Advanced Microwave Guided-Structures and Analysis

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Lecture No. 21

Radiation from an Electric Current Source (Contd.)

Welcome to this lecture on the continuation of the radiation from an electric current source in a homogeneous medium. We have covered the derivation of Helmholtz equation from the electric current source, through the mediary of the vector potential. Now, we are going to investigate how the Helmholtz equation, which is a very central equation in electromagnetics, helps us to calculate the electric and magnetic fields radiated by the current source.

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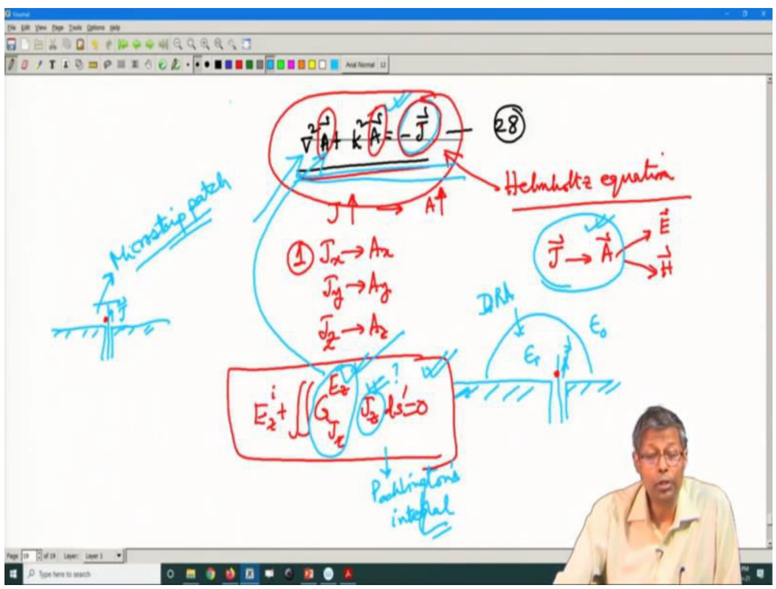

So, we found that the Helmholtz equation is given by $\nabla^2\vec{A}+k^2\vec{A}=\vec{J}$; this one was equation 28. We derived it in the last lecture. Now, you see one of the important characteristics of Helmholtz equation is that the polarization of **J** and the direction of **A**, they are the same.

So, **J** and **A** must point in the same direction because of the nature of this equation. If **J** is x directed, **A** will be x directed; if **J** is y directed, **A** will be y directed; and if **J** is z directed, **A** will be z directed, this is the meaning of this. That is the first characteristics of the Helmholtz equation, which you should remember.

Now, in a field computation problem, as you can see in this architecture that from **J** to **A**, **A** to electric and magnetic field. As I told you this is the step, which takes the maximum amount of resource. In fact, this step is the solution to the Helmholtz equation. So, this Helmholtz equation is actually an equation; this Helmholtz equation is solved under the imposition of the boundary conditions to the problem.

So, therefore, **A** is found from this current source **J**, by the imposition of boundary conditions; frequently, this **J** however is not explicitly known. For example, we have told you that for, for instance in this kind of problem. A hemispherical dielectric resonator antenna is fed by a coaxial probe; this is resting on a ground plane. This is the current source **J**, which is not known to me; so is also the case in a microstrip patch.

This is the ground plane, this is the current source **J**; this **J** is not known to me. So, what we will do? We will take a point source here and find out the response of this point source, which we already discussed in one of our previous lectures; and we call that the Green's function. So, therefore, we substitute for this current source **J,** a point source; and we find out the value of this vector potential **A**; that is the Green's function corresponding to the vector potential **A**.

What does it contain? It contains all information about the boundaries, or it contains all the information about the boundary condition surrounding the problem. We have assumed the source and we have kept the boundary intact. The solution to the vector potential A from the Helmholtz equation contains all the information about the boundary conditions to the problem.

Once that Green's function is obtained, the Green's function corresponding to the vector potential. Once that is that is obtained, we can apply the Pocklington's integral as we said before to find out the fields which is for here, it will be Ezi**,** plus double integral G_{Jz}EzJz ds' equal to 0.

So, once this Green's function is obtained, we can evaluate these true currents, by the imposition of the Pocklington's integral. This is not a major problem, because we can easily find out the value of Jz through the method of moments. The main problem is finding out this; and for this, the main problem is finding out the value of **A**. Because once the value of **A** is found out, we can find out all the electric and magnetic fields, which we will soon see.

So therefore, the solution to an electromagnetic field problem, basically is the solution to the Helmholtz equations along with all the boundary conditions for a unit impulse source, which is the Green's function to the problem. Because of this many scholarly works, many papers, many many scripts in this area will title their work as Green's function for this situation. Green's function for a coaxial probe excited dielectric resonator antenna, which is this one.

We stop at the Green's function, because once the Green's function is found out; the true current here can be easily found out. Therefore, Helmholtz solution as I said solution of Helmholtz equation is of ultimate significance in electromagnetics.

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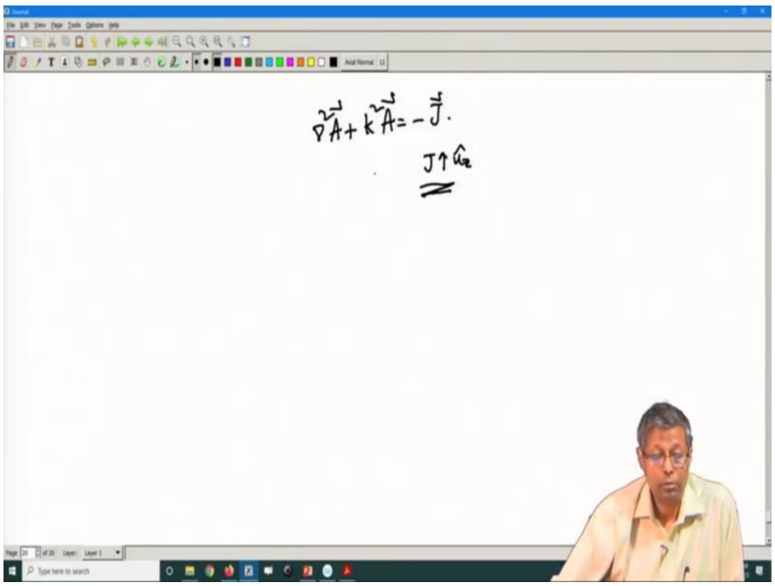

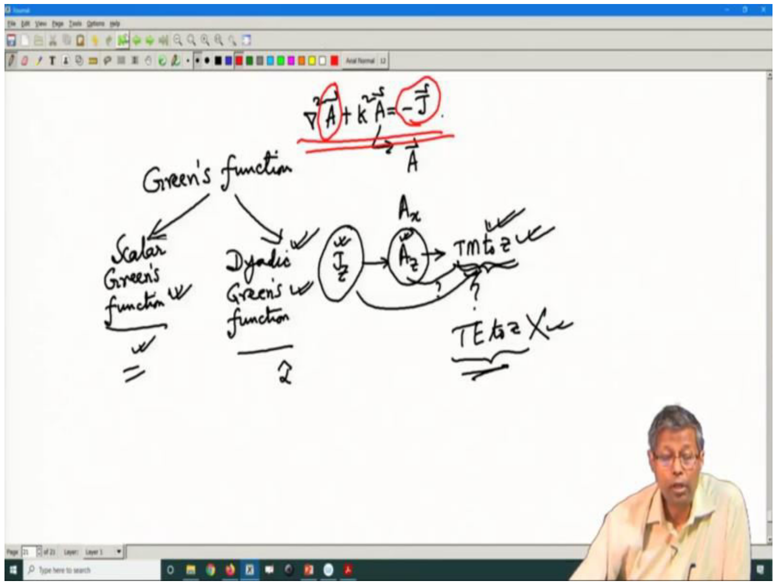

The other important significance of Helmholtz equation comes from the fact that for a particular polarization of this source. Say, if this source is z directed; it will generate a particular nature of field, so I repeat this. If **J** is oriented in a particular way, if **J** is z directed; it will generate a particular nature of field. Let us say a transverse magnetic to z mode, so a particular mode. So, therefore, the orientation of **A,** which is reflecting the orientation of **J** will generate a particular mode.

Let me write down the Helmholtz equation once again. The other significance of Helmholtz equation is that a particular orientation of **A** is going to generate a particular orientation of field, or a particular type of field. So, for example, if **A** is z directed; it will generate the transverse magnetic to z mode. We will come to this. But, **A** z directed means also that **J** is z directed.

So, z directed **J** generates a z directed **A**; and z directed **A** generates TM to z mode. How!! we do not yet know? But let us assume that to be true. We will come to that later on. But, the most important thing to appreciate at this point is that there is a linkage between the direction of **A** and the type of mode.

Why is this significant? This is significant, because given a direction of **J**; I know it will excite a particular orientation of **A**. And I know a priori that this kind of mode is going to be generated. So, I need not compute the amplitude of the TE to z mode, if **J** is z directed; so, this need not to be computed. So, I get away by computation of only the TM to z mode.

This in fact breaks down the Green’s function group into two parts. One is called the scalar Green's function, and another is called the dyadic Green's function. People working on the scalar Green's functions always try to take advantage of the fact, that a particular orientation of the source will generate a particular nature of modes.

Their main purpose is to look at the polarization of the source, and to see how a few number of modes they can compute. So, a priori if they know that these modes are not generated; they will not compute these modes. So, they always try to exploit the behavior of polarization of the source.

The people working on the dyadic Green's function group do not care about this behavior; given a source they will evaluate all possible modes. And the reason they would do that is that their code will become much more general. So, that now if my source polarization changes, the scalar potential group will have to recompute the fields; because my source polarization has changed.

So, all my modes, which I might have neglected before, now have to be computed; so, my problem has to start from scratch. But, the dyadic Green's function group because they have taken into account all the modes; their code is general. They do not care if their sources, their source polarization has changed. In fact, they deal with an alpha directed source.

So, while dyadic Green's function group, the code written by them is general; the scalar Green's function group, their code written by them is quicker. The computation time is less; because they do not calculate other modes which are not relevant. There is no question about which is good and which is bad. It is dependent on what you want, which situation you want. Whether you want your code to be more general, or you want your code to be fast.

So, the essential importance of the Helmholtz equation lies in the fact that our sole purpose now is to calculate **A**. Given a current source **J** or a point source **J,** under the given boundary conditions. But, now let us find out how we calculate the fields from this **A**?

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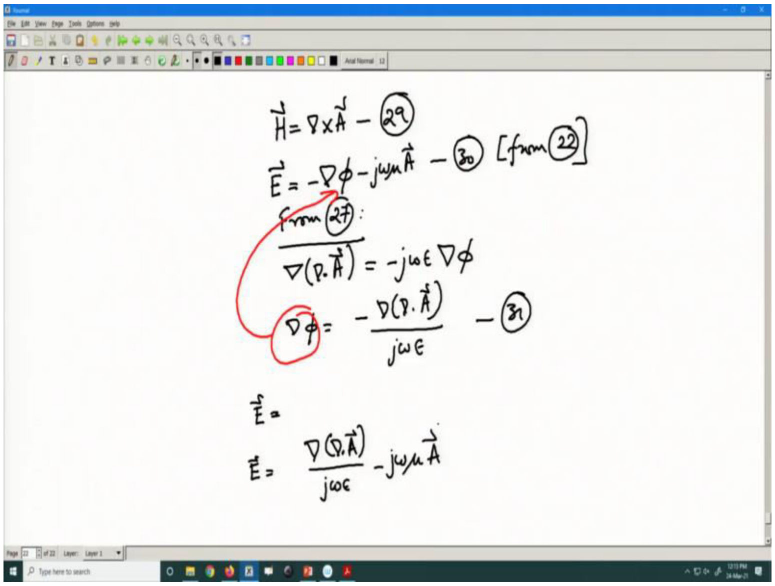

So, for that we first of all note that **H** is already given by curl of **A**. So, the value of **H** is immediately calculated, once the value of **A** is known. The value of **E** is also calculated because **E** is given by minus grad of phi, minus j omega mu **A**. This we noted from 22 and **A**, and phi are linked through 27. So, from 27, we have grad of divergence **A** equal to minus j omega epsilon grad of phi.

Therefore, grad of phi will be equal to minus grad of divergence **A**, divided by j omega epsilon; we call this 31. And then all we have to do is substitute this grad of phi from here to there. So, my electric field will be grad of divergence **A** by j omega epsilon, minus j omega mu **A**.

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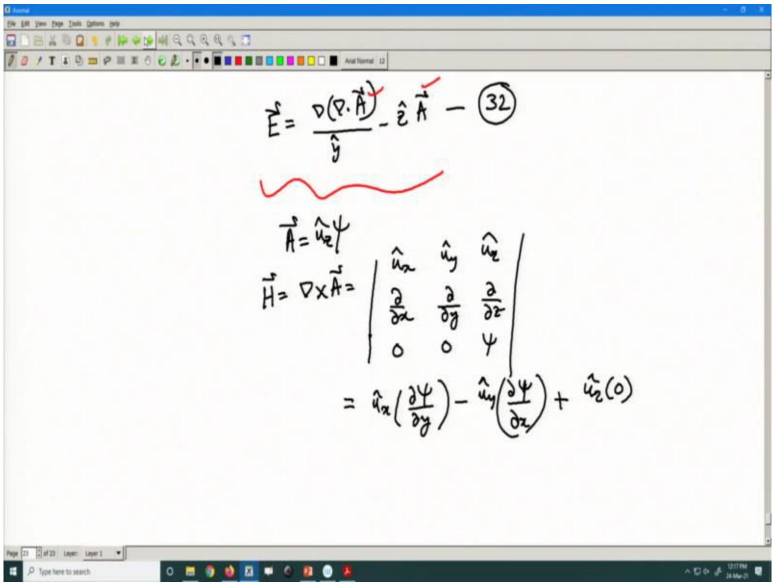

Or I can write this as **E** equal to grad of divergence **A** by y cap, minus z cap **A**; so we call this equation 32. So, we immediately find that the electric and magnetic field can be found out readily from A, from equation 32 for the electric field, and from equation 29 for the magnetic field. That is an immediate process.

Now, you can appreciate that once the vector potential **A** is found out the electric field can be readily found out. So, now you appreciate that the main step in Helmholtz equation is finding out the magnetic vector potential given the current source **j**, under the given boundary conditions surrounding the problem and surrounding Helmholtz equation.

Now, as we were saying regarding the step that when **A** has a particular orientation. It has a linkage to the type of mode it excites. How this is verified and where is this relevant? So, let us suppose in that regard that A has only a z component; **A** equal to uz psi. Then, we can calculate **H** as curl of **A** readily and that will be ux, uy, uz; del del x, del del y, del del z, 0, 0, psi. And that will be ux del psi del y, minus uy del psi del x, plus uz times 0; so this is chi.

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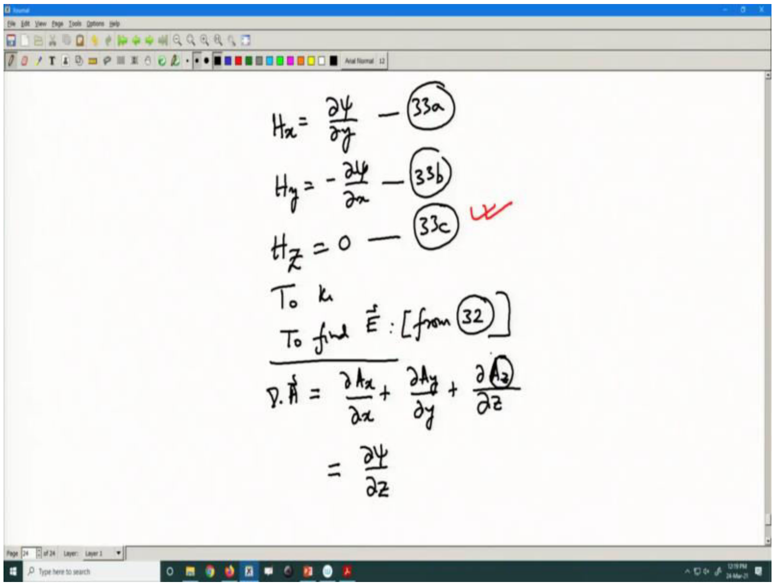

So, therefore from here we get Hx is del psi del y; we call this 33a. Hy is minus del psi del x; we call this 33b; and Hz is 0, we call this 33c. Note this particularly that the z component of **H** is 0; so we similarly note that to find out, we now note that to find **E**, we have to use equation 32; and the first thing we have to do is find out divergence of **A**. And that is del Ax del x, plus del Ay del y, plus del Az del z; and that is equal to del psi del z, because A has only z component.

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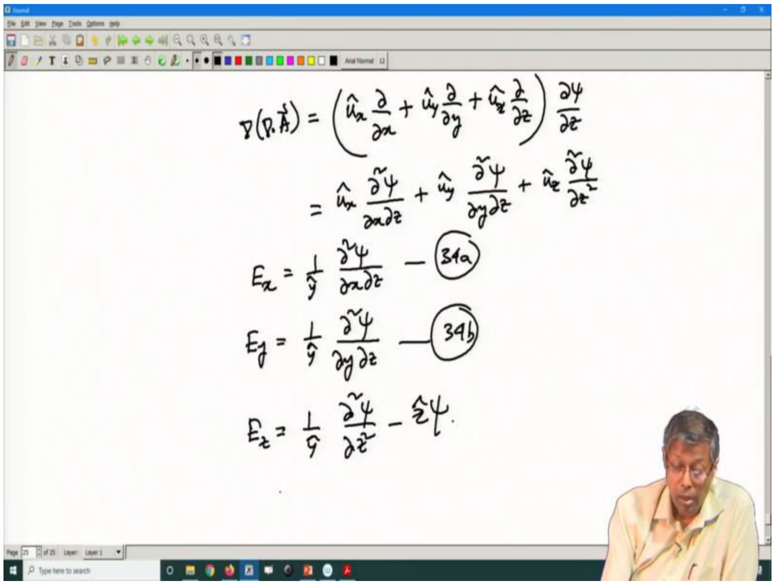

Similarly, grad of divergence of **A** is ux del del x, plus uy del del y, plus uz del del z for the grad part; and the divergence **A** is del psi del z. So, that is going to be ux del square psi del x del z, plus uy del square psi del y del z, plus uz del square psi del z square. So, therefore, from 32 we get Ex equal to 1 by y cap, del square psi del x del z, which we call 34a. We get Ey as 1 by y cap del square psi del y del z; we call this as 34b. And we get Ez as 1 by y cap, del square psi del z square minus z cap psi; because it is z cap **A** and **A** has only a z component.

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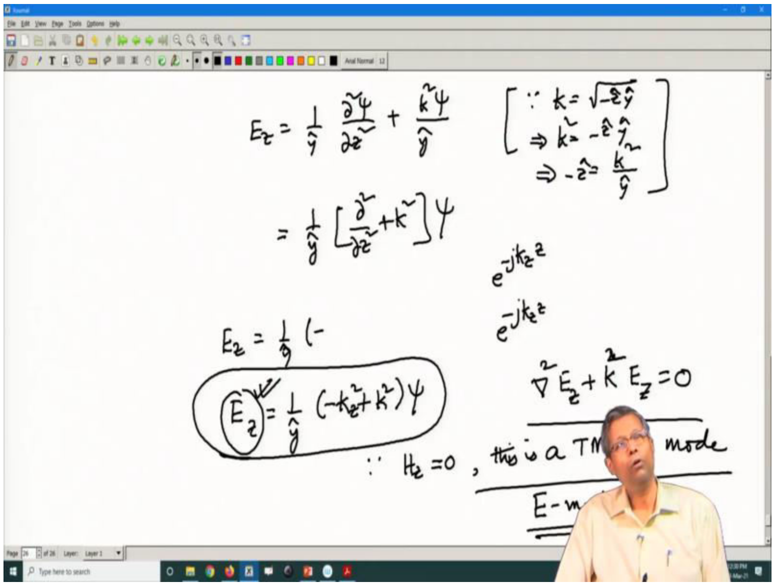

So, Ez can be further simplified as 1 by y cap, del square psi del z square, plus k square psi by y cap. Since, k is equal to minus z cap y cap, which means k square is minus z cap y cap; and which means minus z cap equal to k square by y cap. So, Ez becomes equal to 1 by y cap del square del z square, plus k square psi. Now, if I assume e to the power minus j kz z variation then Ez can be written as 1 by y cap minus kz square plus k square psi.

Now, we first note that as we noted previously that Hz is 0, out of all the six components Hz is 0. Therefore, since Hz is 0, this is a TM to z mode. It is also called the E mode because this component of the electric field is directly proportional to psi. It will independently satisfy the source free wave equation; because psi satisfies the source free wave equation. It is a solution to the source free wave equation; therefore, Ez is also going to satisfy the source free wave equation. So, therefore we find that the component Ez, because it satisfies the source free wave equation; it is also called the TM to z mode, is also called E-mode.

So, now we see that for a z directed current source, which generates z directed magnetic vector potential; we get our TM to z mode. We will now see the practical application of this, other than a simplification resulting from the no necessity to compute irrelevant modes, when we are given a particular orientation of the source.

Other than this, when we apply this to a guided structure, we first of all try to find out whether for the given boundary condition of the problem in a source free case the fields given by the TM to z or the TE to z. We will see that in the rectangular waveguide for instance, the TM to z modes, and the TE to z modes form valid mode sets. What do we understand by valid mode sets? That they satisfy the boundary conditions of the rectangular waveguide.

But, the same modes the TE to z of the TM to z will not satisfy the boundary conditions for a partially filled dielectric wave. There we have to invoke another type of mode set, depending on the type of the discontinuity or the type of the boundary condition of the dielectric.

So, like the TM to z mode, if I would have chosen **A** is equal to ux psi I would have come to the TM to x mode, where Ex is 0. Or, if I if I had chosen **A** equal to uy psi; I would have come across the fields of the type which are TM to y, in which Hy is 0.

So therefore these boundary with these types of sources, the existence of only Az, which generates a TM to z mode. The existence of only Ax which generates a TM to x mode; or the existence of only Ay, which generates a TM to y mode. So, these modes we have to analyze a given guided structure; apply the boundary conditions on the fields for these modes. And then deduce whether that kind of mode, whether it is the TM to z or the TM to x, or the TM to y; they form valid mode sets. So, this is the significance of Helmholtz equation.

So, we stop our lecture here; we will continue from here.