Advanced Microwave Guided-Structures and Analysis
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Lecture No. 14
Instantaneous Form of Maxwell's Equations Tutorials (Contd.)

Welcome back to the next tutorial class, so we are doing the problems on the Pointing vector.

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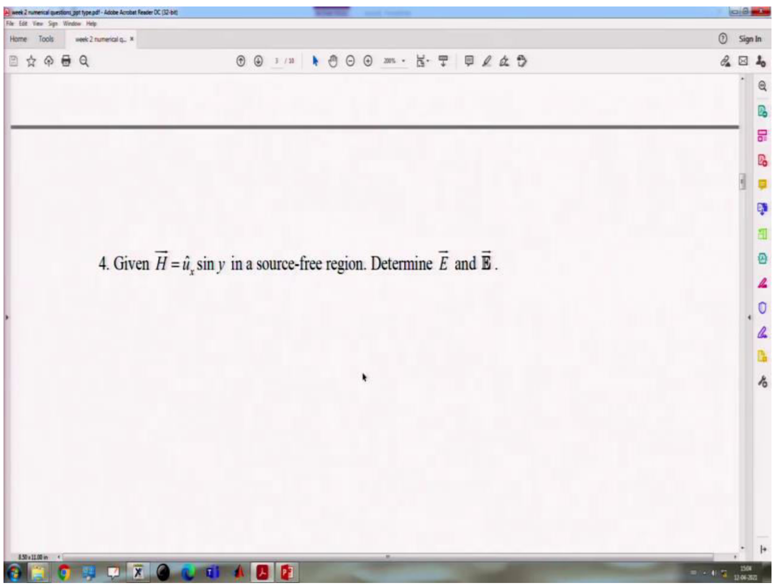

So, next problem is,

4. Given $\vec{H} = \hat{u}_x \sin y$ in a source-free region. Determine $\vec{E}$ and $\vec{\mathbb{E}}$.

So, first is the complex and second one is instantaneous electric field.

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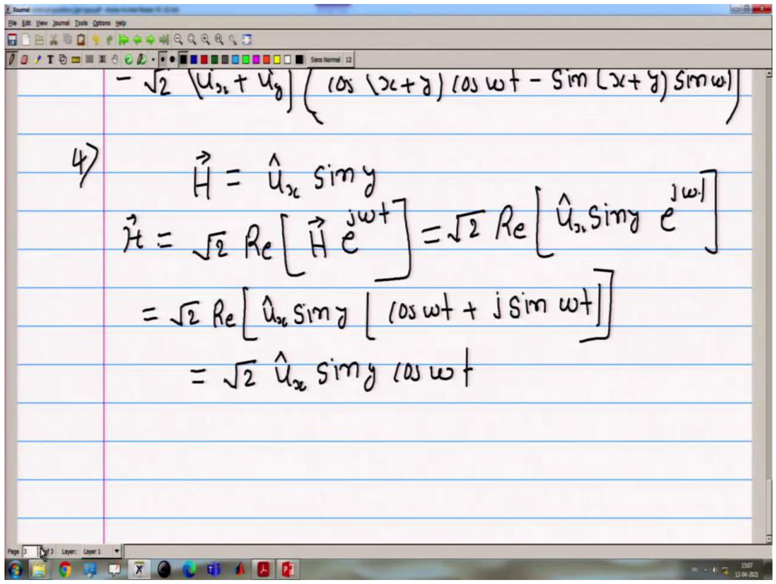

So, given is, so this is a forth problem.

$$\vec{H} = \hat{u}_x \sin y$$

$$\vec{\mathcal{H}} = \sqrt{2}\, Re\left[\vec{H} e^{j\omega t}\right] = \sqrt{2}\, Re\left[\hat{u}_x \sin y\, e^{j\omega t}\right]$$

$$= \sqrt{2}\, Re\left[\hat{u}_x \sin y \left[\cos \omega t + j \sin \omega t\right]\right]$$

$$= \sqrt{2}\, \hat{u}_x \sin y \cos \omega t$$

So, this is instantaneous forms of magnetic field and from the Maxwell's equation we can write.

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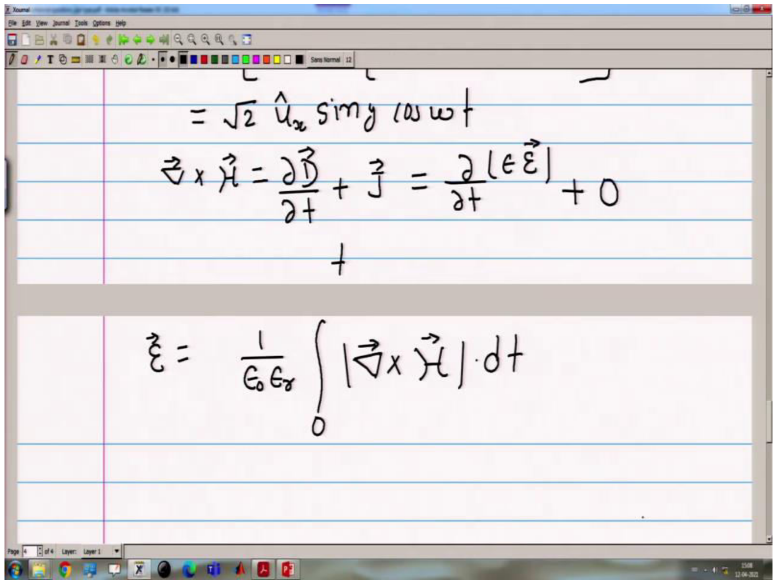

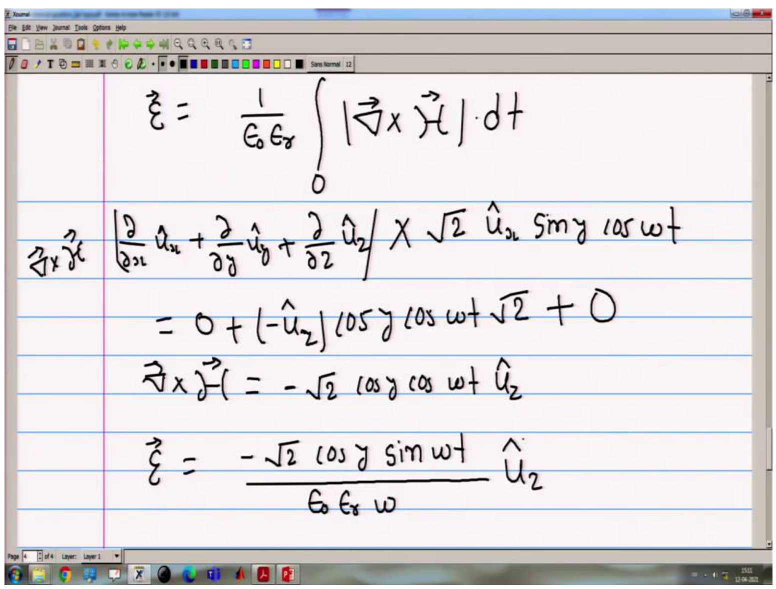

$$= 0 + (-\hat{u}_z)\cos y \cos\omega t \ldots$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = -\sqrt{2}\cos y \cos\omega t \, \hat{u}_z$$

$$\vec{\mathcal{E}} = \frac{-\sqrt{2}\cos y \sin\omega t}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

$$\vec{\mathcal{E}} = \sqrt{2}\, Re\left[\vec{E} e^{j\omega t}\right]$$

$$\vec{E} = \frac{j\cos y}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

So, because here it is source free, in the problem it is written it is source free region, so **J** will be equal to 0.

$$\vec{\nabla} \times \vec{\mathcal{H}} = \frac{\partial \vec{D}}{\partial t} + \vec{J} = \frac{\partial}{\partial t}(\epsilon \vec{\mathcal{E}}) + 0$$

Because epsilon is constant it will come outside. And because there is differentiation on E it can be taken this side and it will be integration.

$$\vec{\mathcal{E}} = \frac{1}{\epsilon_0 \epsilon_r} \int_0 (\vec{\nabla} \times \vec{\mathcal{H}}) \cdot dt$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = \left(\frac{\partial}{\partial x}\hat{u}_x + \frac{\partial}{\partial y}\hat{u}_y + \frac{\partial}{\partial z}\hat{u}_z\right) \times \sqrt{2}\, \hat{u}_x \sin y \cos\omega t$$

$$= 0 + (-\hat{u}_z)\cos y \cos\omega t \sqrt{2} + 0$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = -\sqrt{2}\cos y \cos\omega t \, \hat{u}_z$$

$$\vec{\mathcal{E}} = \frac{-\sqrt{2}\cos y \sin\omega t}{\epsilon_0 \epsilon_r \omega} \hat{u}_z$$

now we have to compare and we have to find out, we know the left side, now will have to find out as:

$$\vec{\mathcal{E}} = \sqrt{2}\, Re\left[\vec{E}\, e^{j\omega t}\right]$$

$$\vec{E} = \frac{j\cos y}{\epsilon_0 \epsilon_r \omega}\,\hat{a}_z$$

So, we can go to the next problem. Next problem is.

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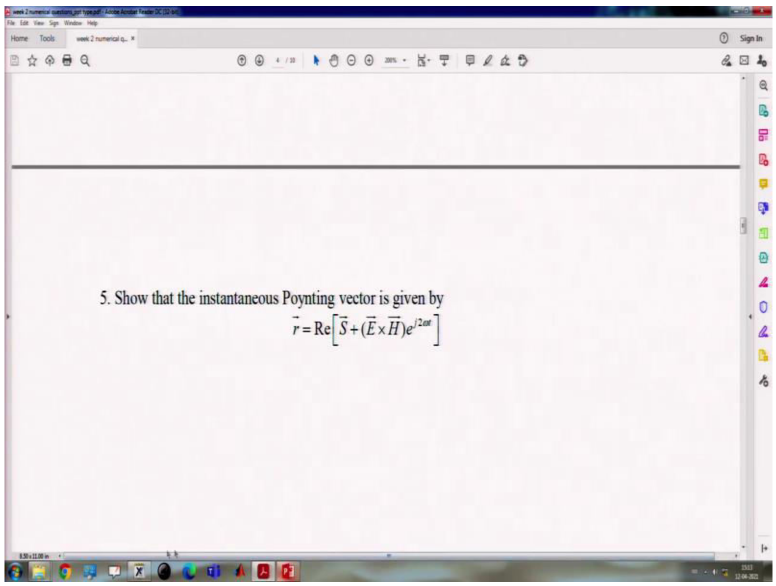

Show, that the instantaneous Poynting vector is given by

5. Show that the instantaneous Poynting vector is given by

$$\vec{r} = \mathrm{Re}\left[\overline{S} + (\vec{E}\times\vec{H})e^{j2\omega t}\right]$$

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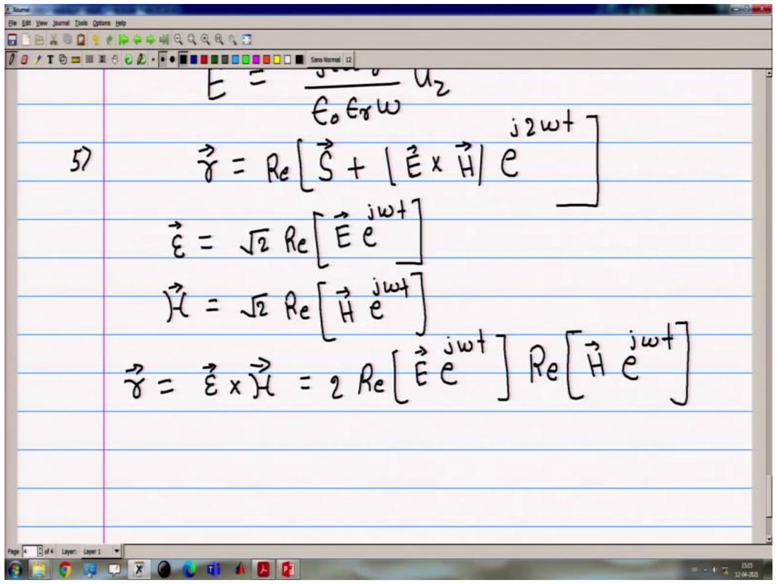
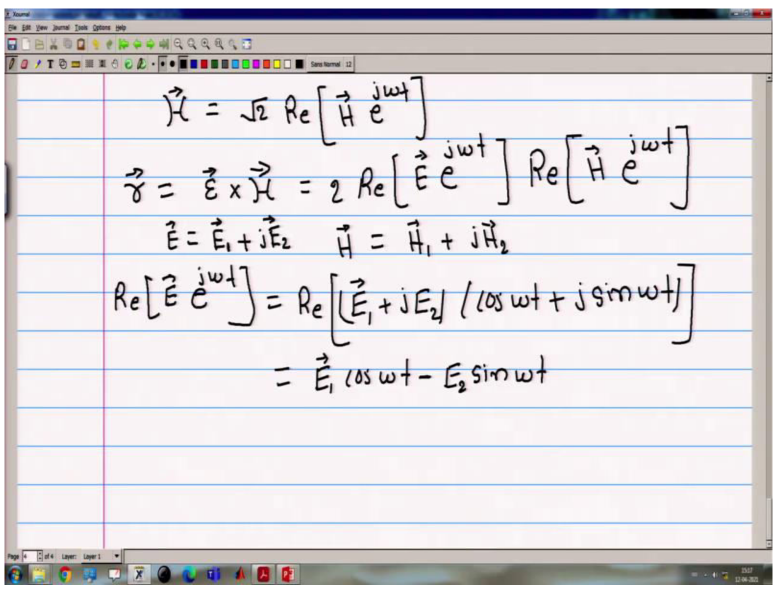
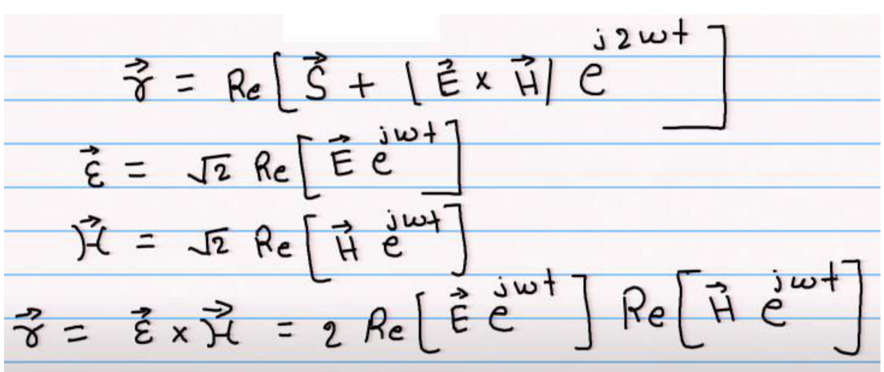




So, for the fifth problem here it is



so we can define

$$\vec{E} = \vec{E}_1 + j\vec{E}_2 \qquad \vec{H} = \vec{H}_1 + j\vec{H}_2$$

So we can write:

$$Re\left[\vec{E}\, e^{j\omega t}\right] = Re\left[(\vec{E}_1 + jE_2)(\cos\omega t + j\sin\omega t)\right]$$

$$= \vec{E}_1 \cos\omega t - E_2 \sin\omega t$$

$$Re\left[\vec{H}\, e^{j\omega t}\right] = Re\left[(\vec{H}_1 + j\vec{H}_2)(\cos\omega t + j\sin\omega t)\right]$$

$$= H_1 \cos\omega t - H_2 \sin\omega t$$

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$$Re\left[\vec{H}\, e^{j\omega t}\right] = Re\left[(\vec{H}_1 + j\vec{H}_2)(\cos\omega t + j\sin\omega t)\right]$$

$$= H_1 \cos\omega t - H_2 \sin\omega t$$

$$\vec{\gamma} = 2\,(\vec{E}_1 \cos\omega t - \vec{E}_2 \sin\omega t)(\vec{H}_1 \cos\omega t - \vec{H}_2 \sin\omega t)$$

$$= 2\left[\vec{E}_1\vec{H}_1 \cos^2\omega t - \vec{E}_2\vec{H}_1 \sin\omega t\cos\omega t - \vec{E}_1\vec{H}_2 \sin\omega t \cos\omega t + \vec{E}_2\vec{H}_2 \sin^2\omega t\right]$$

$$= 2\left[\vec{E}_1\vec{H}_1\cos^2\omega t - \vec{E}_2\vec{H}_1\sin\omega t\cos\omega t - \vec{E}_1\vec{H}_2\sin\omega t\cos\omega t + \vec{E}_2\vec{H}_2\sin^2\omega t\right]$$

$$= \vec{E}_1\vec{H}_1(1+\cos 2\omega t) - \vec{E}_2\vec{H}_1\sin 2\omega t - \vec{E}_1\vec{H}_2\sin 2\omega t + \vec{E}_2\vec{H}_2(1-\cos 2\omega t)$$

$$= \vec{E}_1\vec{H}_1 + \vec{E}_2\vec{H}_2 + \cos 2\omega t\left[\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2\right] - \sin 2\omega t\left[\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2\right]$$

$$+ \vec{E}_2\vec{H}_2(1-\cos 2\omega t)$$

$$= \vec{E}_1\vec{H}_1 + \vec{E}_2\vec{H}_2 + \cos 2\omega t\left[\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2\right] - \sin 2\omega t\left[\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2\right]$$

$$\zeta = \vec{E}\times\vec{H}^* = (\vec{E}_1 + j\vec{E}_2)\times(\vec{H}_1 - j\vec{H}_2)$$

$$= \vec{E}_1\vec{H}_1 + \vec{E}_2\vec{H}_2 + j(\vec{E}_2\vec{H}_1 - \vec{E}_1\vec{H}_2)$$

Now, to calculate the complex pointing vector

$$\vec{\gamma} = 2(\vec{E}_1\cos\omega t - \vec{E}_2\sin\omega t)(\vec{H}_1\cos\omega t - \vec{H}_2\sin\omega t)$$

$$= 2\left[\vec{E}_1\vec{H}_1\cos^2\omega t - \vec{E}_2\vec{H}_1\sin\omega t\cos\omega t - \vec{E}_1\vec{H}_2\sin\omega t\cos\omega t + \vec{E}_2\vec{H}_2\sin^2\omega t\right]$$

$$= \vec{E}_1\vec{H}_1(1+\cos 2\omega t) - \vec{E}_2\vec{H}_1\sin 2\omega t - \vec{E}_1\vec{H}_2\sin 2\omega t + \vec{E}_2\vec{H}_2(1-\cos 2\omega t)$$

On simplification we obtain:

$$\vec{E}_1\vec{H}_1 + \vec{E}_2\vec{H}_2 + \cos 2\omega t\,[\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2] - \sin 2\omega t\,[\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2]$$

so this will be the instantaneous Pointing vector and we can go now to the right side, so S will be

$$\vec{S} = \vec{E} \times \vec{H}^* = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 - j\vec{H}_2)$$
$$= \vec{E}_1\vec{H}_1 + \vec{E}_2\vec{H}_2 + j(\vec{E}_2\vec{H}_1 - \vec{E}_1\vec{H}_2)$$
$$\vec{E} \times \vec{H} = (\vec{E}_1 + j\vec{E}_2) \times (\vec{H}_1 + j\vec{H}_2)$$
$$= \vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2 + j(\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2)$$

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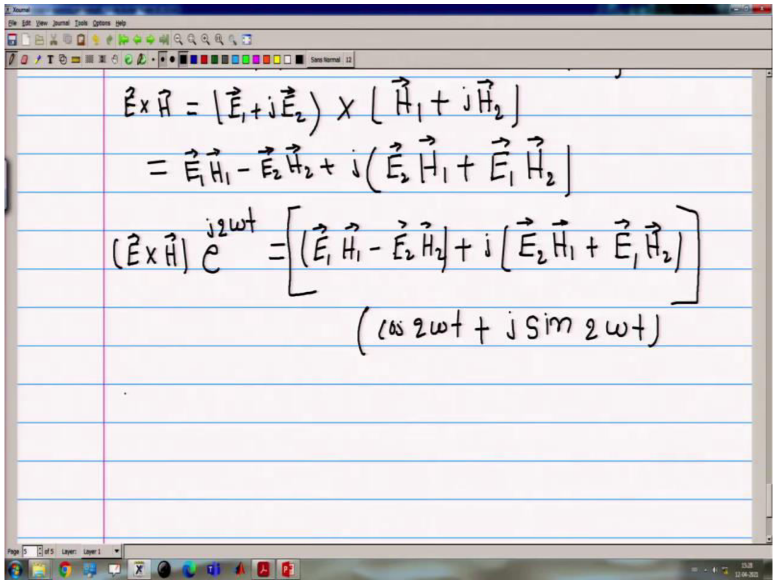

So the other part,

$$(\vec{E}\times\vec{H})\, e^{j2\omega t} = \left[(\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2) + j(\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2)\right](\cos 2\omega t + j\sin 2\omega t)$$

$$= (\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2)\cos 2\omega t - (\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2)\sin 2\omega t$$
$$+ j\sin 2\omega t(\vec{E}_1\vec{H}_1 - \vec{E}_2\vec{H}_2) + j\cos 2\omega t(\vec{E}_2\vec{H}_1 + \vec{E}_1\vec{H}_2)$$

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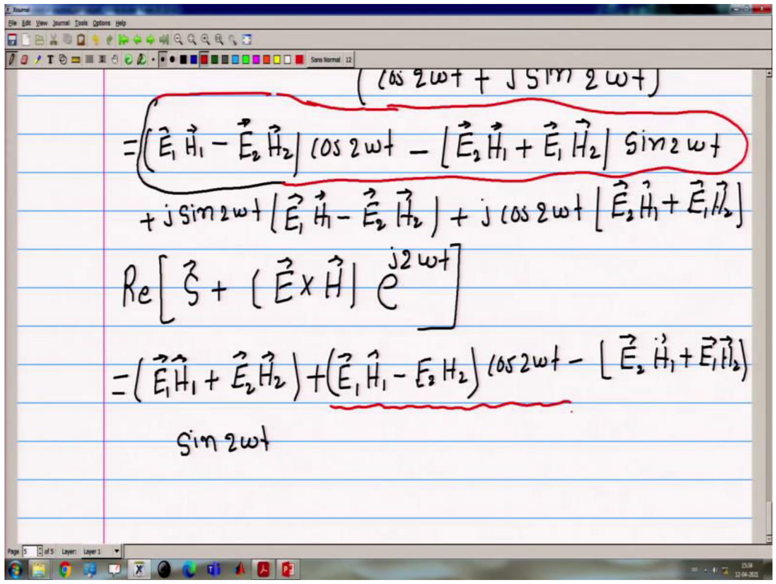
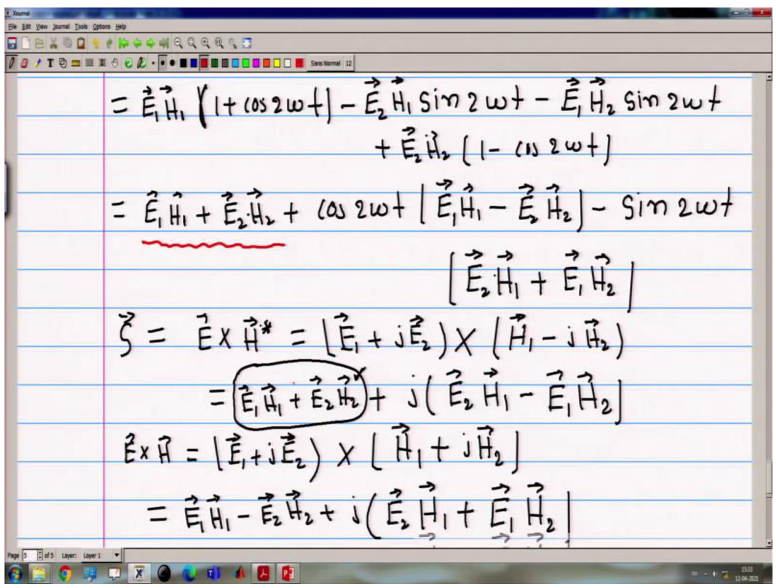
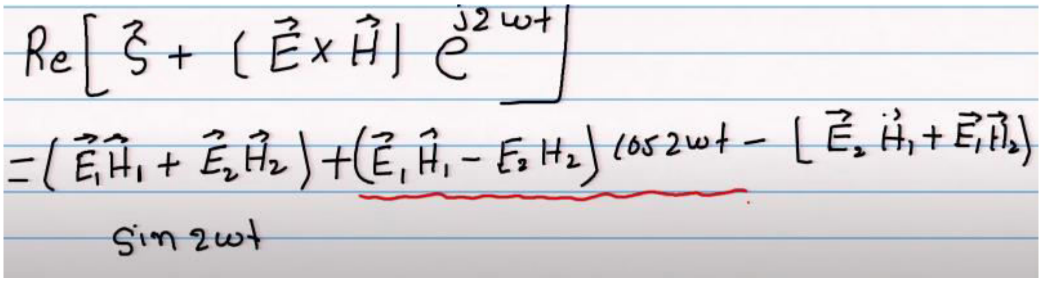




So, this is the final expression, after this we have to find out the real part of S plus E cross H into e to the power j omega t.

For that we have to find out the real part, real part of S plus E cross H into e to the power j 2 omega t.



$$\mathrm{Re}\left[\hat{\xi} + (\vec{E}\times\hat{H})\, e^{j2\omega t}\right]$$
$$= (\vec{E}_1\hat{H}_1 + \hat{E}_2\hat{H}_2) + (\vec{E}_1\hat{H}_1 - E_2H_2)\cos 2\omega t - [\vec{E}_2\dot{H}_1 + \vec{E}_1\vec{H}_2]$$
$$\sin 2\omega t$$

So, like this we can derive the given expression. So, this is a solution for this question.

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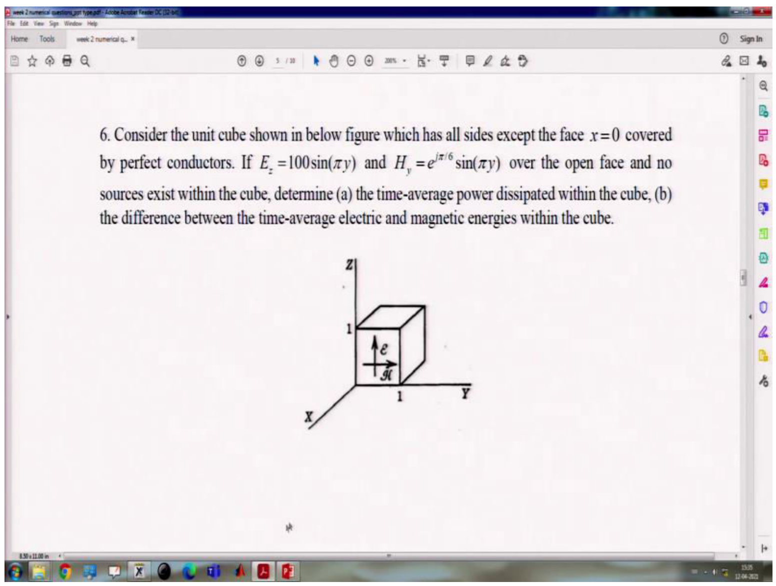
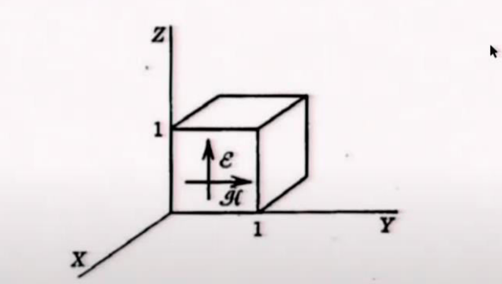


For the next problem.

6. Consider the unit cube shown in below figure which has all sides except the face $x = 0$ covered by perfect conductors. If $E_z = 100\sin(\pi y)$ and $H_y = e^{j\pi/6}\sin(\pi y)$ over the open face and no sources exist within the cube, determine (a) the time-average power dissipated within the cube, (b) the difference between the time-average electric and magnetic energies within the cube.



So, let us do this.

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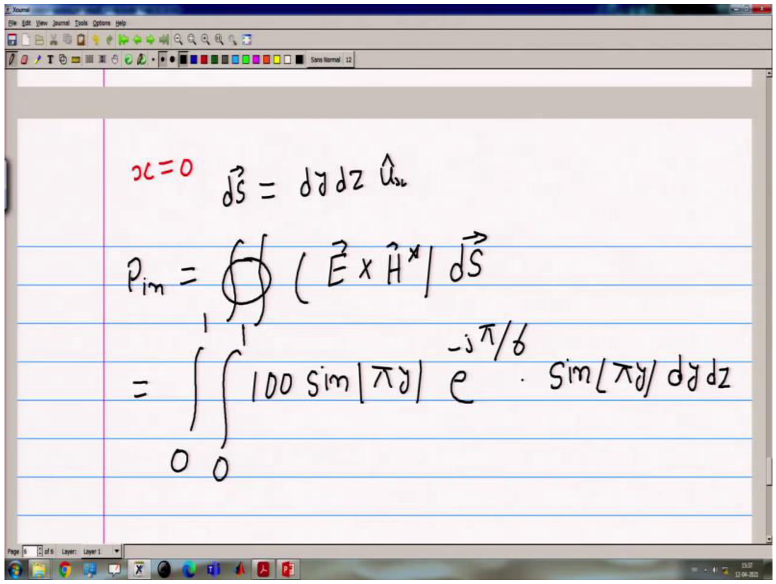
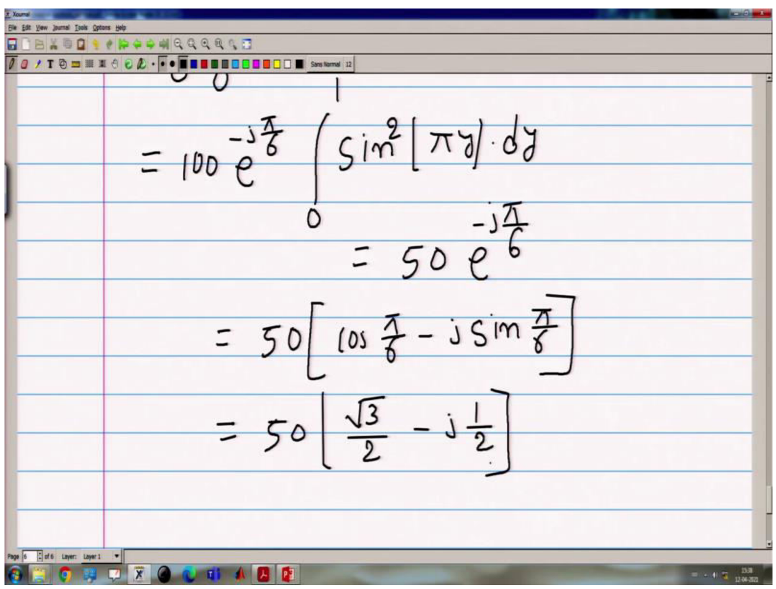
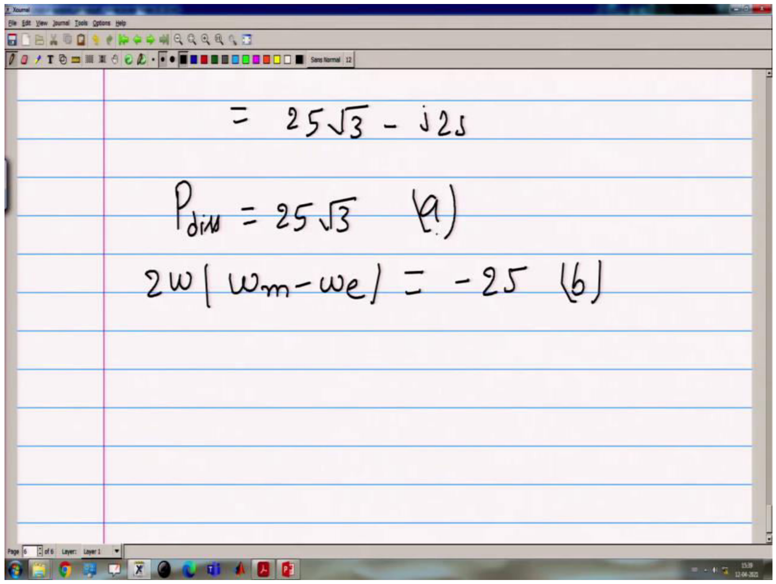
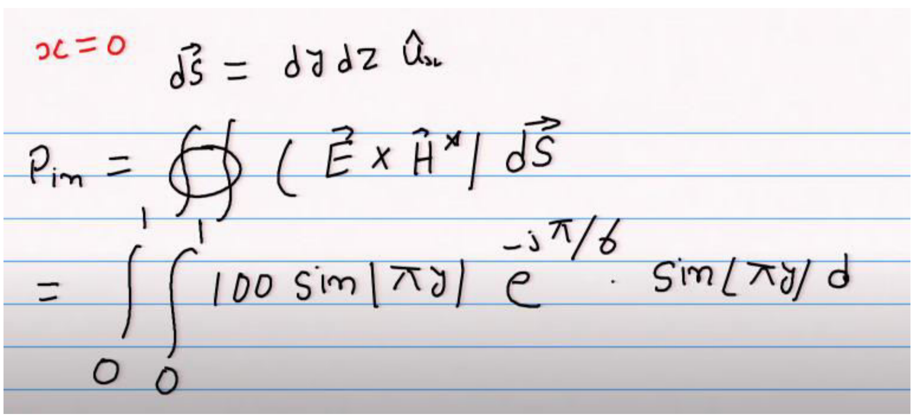


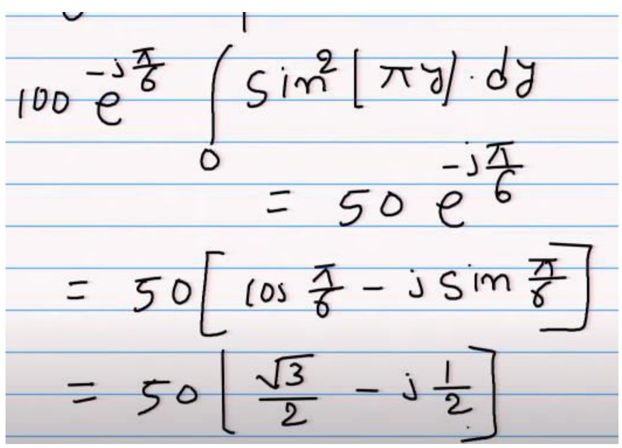




So, this is at, so at x is equal to 0 it is open. So, power, total power in from the interface we can write this like this



On simplifying:



$$= 25\sqrt{3} - j25$$

$$P_{diss} = 25\sqrt{3} \quad (a)$$

$$2W(W_m - W_e) = -25 \quad (b)$$

So, this is our answer for both the parts. So, like this we can solve this problem. Thank you.