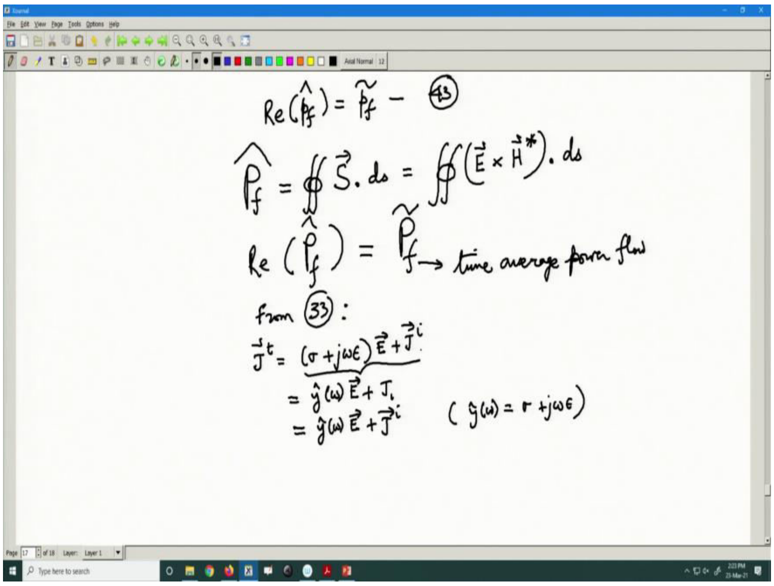
Advance Microwave Guided-Structures and Analysis
Professor Bratin Ghosh
Department of E and ECE
Indian Institute of Technology, Kharagpur
Lecture 16
Time – Harmonic Form of Maxwell's Equations (Contd.)

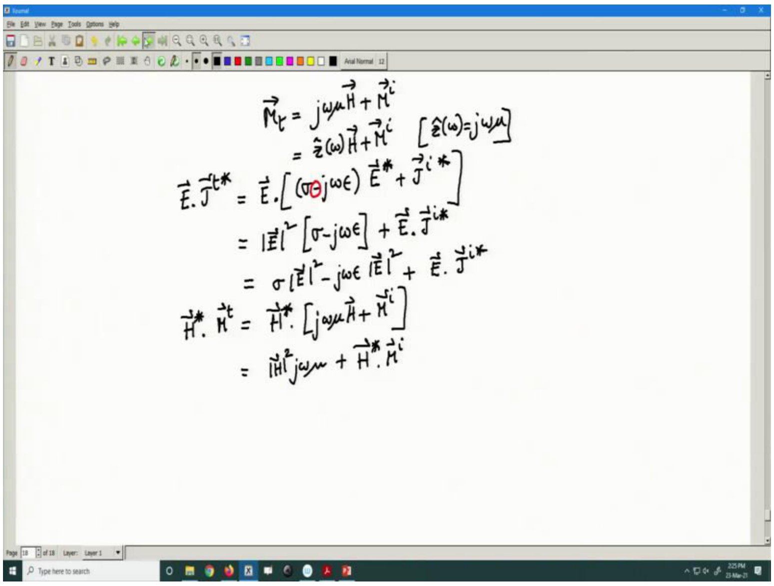
So, welcome to the continuation of the lecture on the Time Harmonic Form of Maxwell's Equations, let us go to the lecture.

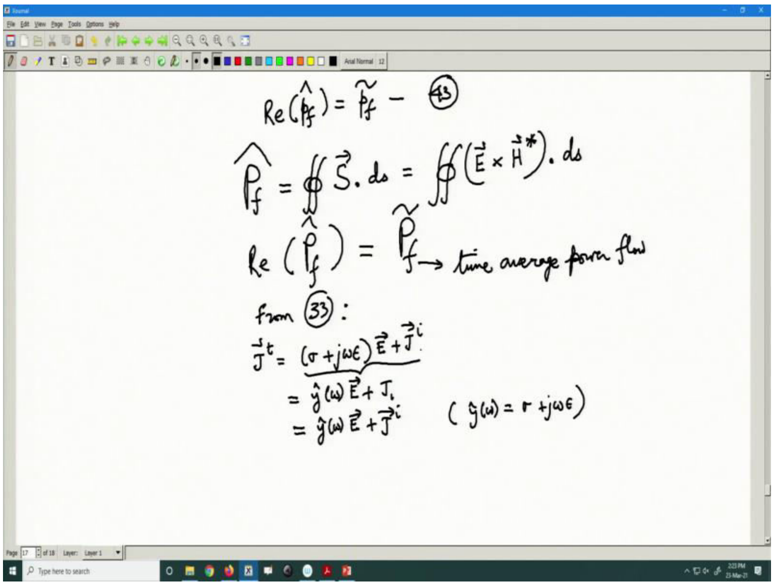
(Refer Slide Time: 00:22)



Now, let us start with the generalised current, we discussed in the instantaneous domain and write down the generalised form of the current in the time harmonic domain. So, we have already written that previously, we will write that down again. So, $\vec{J}^t$ is $(\sigma + j\omega\varepsilon)\vec{E} + \vec{J}^i$. So, we had $\vec{J}^t$ that is equal to $\hat{y}(\omega)\vec{E} + \vec{J}^i$ where $\hat{y}(\omega) = (\sigma + j\omega\varepsilon)$.

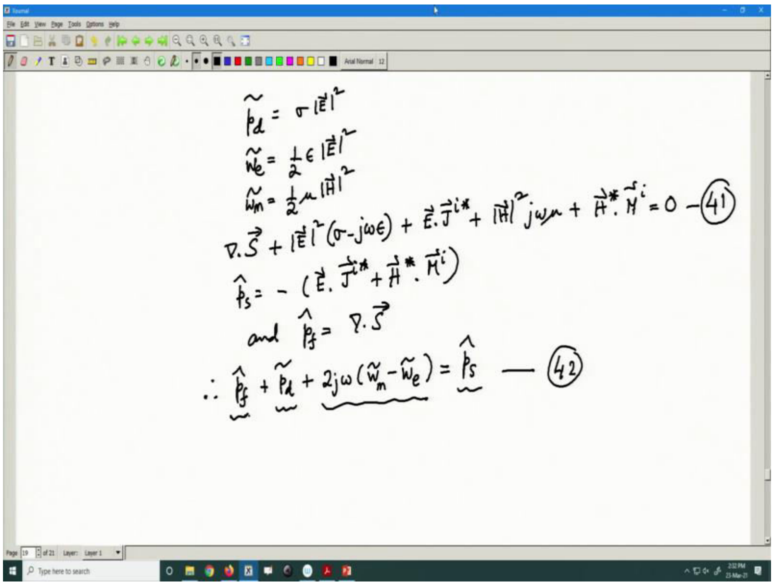
(Refer Slide Time: 01:53)





Next, we have $\vec{M}^t$. We have already written that we are familiar with this $\vec{M}^t$ is $j\omega\mu\vec{H} + \vec{M}^i$, that is equal to $\hat{z}(\omega)\vec{H} + \vec{M}^i$ where $\hat{z}(\omega)$ is $j\omega\mu$. So, if we now perform this operation $\vec{E}\cdot\vec{J}^{t*}$ as we are required to do. We will get $\vec{E}\cdot(\sigma - j\omega\varepsilon)\vec{E}^* + \vec{J}^{i*}$. The minus sign comes as a result of the conjugation operation here. So, then performing $\vec{E}\cdot\vec{E}^*$ it will become $\left|\vec{E}\right|^2(\sigma - j\omega\varepsilon)$ plus $\vec{E}\cdot\vec{J}^{i*}$ and that is equal to $\left|\vec{E}\right|^2\sigma - j\left|\vec{E}\right|^2\omega\varepsilon + \vec{E}\cdot\vec{J}^{i*}$. Similarly, $\vec{H}^*\cdot\vec{M}^t$ will be will be, $\vec{H}^*\cdot(j\omega\mu\vec{H} + \vec{M}^i)$ and that is $|\vec{H}|^2 j\omega\mu + \vec{H}^*\cdot\vec{M}^i$.

(Refer Slide Time: 06:26)

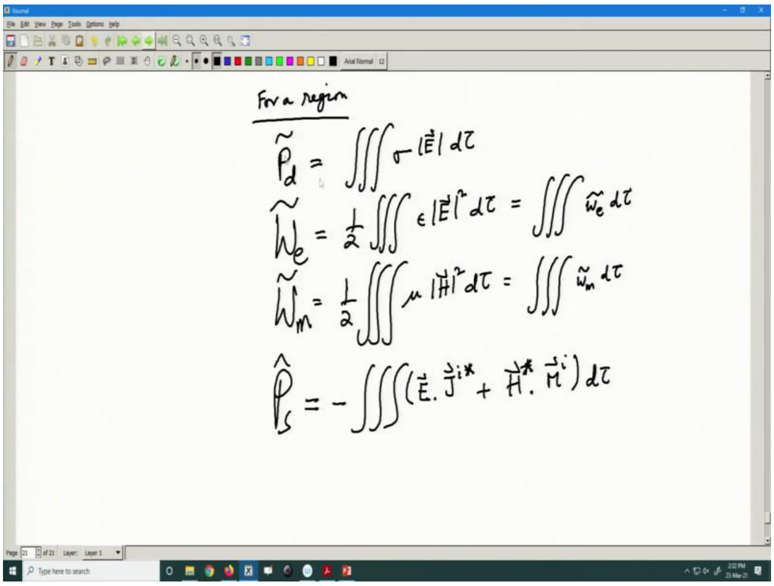


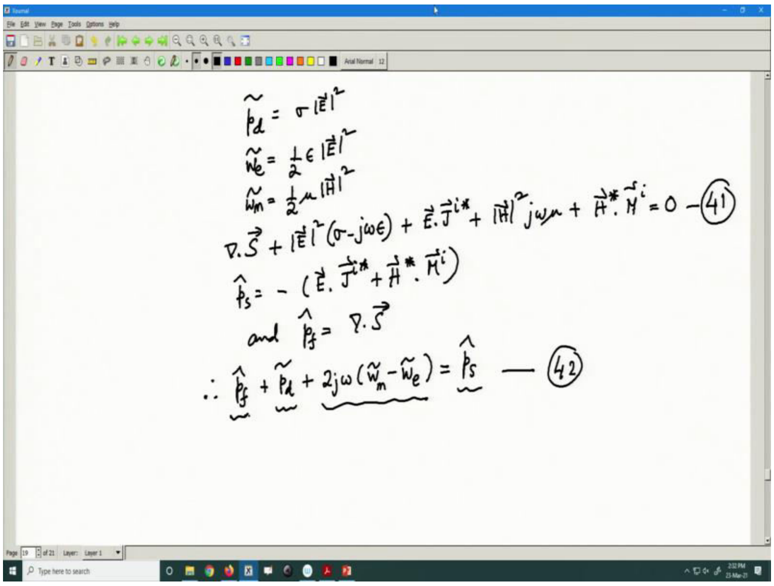
Now, the average dissipated power, the average electric and magnetic energy densities are $\tilde{p}_d$ as $\sigma\left|\vec{E}\right|^2$, $\tilde{w}_e$ as $\frac{1}{2}\varepsilon\left|\vec{E}\right|^2$ and $\tilde{w}_m = \frac{1}{2}\mu\left|\vec{H}\right|^2$. So, equation 40 can be written as finally, $\nabla\cdot\vec{S} + \left|\vec{E}\right|^2(\sigma - j\omega\varepsilon) + \vec{E}\cdot\vec{J}^{i*} + |\vec{H}|^2\ j\omega\mu + \vec{H}^*\cdot\vec{M}^i = 0$, this 41.

So, these are just by direct substitution and this term $\hat{p}_s$ is $-(\vec{E}\cdot\vec{J}^{i*} + \vec{H}^*\cdot\vec{M}^i)$, it is a complex power density supplied by the sources and $\hat{p}_f$ is $\nabla\cdot\vec{S}$. Therefore, we can write $\hat{p}_f$ plus $\tilde{p}_d$ plus $2j\omega(\tilde{w}_m - \tilde{w}_e)$ is equal to $\hat{p}_s$.

So, this is the power flow term, this is a power dissipated term, this is the stored energy term, this is the power supply term. So, this is for the point relationship, this again shows that the power supply term is broken up into the power flow the power dissipated and the stored energy terms.

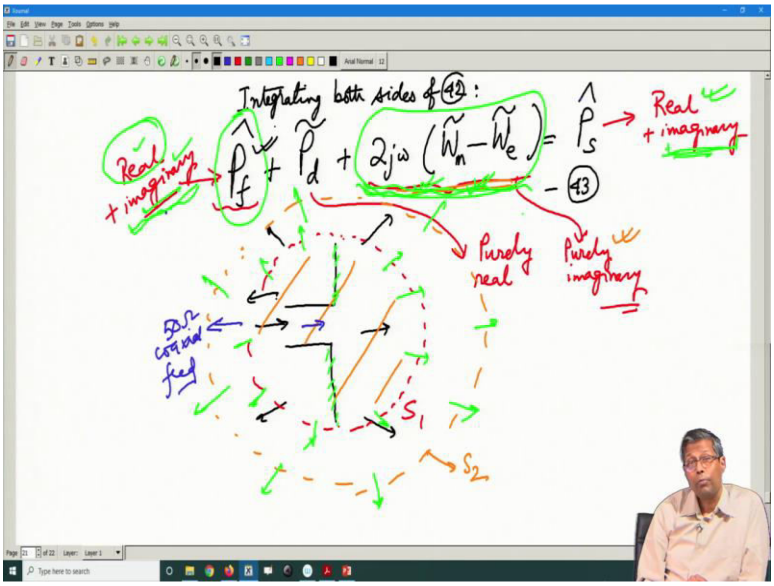
(Refer Slide Time: 10:40)





So, now similarly for a region we have $\tilde{P}_d$ is equal to $\iiint \sigma |\vec{E}|^2 \, d\tau$. We just integrate over the region of space similar to the instantaneous domain. $\tilde{W}_e$ is $\iiint \frac{1}{2} \varepsilon |\vec{E}|^2 \, d\tau$. $\tilde{W}_m$ is $\iiint \frac{1}{2} \mu |\vec{H}|^2 \, d\tau$. And $\hat{P}_s$ is $-\iiint (\vec{E} \cdot \vec{J}^{i*} + \vec{H}^* \cdot \vec{M}^i) d\tau$.

(Refer Slide Time: 13:01)



Integrating both sides of 42 we get $\widehat{P}_f + \tilde{P}_d + 2j\omega(\tilde{w}_m - \tilde{w}_e) = \widehat{P}_s$, this is equation 43. So, what is essentially happening? When we discussed the time harmonic form it is very apparent of the power conservation theorem in a point and in a region so, if I consider for instance a dipole antenna

So, you see it is being fed by let us say our coaxial cable and it is radiating. So, the power supplied by the sources it is saying it is being broken up into this term and this term. Note that this is a purely real term. This term is purely real, as you can well see, this term is purely imaginary and the power supplied by the sources will consist of real and imaginary parts and this will consist of again real plus imaginary parts.

So, as I increase let us imagine a spherical surface. It is an imaginary spherical surface. So, it is S1 so, they surround as they surround this dipole with a spherical surface I try to find out, I try to understand what is happening to the power which is crossing this spherical surface. So, inside this surface in this region inside this region, we have the stored power, this stored power is given by what?

This stored power is given by the volume integral because W_m and W_e are nothing but the volume integrals. So, we are integrating over a volume of space, if I increase the radius of this sphere to let us say S2 the volume enclose will be more and therefore, the stored power will be more.

So, this is called a near field of the antenna. So, as you keep on increasing the volume, the stored power will continue to increase because you are enlarging the volume.

So, this is contributing to the purely imaginary stored energy term. However, you see that this power dissipation is happening in the conductive surface due to ohmic loss in these surfaces, this is where the power dissipation is taking place. So, this is fixed in this region of space, it does not vary with the increase of spherical radius. So, it is happening inside. Also this term is due to the sources inside this region. So, that is also fixed.

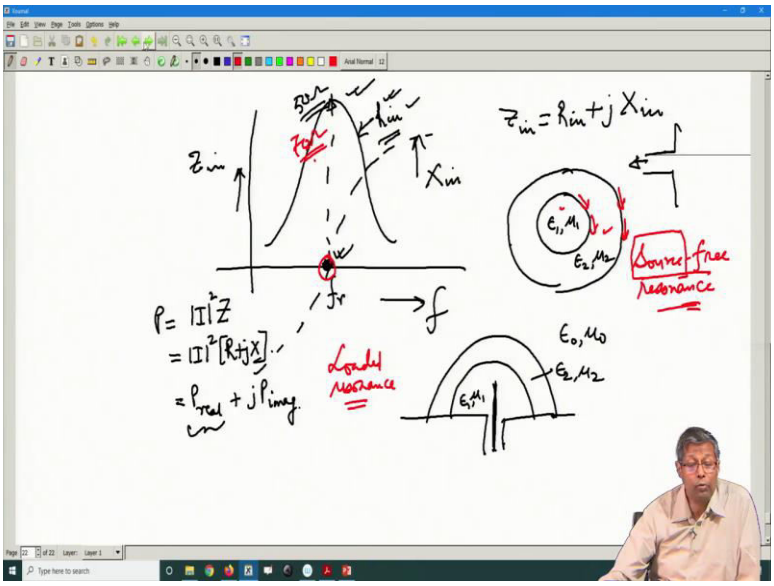
So, what is happening to the other part which is the real and imaginary parts as you cross different surfaces, the real part which is the real power radiated will be the same, the real part of $\hat{p}_f$ will be same as you cross different surfaces, because that is the power radiated. The imaginary part of this term plus this term is going to contribute to the imaginary part of the power supplied. As we go to increase the radius, this power flow becomes purely real and the imaginary part slowly diminishes.

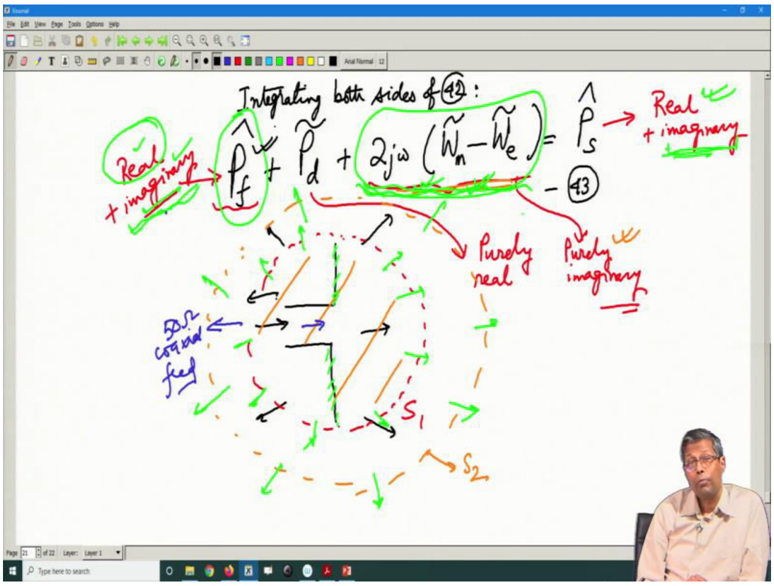
So, this imaginary part is completely taken by this, if you were to increase the sphere to a limiting value, this value will saturate. It will come to a particular limiting value and beyond that it is going to stay virtually constant. You are in the far field region where this power $\hat{p}_f$ is totally real. Factor increasing this would not increase this and this will stay as real.

So, this term you see in the near field and the imaginary part of this term in the near field. It is contributing to the imaginary part of this, why is this important? This is important because we want to match this to the 50-ohm load on the other side, on the other side of this I have a 50-ohm coaxial line. So, this input impedance has to match to the 50-ohm feed.

So, it must therefore be remembered that the reactive part of the power supplied is the same as the stored energy part plus the imaginary part of the power flow, that is contributing to the reactance of the system and this has to be cancelled in order for the system to be matched for this dipole to be matched.

(Refer Slide Time: 21:25)





So, we told you that this matching problem is something like this and let us say so, this is the resonant frequency, this is the X_{in} and this is the R_{in} with Z_{in} equal to $Z_{in} = R_{in} + jX_{in}$, this is the resonance of the system, typically the resonance of any system, let us say if it is a spherical like a multi-layer sphere resonance of this, so this is epsilon 1 mu 1, epsilon 2 mu 2.

A typical example to this might be a system like this where I have like a multi-layer resonator antenna epsilon 1 mu 1 that is epsilon 2 mu 2 and this is epsilon naught mu naught fed by a coaxial probe. So, normally this resonance frequency is the loaded resonant frequency that is with the excitation of the probe, the source included. So, we have included the source in the problem. So, it is the inclusion of the source.

So, the point where the reactance curve crosses the 0 axis is where the resonance occurs, there the input resistance peaks, this is leading to the radiated power. So, we can we can say that we want to maximise the input resistance, we want to maximise the input resistance and for any kind of antenna in order for the antenna to radiate.

But we must also remember that this same input resistance is responsible for the matching because our R_{in} has to match. I said it has to match contribute to the matching with the source. So, it is best that R_{in} should be equal to 50-ohm at its peak. But I told you that the complex power is $|I|^2 z$ and that is equal to $|I|^2 (R + jX)$ and that is equal to power real plus j power imaginary.

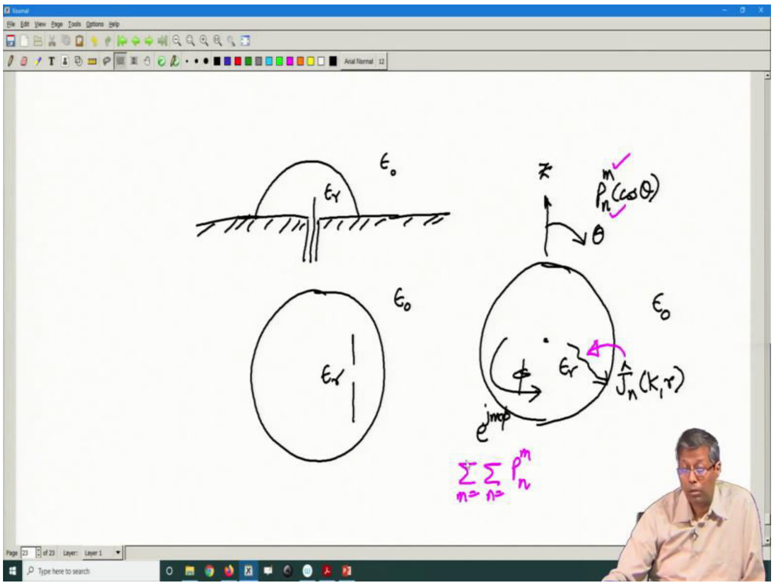
We know our previous analysis shows that this power real which is called the radiated power which is the real part of this term, it depends on the system. It is directly dependent on the amount of real power which will be emitted or radiated by the dipole. So, we have no direct control over these or to ensure that this will correspond to our 50-ohm radiation resistance, we can try to adjust the location of the feed to get a match, we can adjust the location of the feed to get a match such that this peak corresponds to 50 ohm.

If we can get a match with varying this feed location, we can match this point to the this point to the 50 ohm line, if this is 50 ohm, if instead of 50 ohms, this is 70 ohm, because on that number we are pretty much less control little control. So, if that is 70 ohms then we have to use a quarter wave transmission line, in order to match this to a 50 ohm load, but we have to make sure that we are operating at the resonance frequency of the antenna, so that the reactive load is 0.

So, this is called the loaded resonance. The concept of loaded resonance is used to match a antenna or any kind of guidance structure with its feed. It is distinct from what we call source free resonance, this can be well illustrated by this kind of antenna structure.

For example, if I were to compute the source free resonance of this I would say because it is source free it is independent of the source there is no source which is exciting this structure, we would just match the electric and the magnetic fields here and we will match the electric and magnetic fields there.

(Refer Slide Time: 27:48)

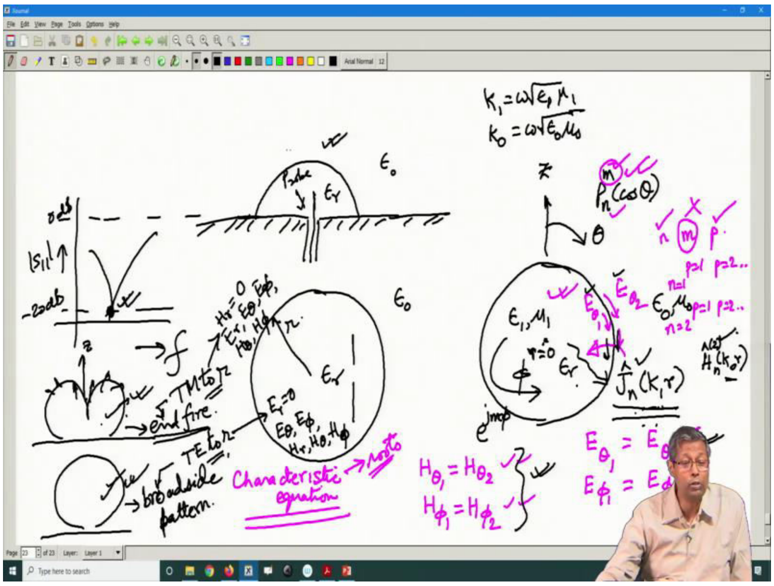


In fact, we can we can work with even a single layer. We can take a single layer of antenna epsilon naught and we have a single layer by image theory we replace the ground plane, so this is my ground plane which is theoretically infinite in extent by image theory we replace the ground plane then have the probe and this is epsilon 0.

Now, when we are dealing about the source free resonance, we consider only this sphere. So, this is epsilon r this is epsilon naught. So, we will come to that, but we what we do is that we expand the fields inside this sphere. So, let us say this is the z axis, so, this is the theta direction and this is the phi direction.

So, the potential functions inside this sphere will be described by these kind of functions $P_n^m(\cos\theta)$ along the theta direction, $e^{im\phi}$ around the phi direction and along the radial direction will be $\hat{j}_n(k_1 r)$ that will be its variation here. And we will sum up what this m and n, so it is going to be a summation over m and n something and then we are going to say.

(Refer Slide Time: 30:21)



So, we are going to sum up over m and n and we are going to match the fields here that is the E theta 1 will be equal to E theta 2 where E theta 1 will be the tangential fields here will be equal to E theta 2, E phi 1 will be equal to E phi 2 then we will have H theta 1 equal to H theta 2 and we will have H phi 1 equal to H phi 2.

So, we will match these four sets of fields and once we match them, we will come to an equation called a characteristic equation. We will explicitly go through this procedure when we do the resonator. Therefore, I am not explicitly writing the explicit equations for this here but we will come to the characteristic equations by matching the fields in this case, in this solution we assume that the source is absent that the fields have been established by the source but the source itself is absent.

So, the roots of these characteristic equation, will yield the infinite modes of this spherical ball, so we label the roots in terms of n, m and p. In fact, those roots will be independent of m, but will depend on n only. And p for a given value of n the roots will be marked as p equal to 1, p equal to 2, p equal to 3 like that. So, n equal to 1, p equal to 1, p equal to 2 and so on, n equal to 2 it will be again p equal to 1, p equal to 2 and so on.

But it must be remembered that though the resonant frequencies independent of m the fields will depend on m. Why we do we do this exercise? Why it is important to us? It is important to us because through the source free modes let us say we have obtained a match with this system with the coaxial probe with this probe, we have obtained a match at this frequency minus 20 dB and that is 0 dB and this is reflection coefficient mod S11 we have obtained a match at this frequency.

So, we want to know which mode is responsible for this resonance. As I told you, this mode is found by matching the electric fields and the magnetic fields and by the way, the inside radial function is going to be this is called the schkelkunoff type Bessel function and outside in this region, the function will be the schkelkunoff type Hankel function.

So, if this is epsilon 1 mu 1 and this is epsilon naught mu naught. So, k1 is going to be omega root epsilon 1 mu 1 and k0 will be equal to omega root epsilon naught mu naught. So, this is k0 and that is k1 because this is located in this region, this is located in this region. The reason why we choose different functions is that the schkelkunoff of type Bessel function is the only function which is finite at r equal to 0, if the point of computation includes the point r equal to 0 the schkelkunoff type Bessel function is the only function which is finite.

Similarly, the schkelkunoff type outgoing Hankel function is $\hat{H}_2$ denoting the outgoing Hankel function, it is exponentially decaying and therefore, satisfies the radiation condition at r is equal to infinity. So, therefore when we match these two sets of fields the electric fields and the magnetic fields using these potential functions. From those potential functions we find out the fields explicit forms. We will discuss when we go to the waveguide problem or the resonator problem.

Here we are giving an outline of the resonance phenomenon and how the loaded resonance as we saw through our power conservation is different from the source free resonance and why the source free resonance is important. So, using the source free resonance we can know the particular mode which is responsible for this resonance.

And if we know the mode, we can better design our probe or our exciting structure to better couple to that mode. In fact, it works a little bit reverse way that the vendors give us a specification that I want my let us say my antenna to radiate in this way which can be called end fire direction.

So, this is the end fire direction or it is called a vertically polarised pattern because the polarisation is like this it is vertical or I want to radiate in this one or we just call a broadside pattern. So, this kind of pattern is typically used in vehicular communication, a ship to ship communication or a vehicle to vehicle communication when we want to communicate along this direction along the horizon and this kind of pattern is used for communicating with sky bound targets satellites, rockets, space vehicles, aircrafts. So, this kind of pattern is useful.

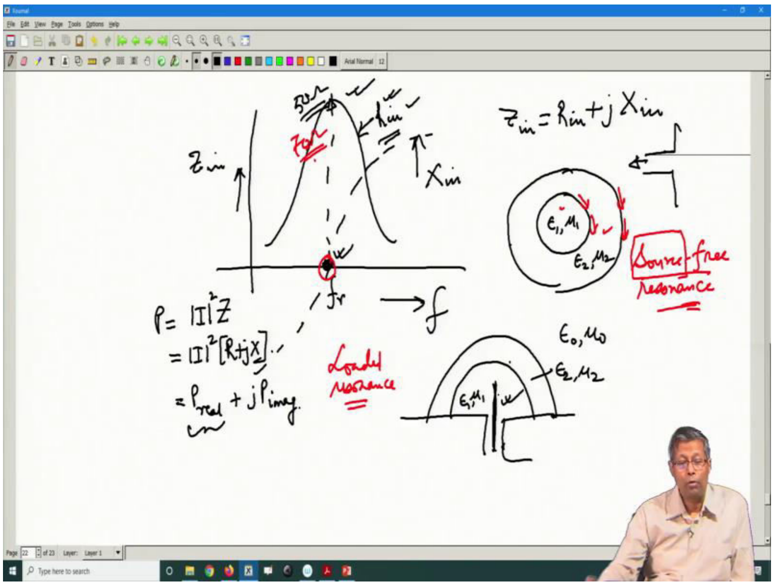
So, typically, the vendor will give you a specification that I want end fire pattern or I want a broadside pattern, at a particular resonance. So, then you have to identify which kind of modes lead to this kind of pattern. Typically, but not always TE to r modes lead to broadside pattern and TM to r modes lead to end fire pattern. What I mean by TE to r is that if this is the radial direction this mode means Er equal to 0.

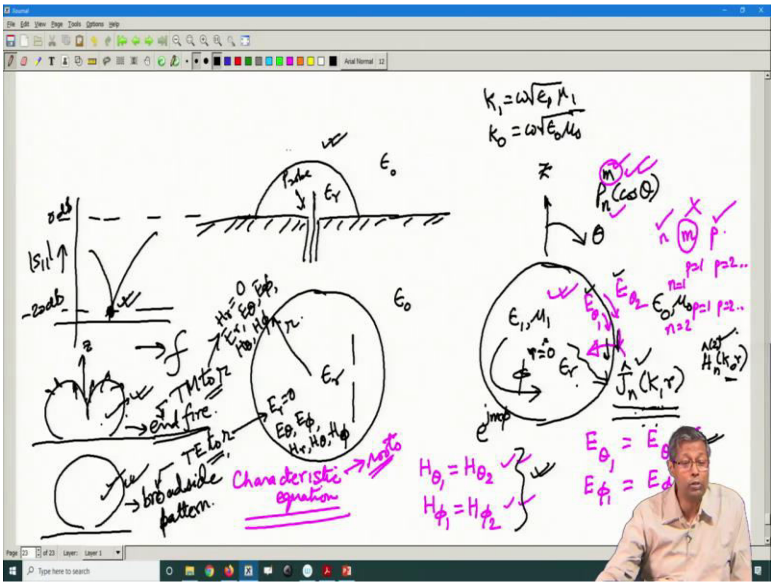
That means the only relevant component for this mode for the TE to r is E theta, E phi, Hr, H theta and H phi. Similarly for this mode it is TM to r transverse magnetic to r, that means it is Hr is 0 Hr component, the radial component is 0 therefore, its field components are Er, E theta, E phi, H theta and H phi. So, typically Tm to r, will give an end fire pattern and TE to r will give broadside pattern.

But there are TE to r modes which give end fire pattern so be careful. So, typically the vendor will give you the specification this is the pattern I want. So, I want to start from that mode I want to excite this mode then you want to design your feed that where will you put, how will you put the feed. And find the resonant frequency from the characteristic equations which is obtain from here.

So, the source free resonance is very-very meaningful, because it helps us to design the system, it helps us to give a physical insight to the system. Where does the loaded resonance enables me to find a match, there most important concept of the loaded resonance is getting a match because the loaded resonance is intimately linked with the concept that I want to match my antenna.

(Refer Slide Time: 42:09)





I want to match my probe to the feed line here or to match my DRA to the probe Br. So, the concept of loaded resonance is deeply embedded to this match, so we will continue let us stop here, we will continue on the significance of the power conservation, thanks.