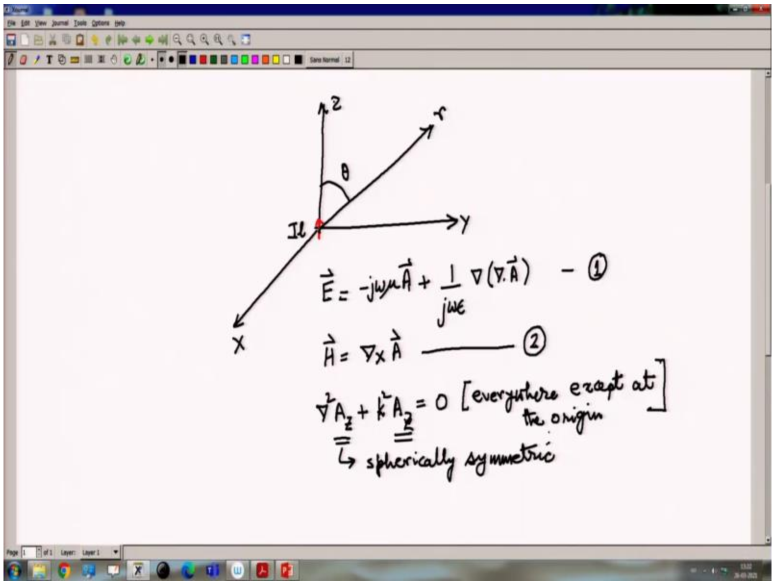
Advanced Microwave Guided-Structures and Analysis
Professor. Bratin Ghosh
Department of E & ECE
Indian Institute of Technology, Kharagpur
Lecture No. 22
Radiation from an Electric Current Source (Contd.)

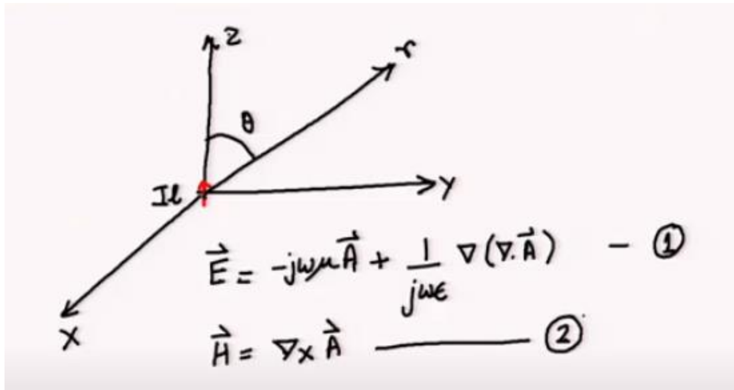
Welcome to the session; it is a continuation session on the radiation from an electric current source in a homogeneous medium. We had been discussing about that and let us now derive the fields radiated by a current source, the spherical components of the field radiated by the current source in a homogeneous medium.

(Refer Slide Time: 0:40)



So, we consider a xyz coordinate system; so this is my x axis, this is my y axis, this is my z axis. This is my point source, a point electric current source Il, this is the radial direction, and this is my angle theta.

Let us write down the equations for the electric and magnetic fields, that we derived in terms of the magnetic vector potential.



let us call this equation number 2.

So, now because the current source Il is z directed as we discussed before; the magnetic vector potential A is also be z directed. And it will satisfy the source free equation everywhere except at the origin; because at the origin, the source Il is there; the electric current source Il is there. So, everywhere except the origin, the magnetic vector potential A which will be z directed, is going to satisfy the source free wave equation.

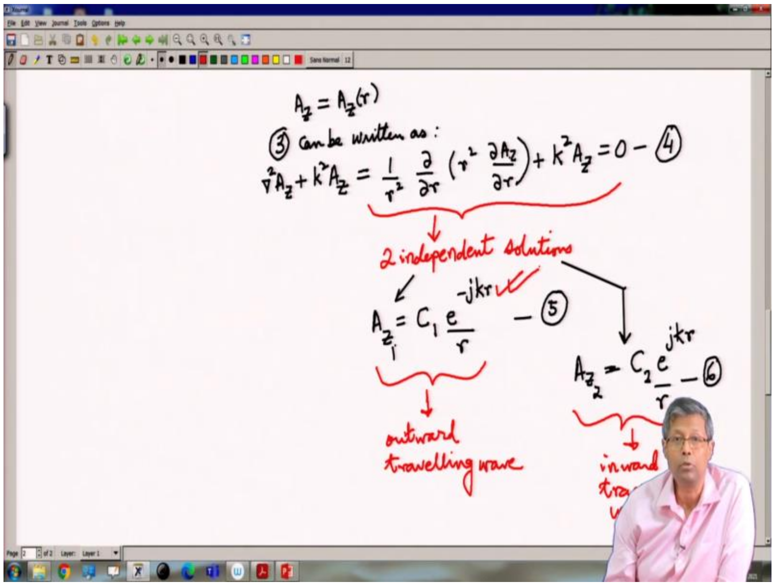
So, we will have

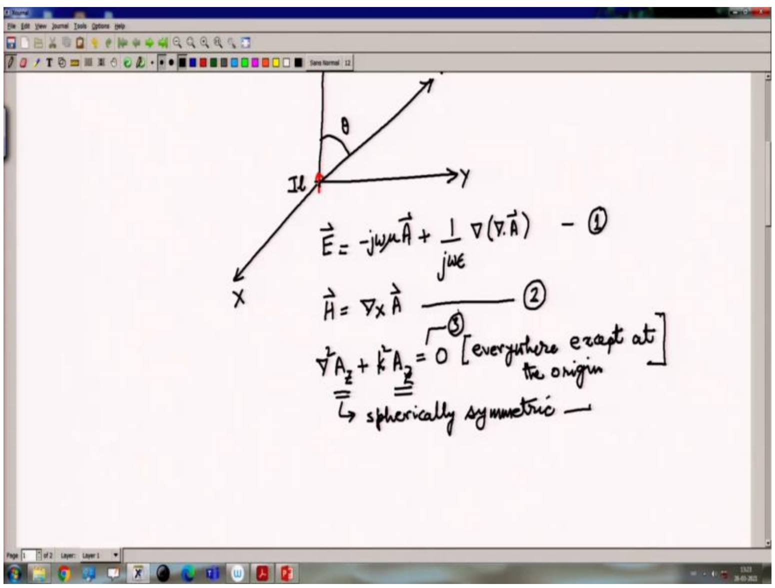
$$\nabla^2 A_z + k^2 A_z = 0 \quad \text{[everywhere except at the origin]}$$

↳ spherically symmetric

so it is going to be spherically symmetric.

(Refer Slide Time: 5:43)





Let us therefore call

$$A_z = A_z(r)$$

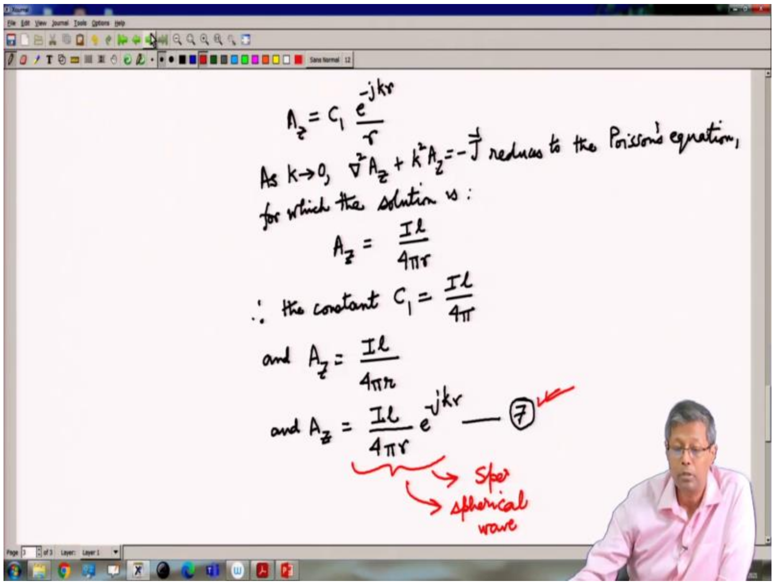
③ can be written as :

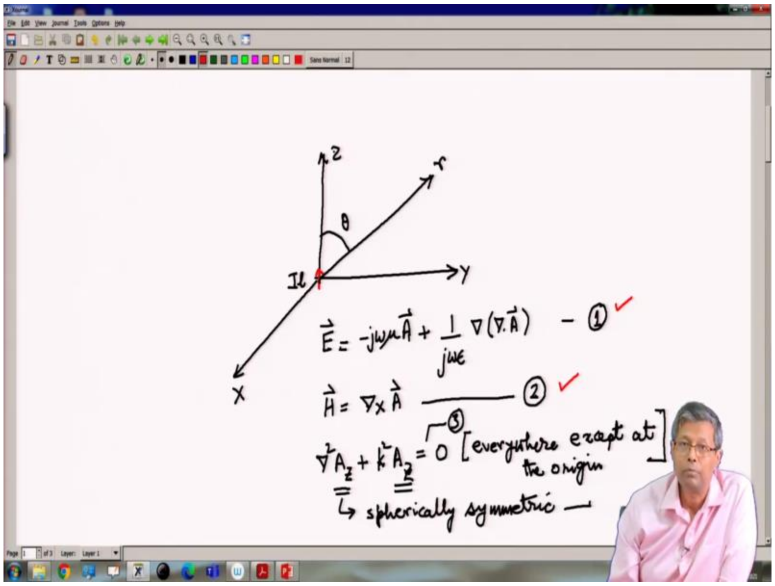
$$\nabla^2 A_z + k^2 A_z = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial A_z}{\partial r}\right) + k^2 A_z = 0 \quad - (4)$$

We call this equation 4. So, this differential equation-4 has 2 independent solutions.

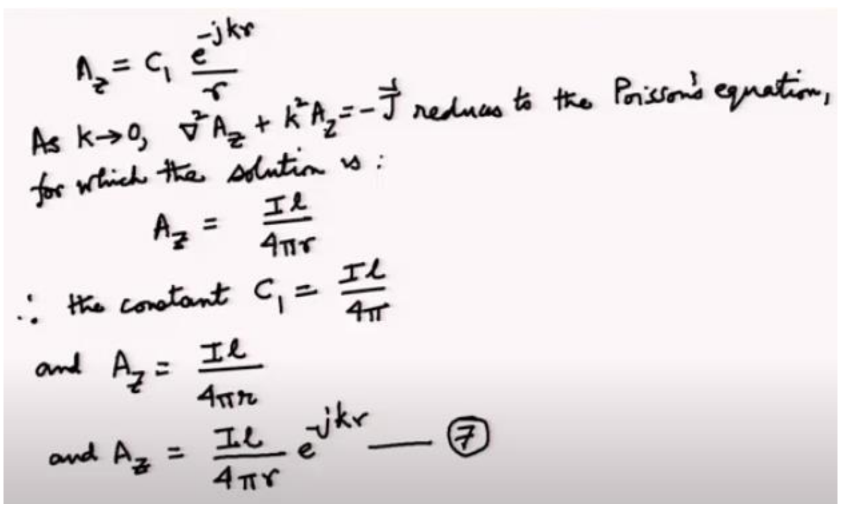
It has 2 independent solutions; one is Az1 is $A_{z1} = C_1 \frac{e^{-jkr}}{r}$; we call this equation 5. And another is Az2 as $A_{z2} = C_2 \frac{e^{+jkr}}{r}$; we call this equation 6. So, this solution represents an outward travelling wave; and this solution represents an inward travelling wave. So, because we are dealing with the outward travelling wave or radiation from the point source, then we will choose the solution $A_{z1} = C_1 \frac{e^{-jkr}}{r}$.

(Refer Slide Time: 10:56)





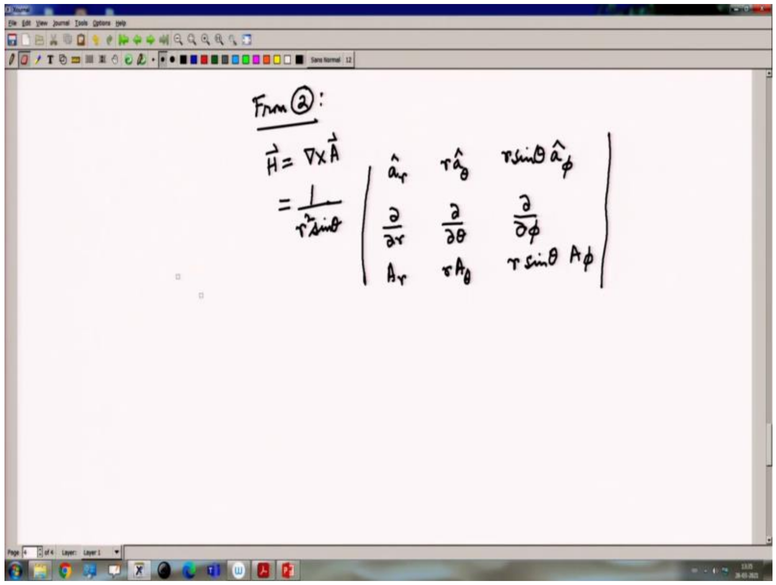
So, therefore, Az is $A_{z1} = C_1 \frac{e^{-jkr}}{r}$. Now, as k tends to 0, this equation



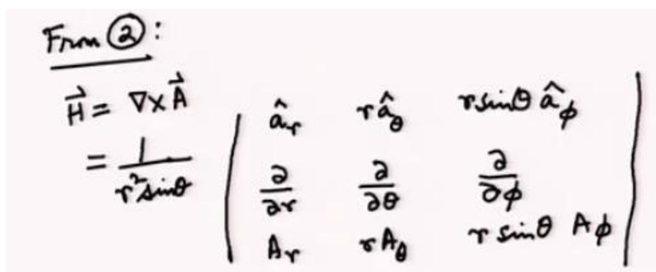
So, this equation 7 is the equation of a spherical wave as the surfaces of constant phases are spheres.

So, this is the equation for a spherical wave; so, this outward travelling wave is a spherical wave. So, now the electric and magnetic fields can be obtained by substituting equation 7 into equations 1 and 2 which denote the relationship between the electric field with the magnetic vector potential and the relationship of the magnetic field with the magnetic vector potential. So, let us obtain how we obtain the radiated fields from this current source.

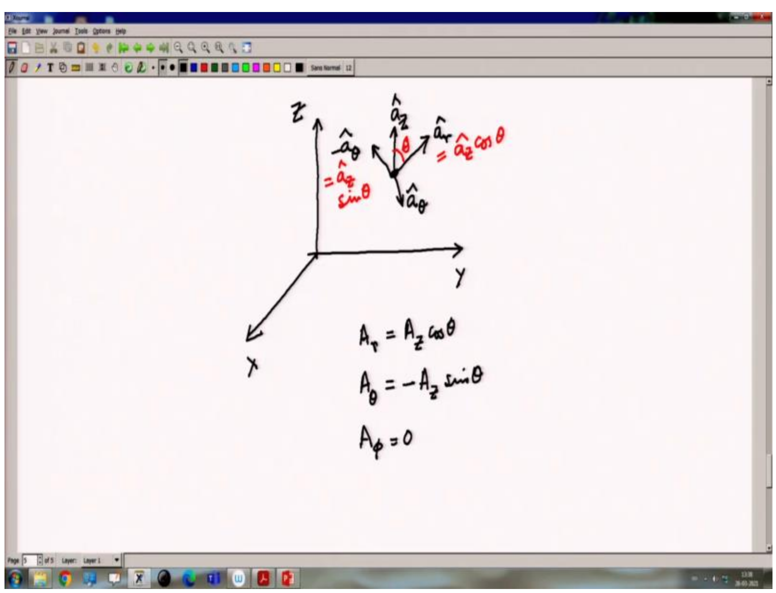
(Refer Slide Time: 16:48)



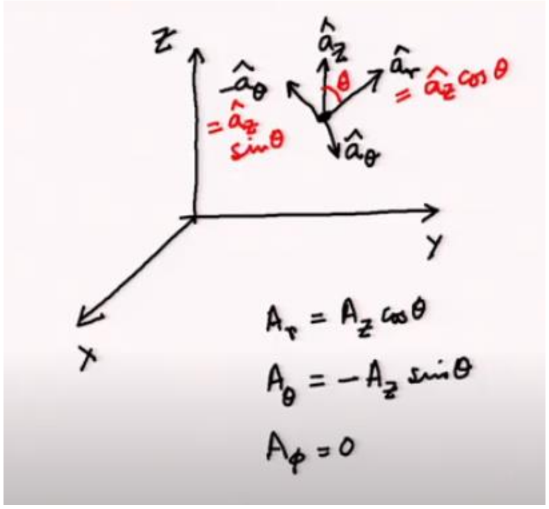
So, we have from 2,



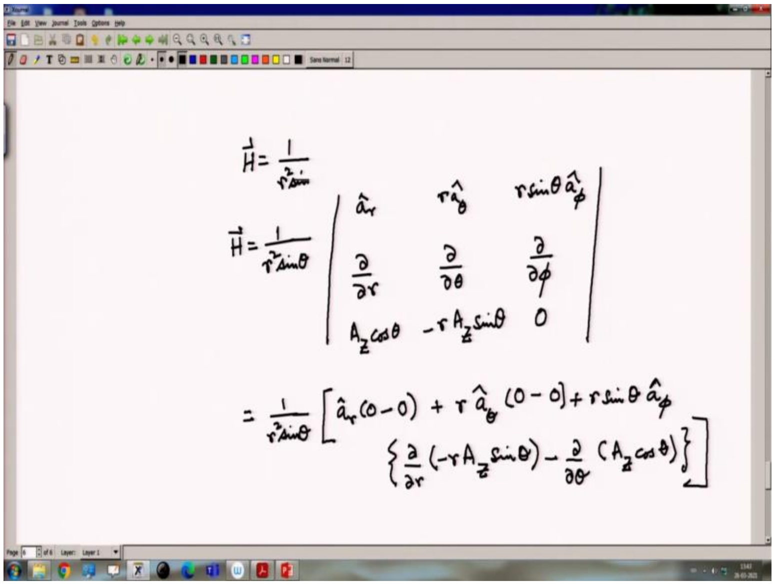
(Refer Slide Time: 18:52)



Now, let us draw the rectangular and the spherical coordinates and find out the relationship between the rectangular and spherical coordinates.



(Refer Slide Time: 22:00)



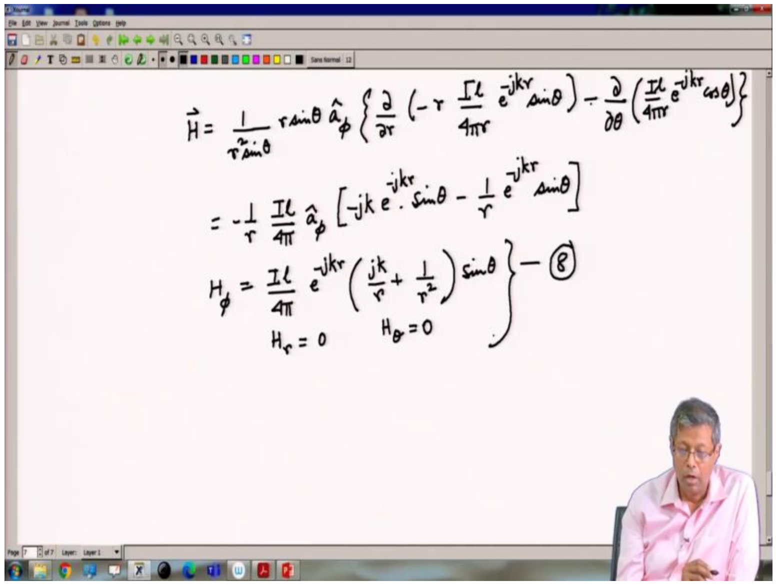
So, now substituting these values into the previous equation we have

$$\vec{H} = \frac{1}{r^2 \sin}$$

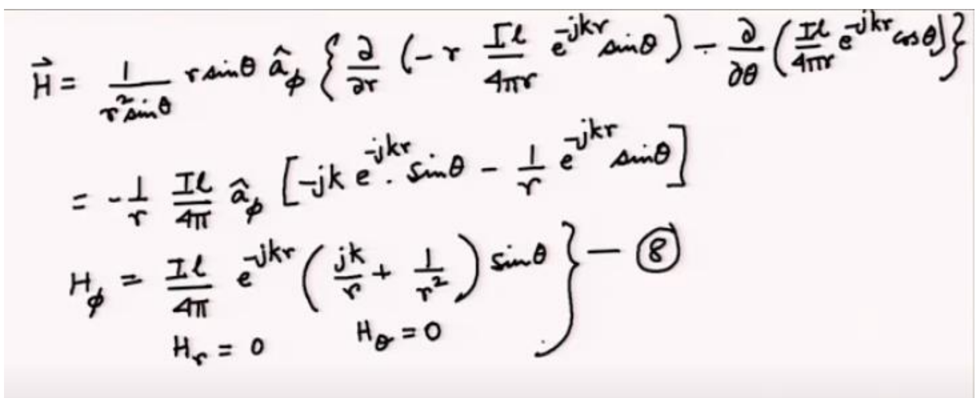
$$\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_z\cos\theta & -rA_z\sin\theta & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\Big[\hat{a}_r(0-0) + r\,\hat{a}_\theta(0-0] + r\sin\theta\,\hat{a}_\phi \left\{\frac{\partial}{\partial r}(-rA_z\sin\theta) - \frac{\partial}{\partial\theta}(A_z\cos\theta)\right\}\Big]$$

(Refer Slide Time: 26:55)



And that can further be written as



So, let us bunch together and call this equation 8.

So, these are the radiated magnetic field components in the spherical coordinates, or the radiated spherical magnetic field components. So, it has only a phi component, because the current is z directed; and both the radial component of the magnetic field. And the elevation of the theta component of the and the elevation or the theta component of the magnetic field is 0.

Now, we proceed to calculate the electric fields via equation number 1, via this equation. And for that we have to calculate this term divergence of A, and grad of divergence. Let us stop here; in the next lecture we will investigate the electric field components.