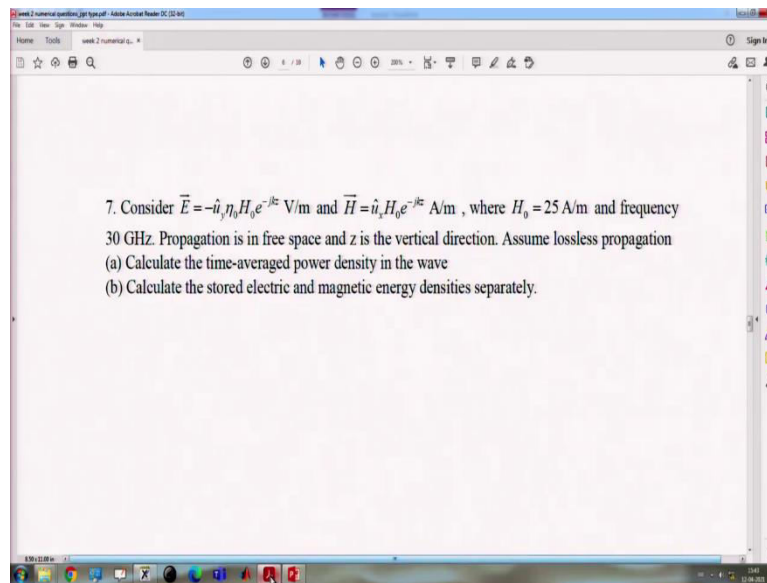


Advance Microwave Guided-Structure and Analysis
Professor Bratin Ghosh
Department of Electronic and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur
Lecture 18
Time – Harmonic form of Maxwell’s equations Tutorials

Welcome to the tutorial class. So, now, in this tutorial class, we will take the numerical problem on complex pointing vector.

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So, this is the first problem.

7. Consider $\vec{E} = -\hat{u}_y \eta_0 H_0 e^{-jkz}$ V/m and $\vec{H} = \hat{u}_x H_0 e^{-jkz}$ A/m, where $H_0 = 25$ A/m and frequency 30 GHz. Propagation is in free space and z is the vertical direction. Assume lossless propagation
(a) Calculate the time-averaged power density in the wave
(b) Calculate the stored electric and magnetic energy densities separately.

So, this is a first problem calculate the time average power density in the wave in the next part calculate the stored electric and magnetic energy density separately. So, let us do the solution.

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$2W | \omega_m - \omega_e | = -2J \quad (b)$

$\vec{H} = \hat{u}_x H_0 e^{-jkz} \text{ A/m}$

$H^* = \hat{u}_x H_0 e^{jkz}$

The time-averaged Poynting vector is

$$P_{av} = \frac{1}{2} \text{Re} [\vec{E} \times H^*]$$
$$= \frac{1}{2} \text{Re} [-\hat{u}_y \eta_0 H_0 e^{-jkz} \times \hat{u}_x H_0 e^{jkz}]$$
$$= \hat{u}_z \frac{1}{2} \eta_0 H_0^2 \text{ W/m}^2$$

The averaged power density is

$$P_{av} = \frac{1}{2} \eta_0 H_0^2 = \frac{1}{2} \times 377 \times 25^2$$
$$= 117.81 \times 10^3 \text{ W/m}^2$$

So, here it is given.

$$\vec{H} = \hat{u}_x H_0 e^{-jkz} \text{ A/m}$$
$$H^* = \hat{u}_x H_0 e^{jkz}$$

The time-averaged Poynting vector is

$$P_{av} = \frac{1}{2} \text{Re} [\vec{E} \times H^*]$$

So, E is given in the question, so above equation becomes:

$$= \frac{1}{2} \operatorname{Re} \left[-\hat{u}_y \eta_0 H_0 e^{-jkz} \times \hat{u}_z H_0 e^{+jkz} \right]$$

$$= \hat{u}_z \frac{1}{2} \eta_0 H_0^2 \quad \text{W/m}^2$$

Time average power density is

The averaged power density is

$$P_{\text{ave}} = \frac{1}{2} \eta_0 H_0^2 = \frac{1}{2} \times 377 \times 25^2$$

$$= 117.81 \times 10^3 \quad \text{W/m}^2$$

because in the question it is given, it is a loss less propagation and progression in the free space. So, free space wave impedance will be 377.

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(b)

$$w_m = \frac{1}{4} \mu_0 \vec{H} \cdot \vec{H}^*$$

$$= \frac{1}{4} \mu_0 \hat{u}_z H_0 e^{-jkz} \hat{u}_z H_0 e^{jkz} = \frac{1}{4} \mu_0 H_0^2$$

$$= \frac{1}{4} \times 4\pi \times 10^{-7} \times 25^2 = 1.963 \times 10^{-3} \quad \text{J/m}^3$$

$$w_e = \frac{1}{4} \epsilon_0 \vec{E} \cdot \vec{E}^* = \frac{\epsilon_0}{4} \left(-\hat{u}_y \eta_0 H_0 e^{-jkz} \right)$$

$$\begin{aligned}
 & \left(-\hat{u}_y \mu_0 H_0 e^{jkz} \right) \\
 &= \frac{\mu_0^2 \epsilon_0 H_0^2}{4} \\
 &= \frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4}
 \end{aligned}$$

$$\begin{aligned}
 & \left(-\hat{u}_y \mu_0 H_0 e^{jkz} \right) \\
 &= \frac{\mu_0^2 \epsilon_0 H_0^2}{4} \\
 &= \frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4} \\
 &= 1.963 \times 10^{-4} \text{ J/m}^3
 \end{aligned}$$

In the next part, stored electric and magnetic energy density are given by:

$$\begin{aligned}
 \omega_m &= \frac{1}{4} \mu_0 \vec{H} \cdot \vec{H}^* \\
 &= \frac{1}{4} \mu_0 \hat{u}_y H_0 e^{-jkz} \hat{u}_y H_0 e^{jkz} = \frac{1}{4} \mu_0 H_0^2 \\
 &= \frac{1}{4} \times 4\pi \times 10^{-7} \times 25^2 = 1.963 \times 10^{-3} \text{ J/m}^3
 \end{aligned}$$

$$\begin{aligned}
 w_e &= \frac{1}{4} \epsilon_0 \vec{E} \vec{E}^* = \frac{\epsilon_0}{4} \left(-\hat{u}_y \eta_0 H_0 e^{-jkz} \right) \\
 &\quad \left(-\hat{u}_y \eta_0 H_0 e^{jkz} \right) \\
 &= \frac{\eta_0^2 \epsilon_0 H_0^2}{4}
 \end{aligned}$$

Substituting the values, we get:

$$\begin{aligned}
 &\frac{377^2 \times 8.854 \times 10^{-12} \times 25^2}{4} \\
 &= 1.963 \times 10^{-4} \text{ J/m}^3
 \end{aligned}$$

So, this is the solution.

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8. In a nonmagnetic medium

$$\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$$

Find

- ϵ_r, η
- The time-average power carried by the wave
- The total power crossing 100 cm^2 of plane $2x + y = 5$

We can go to the next problem,

8. In a nonmagnetic medium

$$\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$$

Find

(a) ϵ_r, η

(b) The time-average power carried by the wave

(c) The total power crossing 100 cm^2 of plane $2x + y = 5$

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8 $\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$
 $\beta = 0.8, \omega = 2\pi \times 10^7, \mu = \mu_0$

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7} = \frac{12}{\pi}$$

$$\epsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7} = \frac{12}{\pi}$$

$$\epsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$= \frac{120\pi}{12} \pi = 10\pi^2 \Omega$$

Here E field is given,

$$\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$$

So, from this we can compare sine omega t minus beta x. So, after comparing

$$\beta = 0.8, \quad \omega = 2\pi \times 10^7, \quad \mu = \mu_0$$

So, this is the omega and beta and because this is a non-magnetic medium, So,

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$= \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7} = \frac{12}{\pi}$$

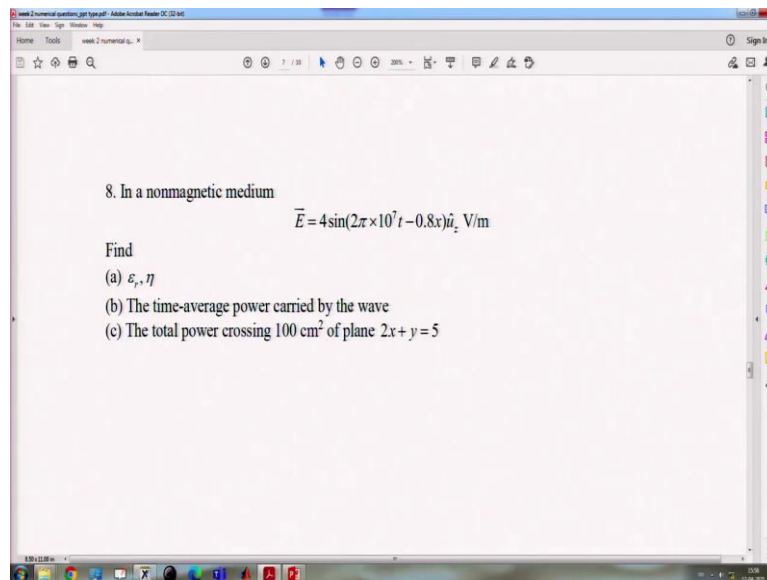
On solving we get:

$$\epsilon_r = 14.59$$

after getting this value, we can find out the wave impedance as:

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$
$$= \frac{120\pi}{12} \pi = 10\pi^2$$

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8. In a nonmagnetic medium

$$\vec{E} = 4\sin(2\pi \times 10^7 t - 0.8x)\hat{u}_z \text{ V/m}$$

Find

- ϵ_r, η
- The time-average power carried by the wave
- The total power crossing 100 cm^2 of plane $2x + y = 5$

Next part, time average power carried by the wave we can find out by:

$$P_{\text{ave}} = \frac{E_0^2}{2\eta} \hat{u}_{zc} = \frac{16}{2 \times 10\pi^2} \hat{u}_{zc}$$
$$= 81 \hat{u}_{zc} \text{ mW/m}^2$$

(Refer Slide Time: 15:00)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$= \frac{120\pi}{12} \pi = 10\pi^2 \Omega$$

$$P_{ave} = \frac{E_0^2}{2\eta} \hat{u}_z = \frac{16}{2 \times 10\pi^2} \hat{u}_z$$

$$= 81 \hat{u}_z \text{ mW/m}^2$$

8

$$\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$$

$$\beta = 0.8, \omega = 2\pi \times 10^7, \mu = \mu_0$$

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$= \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_r}$$

— βc $0.8 \times 3 \times 10^8$ 10

(Refer Slide Time: 16:14)

8. In a nonmagnetic medium

$$\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{u}_z \text{ V/m}$$

Find

- ϵ_r, η
- The time-average power carried by the wave
- The total power crossing 100 cm^2 of plane $2x + y = 5$

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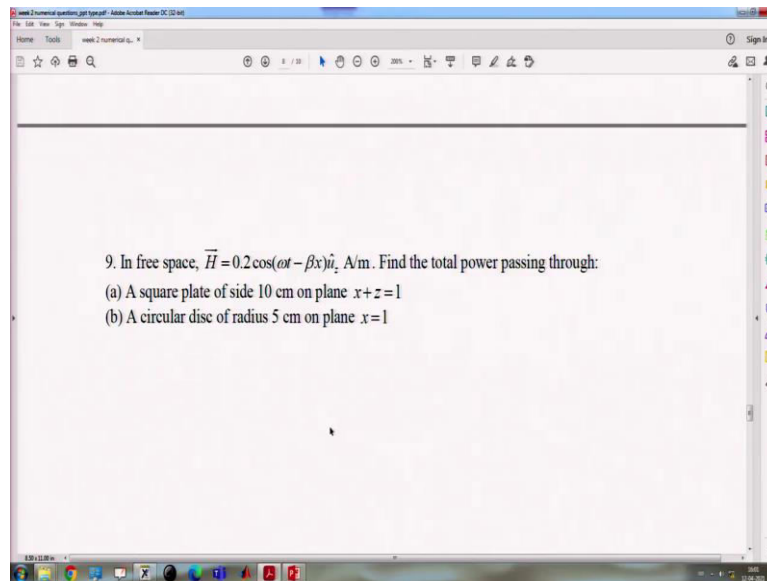
The image shows a handwritten derivation on a digital whiteboard. It starts with the equation of a plane, $2x + y = 5$. The unit normal vector \hat{u}_n is derived as $\frac{2\hat{u}_x + \hat{u}_y}{\sqrt{5}}$. Then, the power P_{ox} is calculated as $81 \times 10^{-3} \hat{u}_{sc} \cdot 100 \times 10^{-4} \cdot \left(\frac{2\hat{u}_x + \hat{u}_y}{\sqrt{5}} \right)$. This is simplified to $81 \times 10^{-3} \times 100 \times 10^{-4} \times \frac{2}{\sqrt{5}} = 724.5 \text{ MW}$.

Next part is total power crossing the crossing 100 a square centimetre of a plane 2x plus y equal to 5.

This image is a duplicate of the handwritten derivation shown above, containing the same equations and steps for finding the unit normal vector and calculating the power P_{ox} .

So, this will come 724.5 Macro Watt. So, this is the answer for the last part, we can go to the next problem.

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9. In free space, $\vec{H} = 0.2 \cos(\omega t - \beta x) \hat{u}_z$ A/m. Find the total power passing through:
(a) A square plate of side 10 cm on plane $x+z=1$
(b) A circular disc of radius 5 cm on plane $x=1$

We need to find out the total power passing through a square plate of side, this is a square plate of side 10 centimetre that means area will be 10 X 10 centimetre square, on the plan $x + z = 1$. So, first we have to find out the unit vector on this plane. So, unit vector on this plane will be.

$$\hat{u}_n = \frac{\hat{u}_x + \hat{u}_z}{\sqrt{2}}$$

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$$\begin{aligned}
 \textcircled{9} \quad |a| \quad \hat{u}_n &= \frac{\hat{u}_{0z} + \hat{u}_2}{\sqrt{2}} \\
 P_+ &= \frac{1}{2} \epsilon_0 H_0^2 \hat{u}_{0z} \cdot \frac{\hat{u}_{0z} + \hat{u}_2}{\sqrt{2}} 100 \times 10^{-4} \\
 &= \frac{1}{2} \times 120\pi \times 0.2^2 \times \frac{1}{\sqrt{2}} \times 10^{-2} = 53.31 \text{ mW}
 \end{aligned}$$

9. In free space, $\vec{H} = 0.2 \cos(\omega t - \beta x) \hat{i}$, A/m. Find the total power passing through:

(a) A square plate of side 10 cm on plane $x+z=1$

(b) A circular disc of radius 5 cm on plane $x=1$

And the total power passing through the square plate of side 10 centimetre:

$$\begin{aligned}
 P_+ &= \frac{1}{2} \epsilon_0 H_0^2 \hat{u}_{0z} \cdot \frac{\hat{u}_{0z} + \hat{u}_2}{\sqrt{2}} 100 \times 10^{-4} \\
 &= \frac{1}{2} \times 120\pi \times 0.2^2 \times \frac{1}{\sqrt{2}} \times 10^{-2} = 53.31 \text{ mW}
 \end{aligned}$$

So, this will come after calculation 53.31 milli watt. So, this is the answer for the first part.

(Refer Slide Time: 22:24)

$$P_t = \frac{1}{2} n_0 H_0^2 \frac{U_z^2}{\sqrt{2}} = 100 \times 10$$
$$= \frac{1}{2} \times 120\pi \times 0.2^2 \times \frac{1}{\sqrt{2}} \times 10^{-2} = 53.31 \text{ mW}$$

b)

$$P_t = \frac{1}{2} n_0 H_0^2 \hat{U}_z \cdot \hat{U}_z \cdot \pi (0.05)^2$$
$$= \frac{1}{2} \times 120\pi (0.2)^2 \pi 0.05^2 = 59.22 \text{ mW}$$

9. In free space, $\vec{H} = 0.2 \cos(\omega t - \beta x) \hat{i}_z$ A/m. Find the total power passing through:

(a) A square plate of side 10 cm on plane $x+z=1$

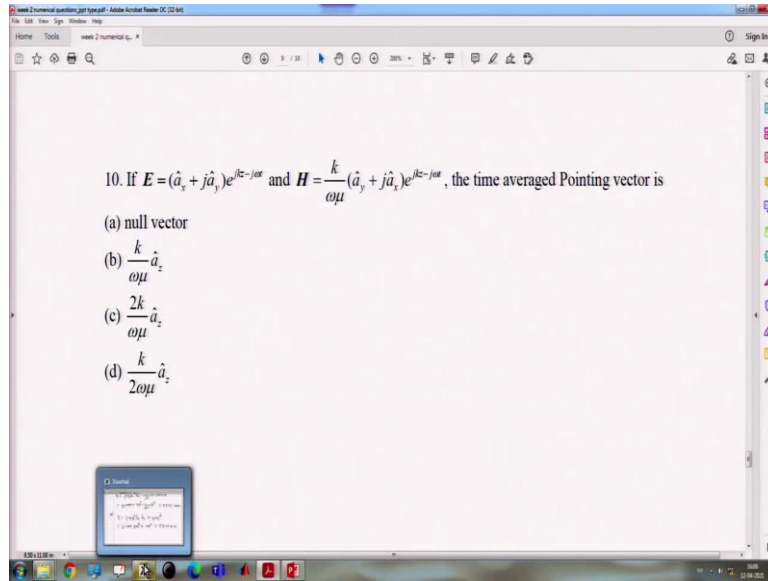
(b) A circular disc of radius 5 cm on plane $x=1$

Similarly, for the next part in the b part, in the b part it is given, a circular disc of radius 5 centimetre on the plane $x = 1$. so, on the plane $x = 1$, we have to find out P_t .

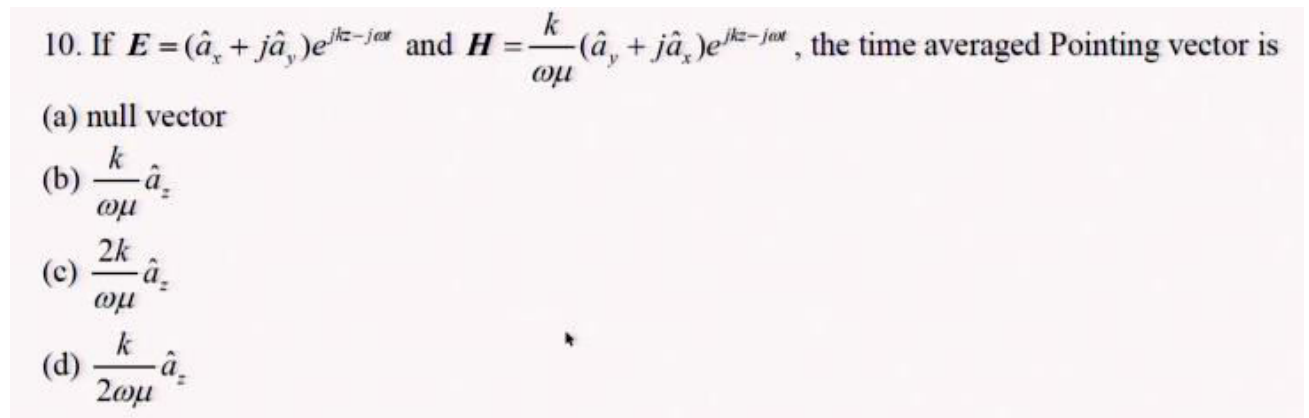
$$P_t = \frac{1}{2} n_0 H_0^2 \hat{U}_z \cdot \hat{U}_z \cdot \pi (0.05)^2$$
$$= \frac{1}{2} \times 120\pi (0.2)^2 \pi 0.05^2 = 59.22 \text{ mW}$$

after simplification it will come 59.22 milli watt. So, this is the answer for the b part.

(Refer Slide Time: 24:15)



So, in the next problem it is given:



So, here options are given, so I have to find out time average pointing vector. So, time average pointing vector is given as:

$$\begin{aligned}
 P_{avg} &= \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^*] \\
 &= \frac{1}{2} \operatorname{Re} \left[(\hat{a}_x + j\hat{a}_y) e^{jkz-j\omega t} \times \frac{k}{\omega\mu} (\hat{a}_y - j\hat{a}_x) e^{-jkz+j\omega t} \right] \\
 &= \frac{1}{2} \operatorname{Re} \left[\frac{k}{\omega\mu} \{ a_z + 0 + 0 - a_z \} \right] = 0
 \end{aligned}$$

(Refer Slide Time: 24:53)

$$\begin{aligned} \textcircled{10} \quad P_{avg} &= \frac{1}{2} \operatorname{Re} \left[\vec{E} \times \vec{H}^* \right] \\ &= \frac{1}{2} \operatorname{Re} \left[(\hat{a}_x + j\hat{a}_z) e^{jkz - j\omega t} \times \frac{k}{\omega\mu} [a_y - ja_z] e^{-jkz + j\omega t} \right] \\ &= \frac{1}{2} \operatorname{Re} \left[\frac{k}{\omega\mu} \right] a_z + ja_z \end{aligned}$$

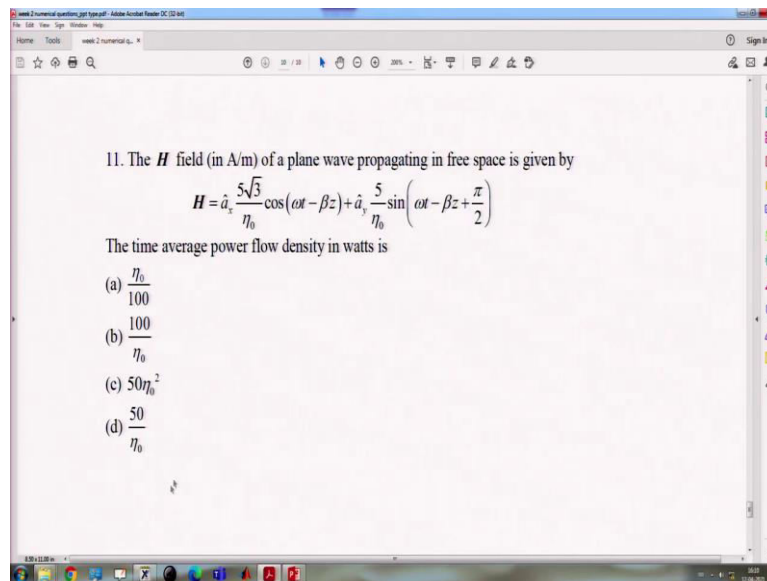
10. If $E = (\hat{a}_x + j\hat{a}_z)e^{jkz - j\omega t}$ and $H = \frac{k}{\omega\mu}(\hat{a}_y + j\hat{a}_z)e^{jkz - j\omega t}$, the time averaged Poynting vector is

- (a) null vector
- (b) $\frac{k}{\omega\mu} \hat{a}_z$
- (c) $\frac{2k}{\omega\mu} \hat{a}_z$
- (d) $\frac{k}{2\omega\mu} \hat{a}_z$

$$\begin{aligned} \textcircled{10} \quad P_{avg} &= \frac{1}{2} \operatorname{Re} \left[\vec{E} \times \vec{H}^* \right] \\ &= \frac{1}{2} \operatorname{Re} \left[(\hat{a}_x + j\hat{a}_z) e^{jkz - j\omega t} \times \frac{k}{\omega\mu} [a_y - ja_z] e^{-jkz + j\omega t} \right] \\ &= \frac{1}{2} \operatorname{Re} \left[\frac{k}{\omega\mu} \left\{ a_z + 0 + 0 - a_z \right\} \right] = 0 \end{aligned}$$

So, option will be a null vector.

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11. The \mathbf{H} field (in A/m) of a plane wave propagating in free space is given by

$$\mathbf{H} = \hat{a}_x \frac{5\sqrt{3}}{\eta_0} \cos(\omega t - \beta z) + \hat{a}_y \frac{5}{\eta_0} \sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

The time average power flow density in watts is

- (a) $\frac{\eta_0}{100}$
- (b) $\frac{100}{\eta_0}$
- (c) $50\eta_0^2$
- (d) $\frac{50}{\eta_0}$

In the neck problem

11. The \mathbf{H} field (in A/m) of a plane wave propagating in free space is given by

$$\mathbf{H} = \hat{a}_x \frac{5\sqrt{3}}{\eta_0} \cos(\omega t - \beta z) + \hat{a}_y \frac{5}{\eta_0} \sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

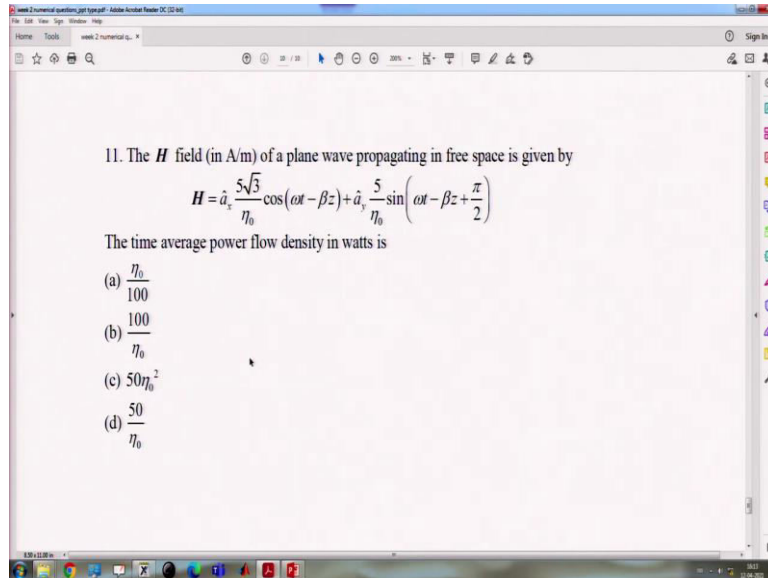
The time average power flow density in watts is

- (a) $\frac{\eta_0}{100}$
- (b) $\frac{100}{\eta_0}$
- (c) $50\eta_0^2$
- (d) $\frac{50}{\eta_0}$

(Refer Slide Time: 28:36)

$$= \frac{1}{2} \operatorname{Re} \left[\frac{1}{\omega \mu} \{ a_2 + 0 + 0 - a_2 \} \right] = 0$$
$$\vec{H} = \hat{a}_x \frac{5\sqrt{3}}{\eta_0} \cos[\omega t - \beta z] + \hat{a}_y \frac{5}{\eta_0} \sin[\omega t - \beta z + \frac{\pi}{2}]$$
$$\rho = \frac{E_0^2}{2\eta_0}, \quad E_0 = \eta_0 H_0$$
$$= \frac{1}{2} \eta_0 H_0^2$$

$$H_0 = \sqrt{\left[\frac{5\sqrt{3}}{\eta_0} \right]^2 + \left[\frac{5}{\eta_0} \right]^2}$$
$$= \frac{1}{\eta_0} \sqrt{75 + 25} = \frac{10}{\eta_0}$$
$$\rho = \frac{1}{2} \eta_0 \times \frac{100}{\eta_0^2} = \frac{50}{\eta_0} \text{ watts.}$$



Given:

$$\vec{H} = \hat{a}_{xc} \frac{5\sqrt{3}}{\eta_0} \cos[\omega t - \beta z] + \hat{a}_y \frac{5}{\eta_0} \sin[\omega t - \beta z + \frac{\pi}{2}]$$

So time average power flow in watt in free space :

$$P = \frac{E_0^2}{2\eta_0}, \quad E_0 = \eta_0 H_0$$

$$= \frac{1}{2} \eta_0 H_0^2$$

So, after simplification we get:

$$H_0 = \sqrt{\left[\frac{5\sqrt{3}}{\eta_0}\right]^2 + \left[\frac{5}{\eta_0}\right]^2}$$

$$= \frac{1}{\eta_0} \sqrt{75 + 25} = \frac{10}{\eta_0}$$

$$P = \frac{1}{2} \eta_0 \times \frac{100}{\eta_0^2} = \frac{50}{\eta_0} \text{ watt}$$

So, we get correct answer as 50 upon eta naught so, option will be d. So, like this, all these problems can be solved. Thank you.