# Advance Microwave Guided-Structure and Analysis

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**Lecture 18**

**Time – Harmonic form of Maxwell's equations Tutorials**

Welcome to the tutorial class. So, now, in this tutorial class, we will take the numerical problem on complex pointing vector.

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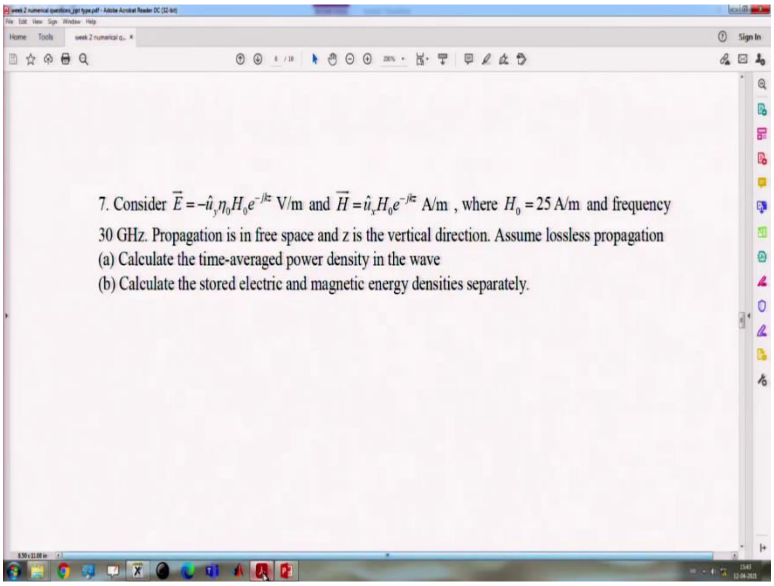


So, this is the first problem.

7. Consider $\overrightarrow{E} = -\hat{u}_y \eta_0 H_0 e^{-jkz}$ V/m and $\overrightarrow{H} = \hat{u}_x H_0 e^{-jkz}$ A/m , where $H_0 = 25$ A/m and frequency 30 GHz. Propagation is in free space and z is the vertical direction. Assume lossless propagation

(a) Calculate the time-averaged power density in the wave

(b) Calculate the stored electric and magnetic energy densities separately.

So, this is a first problem calculate the time average power density in the wave in the next part calculate the stored electric and magnetic energy density separately. So, let us do the solution.

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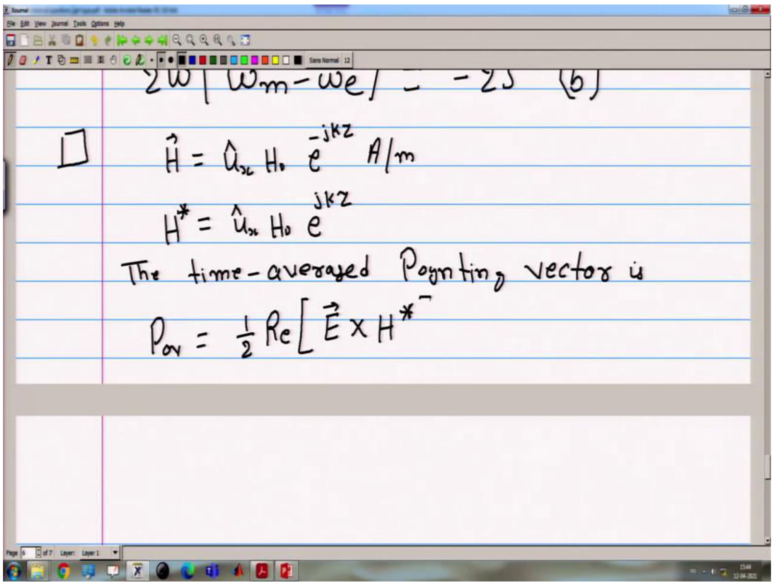
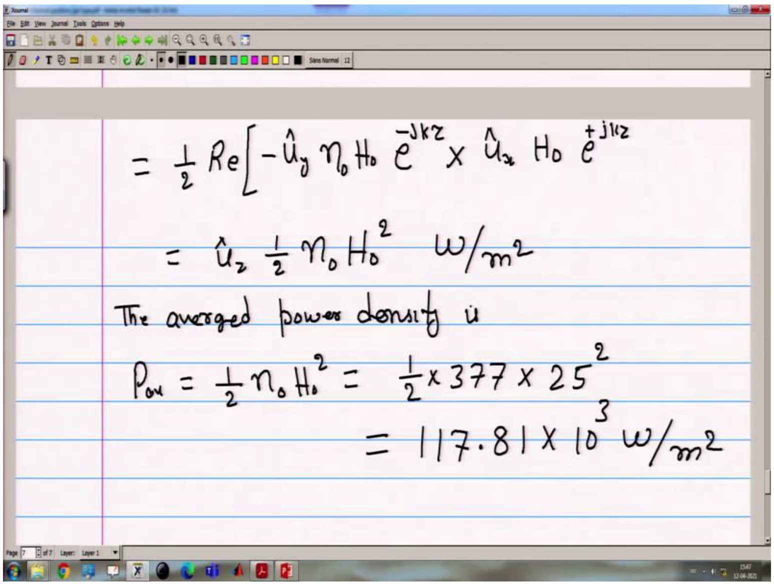
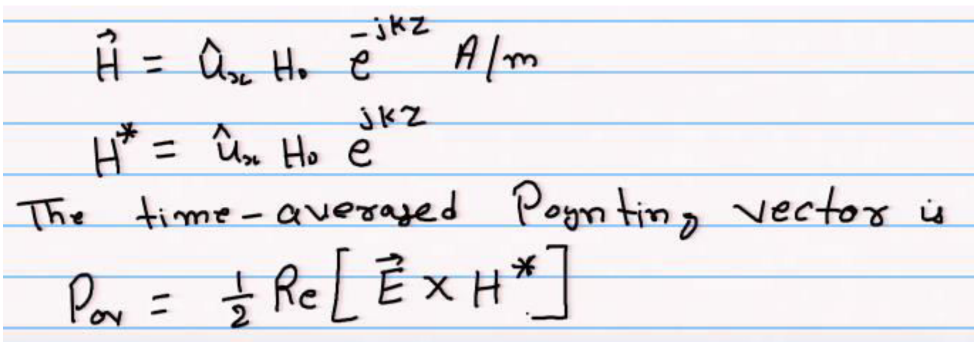



So, here it is given.



So, E is given in the question, so above equation becomes:

$$= \frac{1}{2} Re\left[ -\hat{u}_y \eta_0 H_0 e^{-jkz} \times \hat{u}_x H_0 e^{+jkz} \right.$$

$$= \hat{u}_z \frac{1}{2} \eta_0 H_0^2 \quad W/m^2$$

Time average power density is

The averaged power density is

$$P_{av} = \frac{1}{2} \eta_0 H_0^2 = \frac{1}{2} \times 377 \times 25^2$$

$$= 117.81 \times 10^3 \ W/m^2$$

because in the question it is given, it is a loss less propagation and progression in the free space. So, free space wave impedance will be 377.

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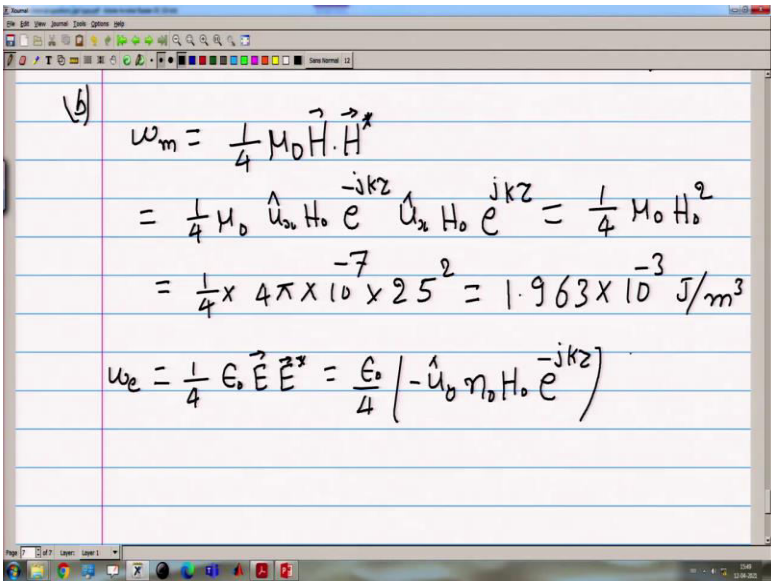

$$\left(-\hat{u}_y \eta_0 H_0 e^{jkz}\right)$$

$$= \frac{\eta_0^2 \epsilon_0 H_0^2}{4}$$

$$= \frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4}$$


$$\left(-\hat{u}_y \eta_0 H_0 e\right)$$

$$= \frac{\eta_0^2 \epsilon_0 H_0^2}{4}$$

$$= \frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4}$$


$$= 1.963 \times 10^{-4} \ J/m^3$$

In the next part, stored electric and magnetic energy density are given by:

$$w_m = \frac{1}{4} \mu_0 \vec{H} \cdot \vec{H}^*$$

$$= \frac{1}{4} \mu_0 \hat{u}_x H_0 e^{-jkz} \hat{u}_x H_0 e^{jkz} = \frac{1}{4} \mu_0 H_0^2$$

$$= \frac{1}{4} \times 4\pi \times 10^{-7} \times 25^2 = 1.963 \times 10^{-3} \ J/m^3$$

$$u_e = \frac{1}{4}\epsilon_0 \vec{E}\vec{E}^* = \frac{\epsilon_0}{4}\left(-\hat{u}_y \eta_0 H_0 e^{-jkz}\right)\cdot\left(-\hat{u}_y \eta_0 H_0 e^{jkz}\right)$$

$$= \frac{\eta_0^2 \epsilon_0 H_0^2}{4}$$

Substituting the values, we get:

$$\frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4} = 1.963 \times 10^{-4}\ J/m^3$$

So, this is the solution.

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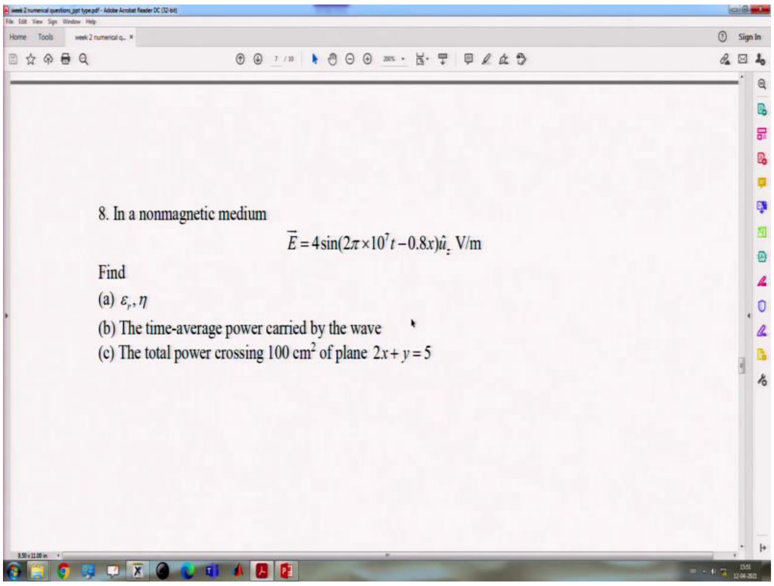


We can go to the next problem,

8. In a nonmagnetic medium

$$\vec{E} = 4\sin(2\pi \times 10^7 t - 0.8x)\hat{u}_z \text{ V/m}$$

Find

(a) $\varepsilon_r, \eta$

(b) The time-average power carried by the wave

(c) The total power crossing 100 cm$^2$ of plane $2x + y = 5$

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$\boxed{8}$ $$\vec{E} = 4\sin(2\pi 10^7 t - 0.8x)\hat{u}_z \text{ V/m}$$

$$\beta = 0.8, \quad \omega = 2\pi \times 10^7, \quad \mu = \mu_0$$

$$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}$$

$$= \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\beta = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7} = \frac{12}{\pi}$$

$$\epsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^{8}}{2\pi \times 10^{7}} = \frac{12}{\pi}$$

$$\epsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$= \frac{120\pi}{12}\pi = 10\pi^2\ \Omega$$

Here E field is given,

$$\vec{E} = 4 \sin(2\pi 10^7 t - 0.8x)\, \hat{u}_z\ \text{V/m}$$

So, from this we can compare sine omega t minus beta x. So, after comparing

$$\beta = 0.8, \quad \omega = 2\pi \times 10^7, \quad \mu = \mu_0$$

So, this is the omega and beta and because this is a non-magnetic medium, So,

$$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$= \omega\sqrt{\mu_0 \epsilon_0}\sqrt{\epsilon_r}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\beta = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^{8}}{2\pi \times 10^{7}} = \frac{12}{\pi}$$

On solving we get:

$$\epsilon_r = 14.59$$

after getting this value, we can find out the wave impedance as:

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$= \frac{120\pi}{12}\pi = 10\pi^2$$

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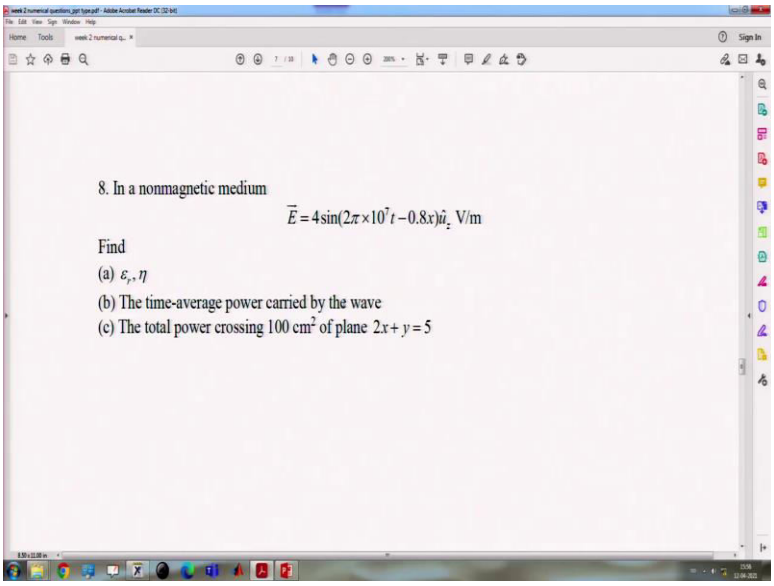


Next part, time average power carried by the wave we can find out by:

$$P_{ave} = \frac{E_0^2}{2\eta}\hat{u}_x = \frac{16}{2 \times 10\pi^2}\hat{u}_x$$

$$= 81\ \hat{u}_x\ mW/m^2$$

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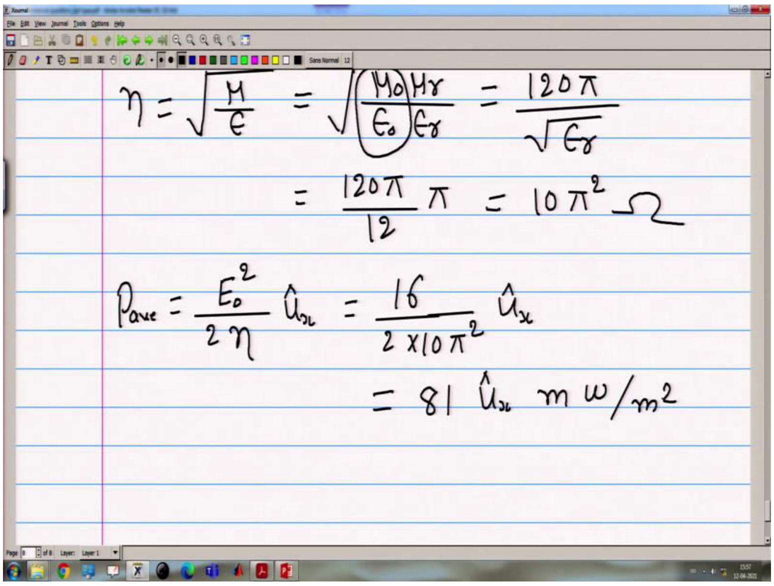


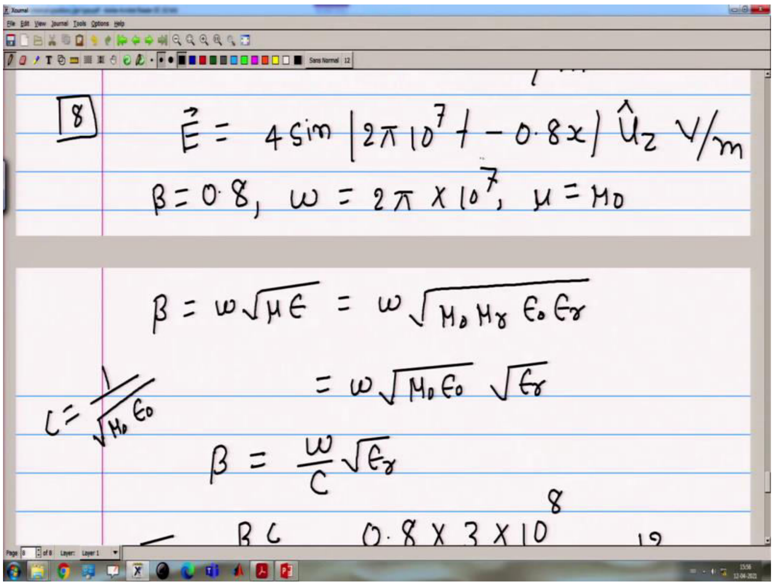


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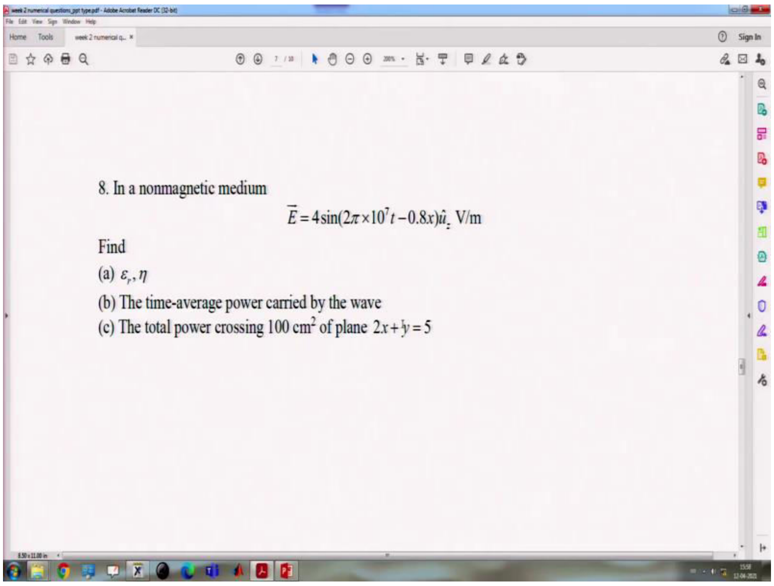

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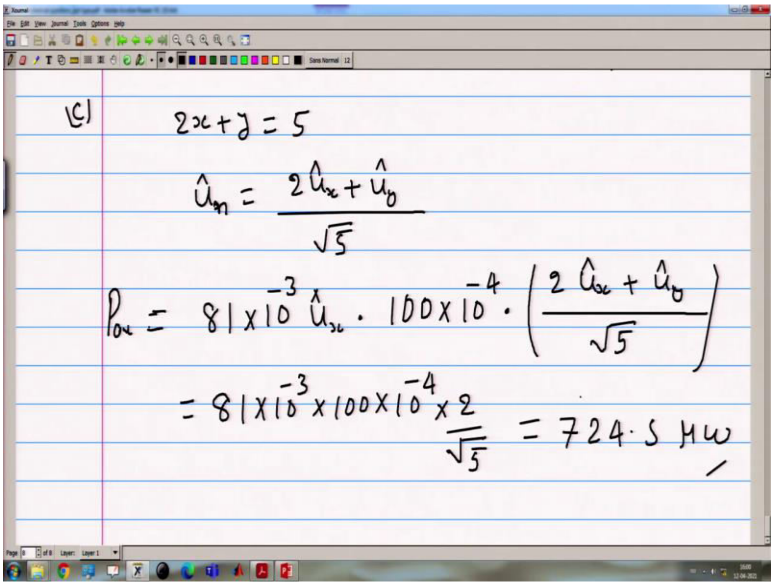
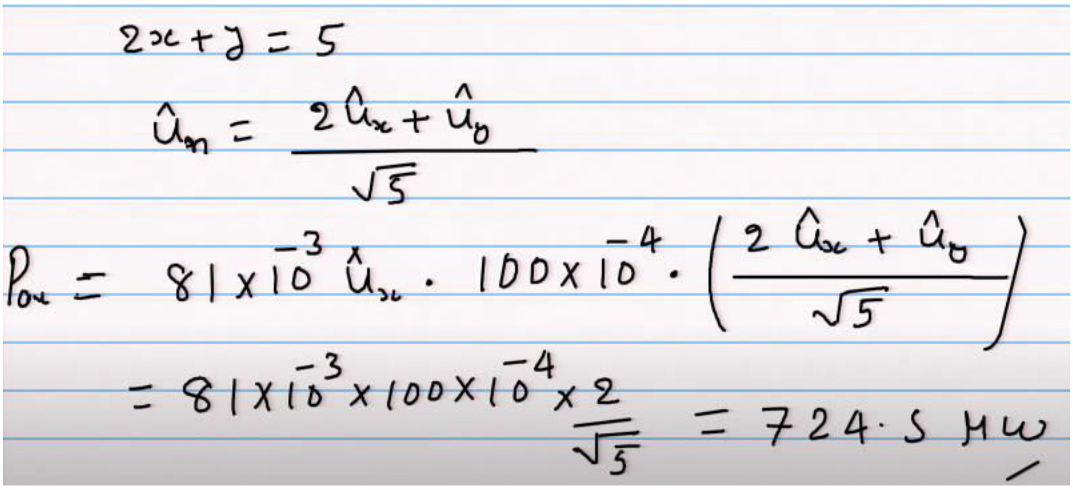



Next part is total power crossing the crossing 100 a square centimetre of a plane 2x plus y equal to 5.




So, this will come 724.5 Macro Watt. So, this is the answer for the last part, we can go to the next problem.

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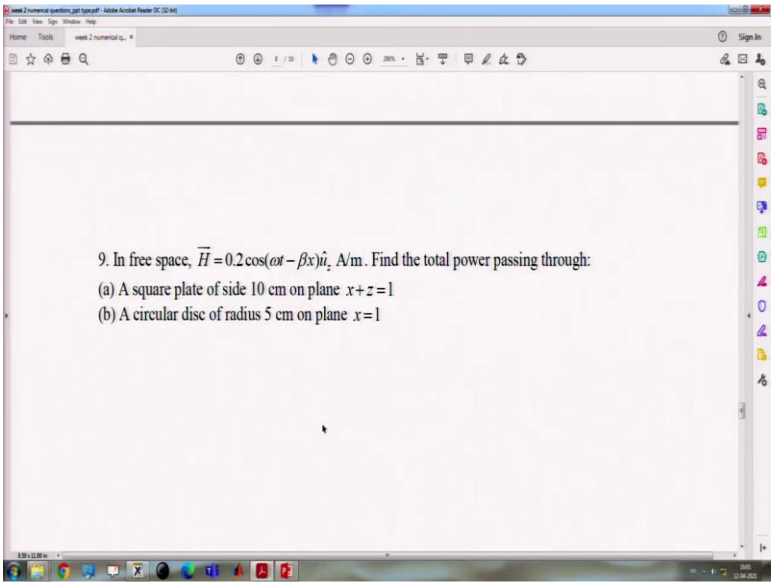


9. In free space, $\vec{H} = 0.2\cos(\omega t - \beta x)\hat{u}_z$ A/m. Find the total power passing through:
(a) A square plate of side 10 cm on plane $x + z = 1$
(b) A circular disc of radius 5 cm on plane $x = 1$

We need to find out the total power passing through a square plate of side, this is a square plate of side 10 centimetre that means area will be 10 X 10 centimetre square, on the plan x + z = 1. So, first we have to find out the unit vector on this plane. So, unit vector on this plane will be.

$$\hat{u}_n = \frac{\hat{u}_x + \hat{u}_z}{\sqrt{2}}$$

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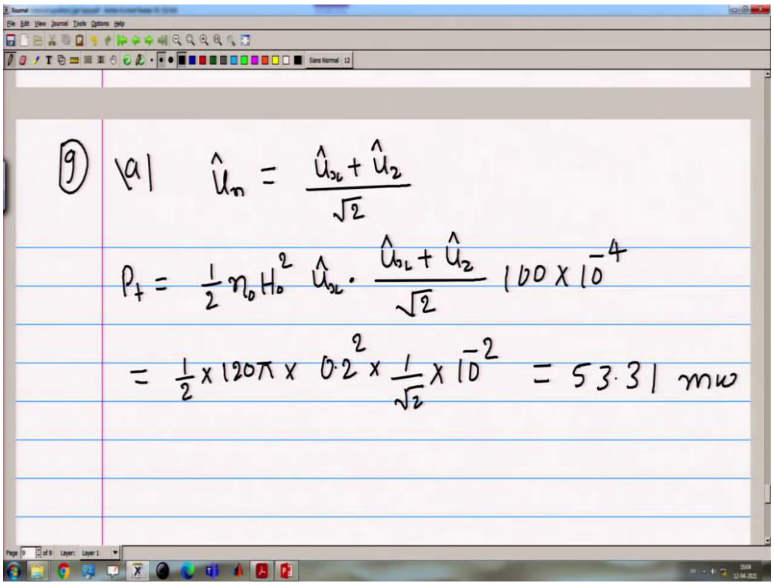


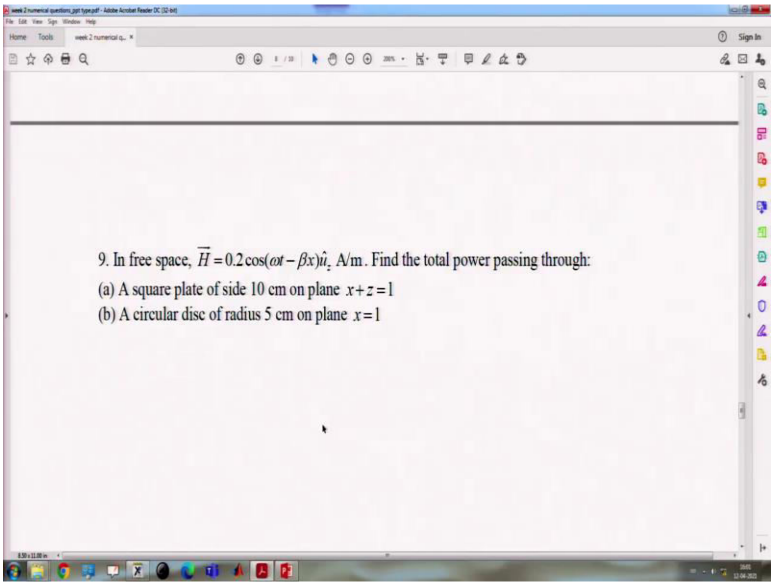


And the total power passing through the square plate of side 10 centimetre:

$$P_t = \frac{1}{2}\eta_0 H_0^2 \, \hat{u}_x \cdot \frac{\hat{u}_x + \hat{u}_z}{\sqrt{2}} \, 100 \times 10^{-4}$$

$$= \frac{1}{2} \times 120\pi \times 0.2^2 \times \frac{1}{\sqrt{2}} \times 10^{-2} = 53.31 \text{ mW}$$

So, this will come after calculation 53.31 milli watt. So, this is the answer for the first part.

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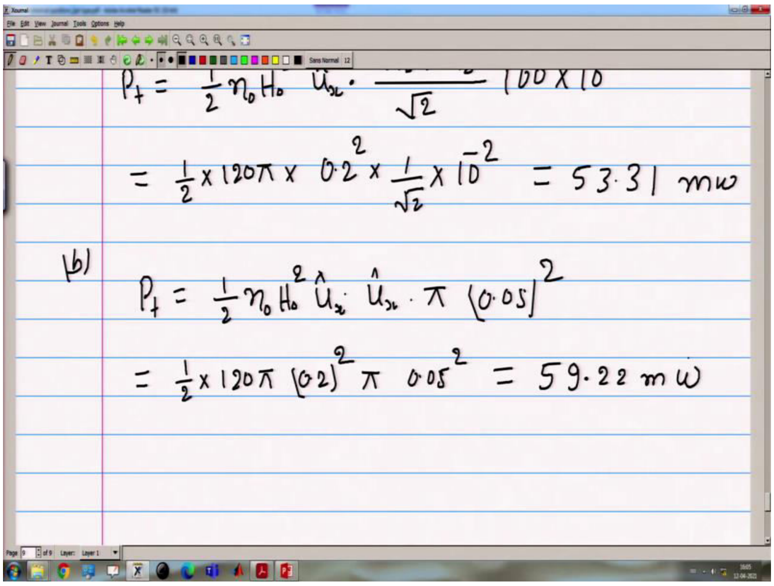


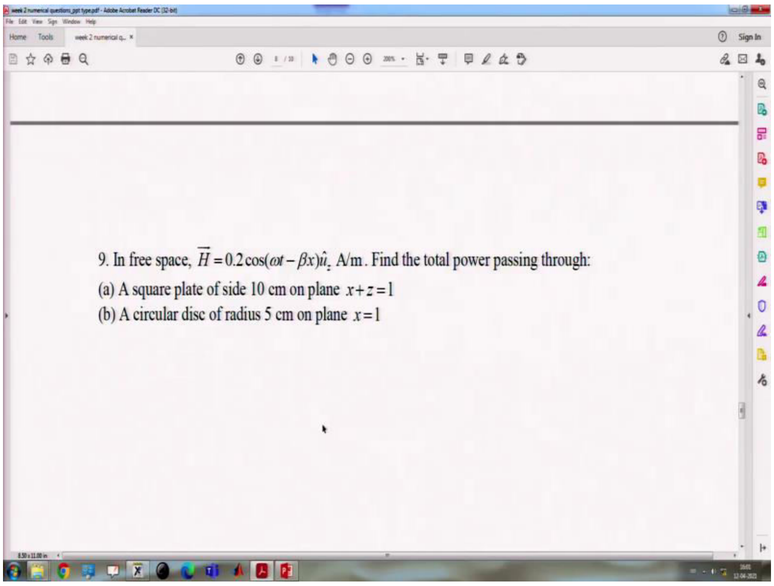


Similarly, for the next part in the b part, in the b part it is given, a circular disc of radius 5 centimetre on the plane x = 1. so, on the plane x = 1, we have to find out Pt.

$$P_t = \frac{1}{2}\eta_0 H_0^2 \hat{u}_x \cdot \hat{u}_x \cdot \pi (0.05)^2$$

$$= \frac{1}{2} \times 120\pi (0.2)^2 \pi \, 0.05^2 = 59.22 \text{ mW}$$

after simplification it will come 59.22 milli watt. So, this is the answer for the b part.

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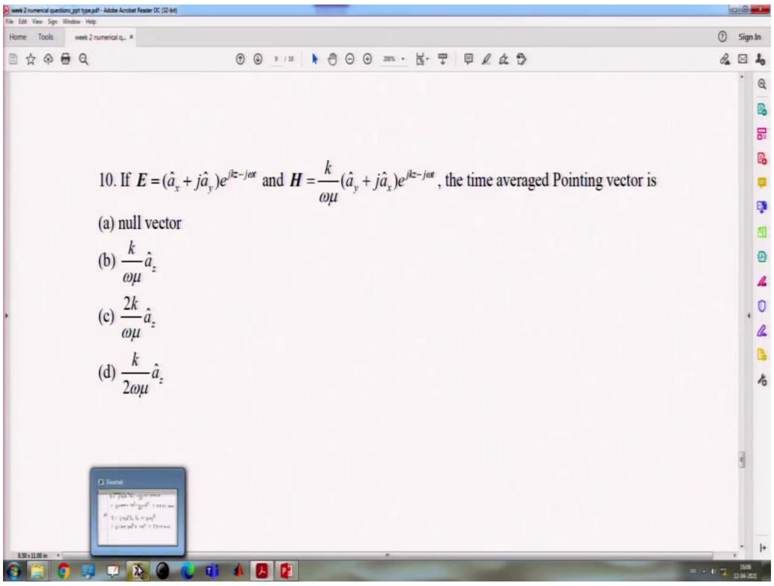

So, in the next problem it is given:

10. If $\boldsymbol{E} = (\hat{a}_x + j\hat{a}_y)e^{jkz - j\omega t}$ and $\boldsymbol{H} = \frac{k}{\omega\mu}(\hat{a}_y + j\hat{a}_x)e^{jkz - j\omega t}$, the time averaged Pointing vector is

(a) null vector

(b) $\frac{k}{\omega\mu}\hat{a}_z$

(c) $\frac{2k}{\omega\mu}\hat{a}_z$

(d) $\frac{k}{2\omega\mu}\hat{a}_z$

So, here options are given, so I have to find out time average pointing vector. So, time average pointing vector is given as:

$$P_{avg} = \frac{1}{2} Re\left[ \vec{E} \times H^* \right]$$

$$= \frac{1}{2} Re\left[ [\hat{a}_x + j a_y] e^{jkz - j\omega t} \times \frac{k}{\omega\mu} [a_y - j a_x] e^{-jkz + j\omega t} \right]$$

$$= \frac{1}{2} Re\left[ \frac{k}{\omega\mu} \left\{ a_z + 0 + 0 - a_z \right\} \right] = 0$$

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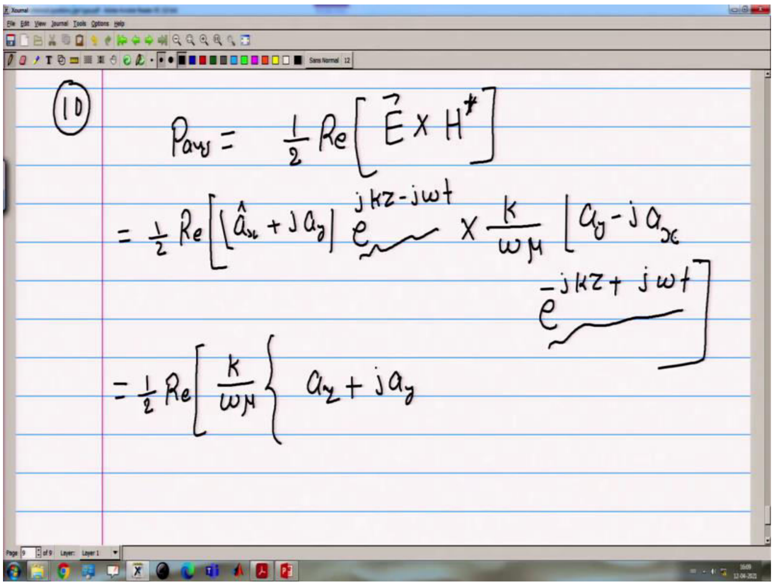


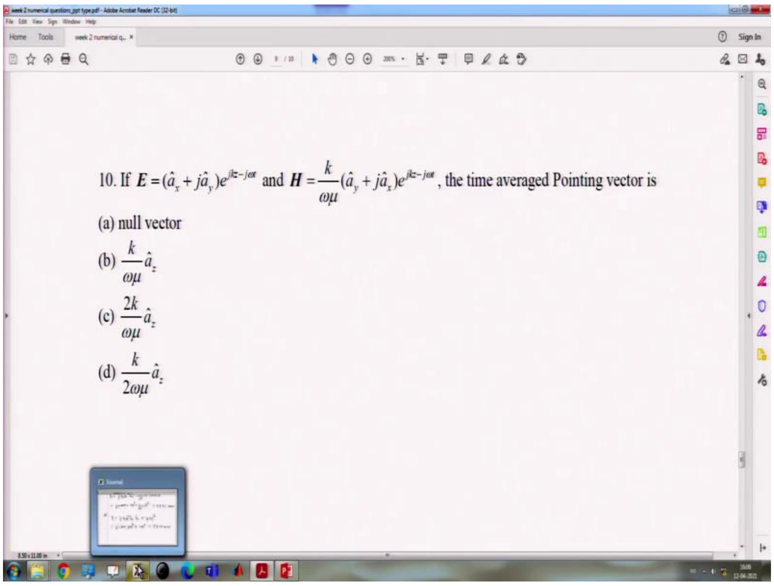


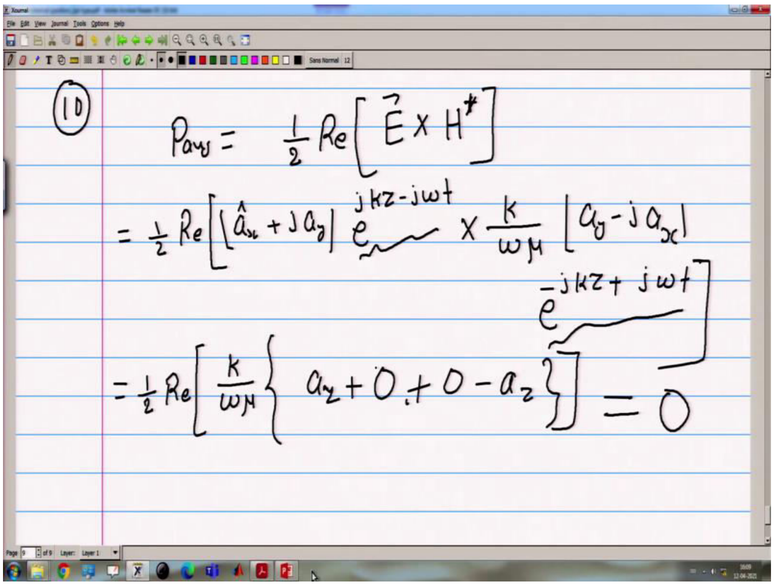

So, option will be a null vector.

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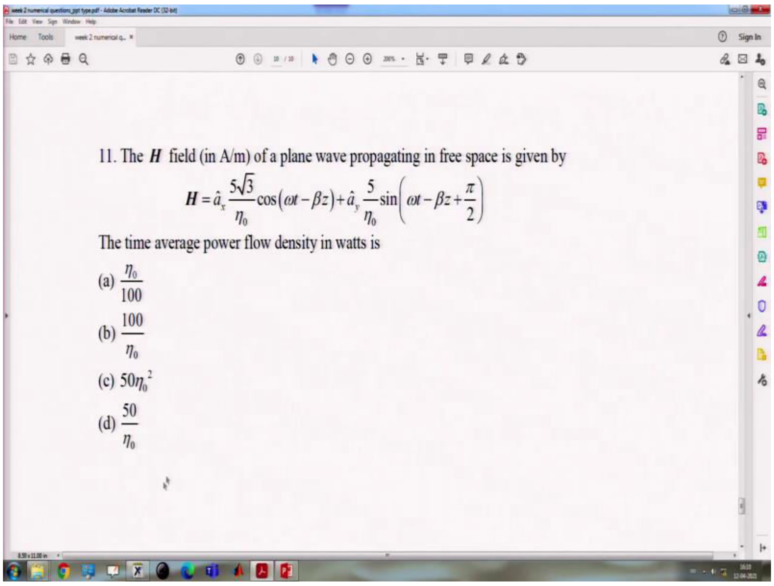


In the neck problem

11. The $\boldsymbol{H}$ field (in A/m) of a plane wave propagating in free space is given by

$$\boldsymbol{H} = \hat{a}_x \frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{a}_y \frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

The time average power flow density in watts is

(a) $\frac{\eta_0}{100}$

(b) $\frac{100}{\eta_0}$

(c) $50\eta_0^2$

(d) $\frac{50}{\eta_0}$

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$$= \frac{1}{2} Re\left[\frac{1}{\omega\mu}\left\{ a_z + 0 + 0 - a_z \right\}\right] = 0$$

$$\vec{H} = a_x \frac{5\sqrt{3}}{\eta_0} \cos(\omega t - \beta z) + a_y \frac{5}{\eta_0} \sin(\omega t - \beta z + \frac{\pi}{2})$$

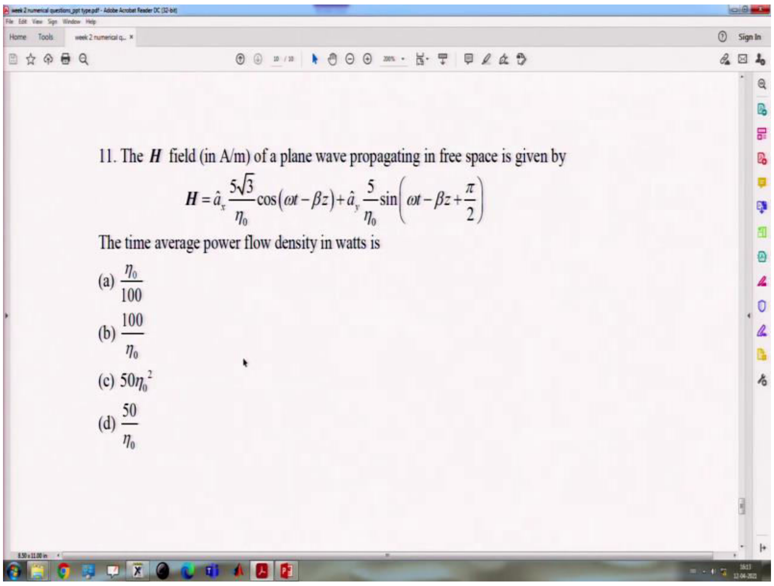
$$P = \frac{E_0^2}{2\eta_0}, \quad E_0 = \eta_0 H_0$$

$$= \frac{1}{2} \eta_0 H_0^2$$

$$H_0 = \sqrt{\left[\frac{5\sqrt{3}}{\eta_0}\right]^2 + \left[\frac{5}{\eta_0}\right]^2}$$

$$= \frac{1}{\eta_0}\sqrt{75 + 25} = \frac{10}{\eta_0}$$

$$P = \frac{1}{2}\cancel{\eta_0} \times \frac{100}{\eta_0^{\cancel{2}}} = \frac{50}{\eta_0} \text{ watts.}$$



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The time average power flow density in watts is

(a) $\frac{\eta_0}{100}$

(b) $\frac{100}{\eta_0}$

(c) $50\eta_0^2$

(d) $\frac{50}{\eta_0}$

Given:

$$\vec{H} = \hat{a}_x \frac{5\sqrt{3}}{\eta_0} \cos[\omega t - \beta z] + \hat{a}_y \frac{5}{\eta_0} \sin[\omega t - \beta z + \frac{\pi}{2}]$$

So time average power flow in watt in free space :

$$P = \frac{E_0^2}{2\eta_0}, \quad E_0 = \eta_0 H_0$$

$$= \frac{1}{2}\eta_0 H_0^2$$

So, after simplification we get:

$$H_0 = \sqrt{\left[\frac{5\sqrt{3}}{\eta_0}\right]^2 + \left[\frac{5}{\eta_0}\right]^2}$$

$$= \frac{1}{\eta_0}\sqrt{75 + 25} = \frac{10}{\eta_0}$$

$$P = \frac{1}{2}\cancel{\eta_0} \times \frac{100}{\eta_0^{\cancel{2}}} = \frac{50}{\eta_0} \text{ watt}$$

So, we get correct answer as 50 upon eta naught so, option will be d. So, like this, all these problems can be solved. Thank you.