2 omega t.

$$\operatorname{Re} \left[ \zeta + (\vec{E} \times \vec{H}) e^{j2\omega t} \right]$$

$$= (\vec{E}_1 \vec{H}_1 + \vec{E}_2 \vec{H}_2) + \underbrace{(\vec{E}_1 \vec{H}_1 - \vec{E}_2 \vec{H}_2)}_{\sin 2\omega t} \cos 2\omega t - (\vec{E}_2 \vec{H}_1 + \vec{E}_1 \vec{H}_2)$$

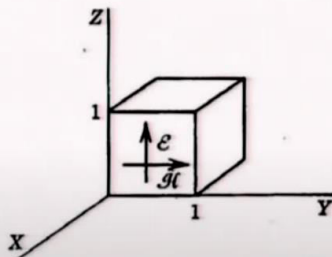
So, like this we can derive the given expression. So, this is a solution for this question.

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6. Consider the unit cube shown in below figure which has all sides except the face  $x=0$  covered by perfect conductors. If  $E_z = 100 \sin(\pi y)$  and  $H_y = e^{j\pi/6} \sin(\pi y)$  over the open face and no sources exist within the cube, determine (a) the time-average power dissipated within the cube, (b) the difference between the time-average electric and magnetic energies within the cube.

For the next problem.

6. Consider the unit cube shown in below figure which has all sides except the face  $x=0$  covered by perfect conductors. If  $E_z = 100 \sin(\pi y)$  and  $H_y = e^{j\pi/6} \sin(\pi y)$  over the open face and no sources exist within the cube, determine (a) the time-average power dissipated within the cube, (b) the difference between the time-average electric and magnetic energies within the cube.



So, let us do this.

(Refer Slide Time: 31:30)

$$\begin{aligned} \text{rc} = 0 \quad d\vec{S} &= dy dz \hat{a}_x \\ P_{im} &= \oint (\vec{E} \times \vec{H}^*) \cdot d\vec{S} \\ &= \int_0^1 \int_0^1 100 \sin^2[\pi y] e^{-j\pi/6} \cdot \sin[\pi y] dy dz \end{aligned}$$

$$\begin{aligned} &= 100 e^{-j\pi/6} \int_0^1 \sin^2[\pi y] \cdot dy \\ &= 50 e^{-j\pi/6} \\ &= 50 \left[ \cos \frac{\pi}{6} - j \sin \frac{\pi}{6} \right] \\ &= 50 \left[ \frac{\sqrt{3}}{2} - j \frac{1}{2} \right] \end{aligned}$$

$$= 25\sqrt{3} - j25$$

$$P_{\text{avg}} = 25\sqrt{3} \quad (a)$$

$$2W(\omega_m - \omega_e) = -25 \quad (b)$$

So, this is at, so at  $x$  is equal to 0 it is open. So, power, total power in from the interface we can write this like this

$x=0 \quad d\vec{S} = dy dz \hat{a}_x$ 

$$P_{\text{im}} = \oint (\vec{E} \times \vec{H}^*) \cdot d\vec{S}$$

$$= \int_0^1 \int_0^1 100 \sin(\pi y) e^{-j\pi/6} \cdot \sin(\pi y) dy$$

On simplifying:

$$100 e^{-j\frac{\pi}{6}} \int_0^1 \sin^2(\pi y) \cdot dy$$

$$= 50 e^{-j\frac{\pi}{6}}$$

$$= 50 \left[ \cos \frac{\pi}{6} - j \sin \frac{\pi}{6} \right]$$

$$= 50 \left[ \frac{\sqrt{3}}{2} - j \frac{1}{2} \right]$$

$$= 25\sqrt{3} - j25$$

$$P_{\text{div}} = 25\sqrt{3} \quad (a)$$

$$2\omega | \omega_m - \omega_e | = -25 \quad (b)$$

So, this is our answer for both the parts. So, like this we can solve this problem. Thank you.