Advanced Microwave Guided-Structures and Analysis

Professor. Bratin Ghosh
Department of E & ECE
Indian Institute of Technology, Kharagpur

Lecture No. 23

Radiation from an Electric Current Source (contd.)

Welcome to this session, we will be continuing with the radiation from an electric current source, the spherical wave components for the radiated fields. Last time we investigated the magnetic field components and we saw that both the hr, the radial component and the elevation comp1nt which is the h theta component of the magnetic field R0, the magnetic field has only an azimuthal component or h file.

Next, we will investigate the electric field components radiated by the current source. Let us go to the lecture. Now, in order to find the electric field components, we need to look again at question number 1.

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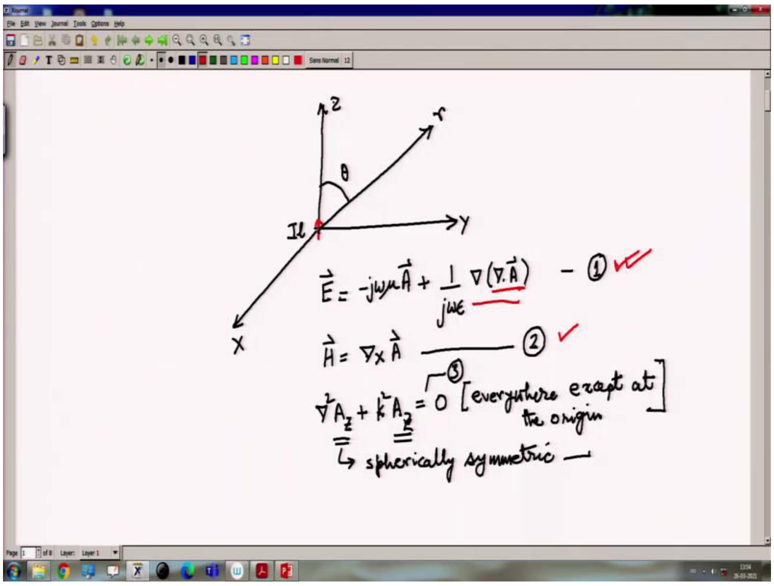

And for calculating the radiated electric field, we need to calculate divergence of A and grad of divergence of A for the spherical wave. So, let us do that.

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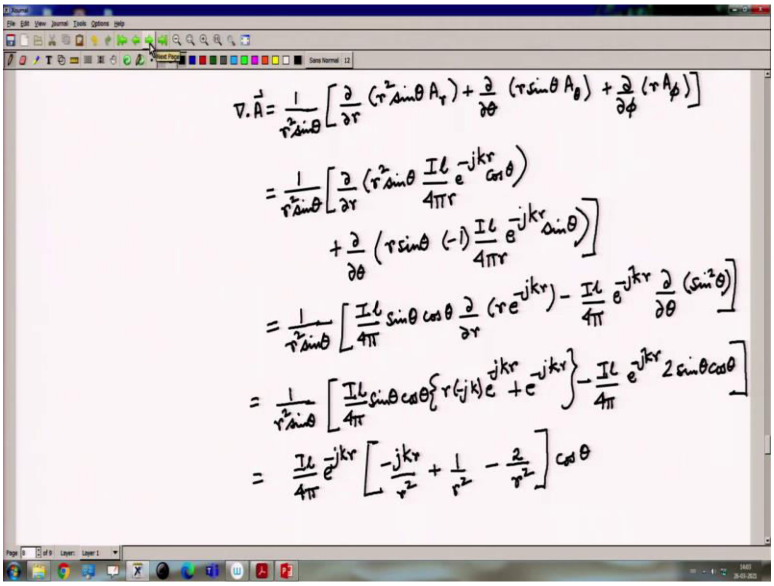

So, divergence of A is 1 by r squared sine theta del del r r square sine theta AR plus del del theta r sine theta A theta plus del del phi r A phi and that is equal to 1 by r square sine theta, we substitute the value of A r A theta in A phi. So, del del r r square sine theta Il by 4 pi r e to the power minus j kr cos theta plus del del theta r sine theta times minus 1 Il by 4 pi r e to the power minus j kr sine theta plus 0 because A phi is 0.

And that is equal to 1 by r square sine theta Il by 4 pi sine theta cos theta del del r of r e to the power minus j kr minus Il by 4 pi e to the power minus j kr. Del del theta of sine square theta and that becomes equal to 1 by r squared sine theta are Il by 4 pi sine theta cos theta r minus j k e to the minus jkr plus e to the power minus j k plus e to the power minus j kr minus Il by 4 pi e to the power minus j kr 2 sine theta cos theta.

And that is equal to Il by 4 pi e to the power minus j kr minus j kr by r squared plus 1 by r square minus 2 by r square cos theta.

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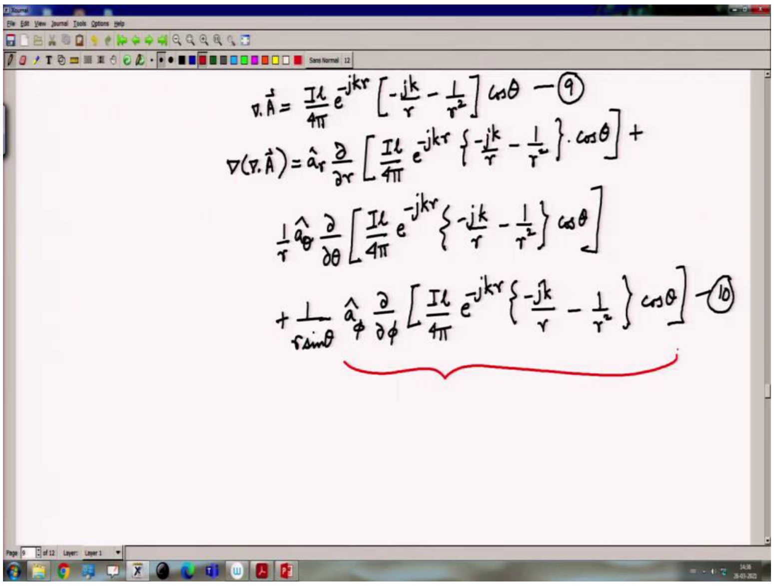

Which can be further simplified as, divergence of A is Il by 4 pi e to the power minus j kr minus jk by r minus 1 by r square, minus 1 by r square times cos theta. We call this equation 9 and then we proceed to calculate grad of divergence A and then we proceed to calculate grad of divergence A and that is equal to ar del del r Il by 4 pi e to the power minus j kr minus jk by r minus 1 by r square cos theta.

Plus 1 by r a theta del del theta Il by 4 pi e to the power minus jkr minus jk by r minus 1 by r squared cos theta plus 1 by r sine theta a phi, del del phi Il by 4 pi e to the power minus j kr the same thing jk, minus jk by r minus 1 by r square cos theta. So, let us call this step this is by the definition of the gradient operation in the spherical coordinate system.

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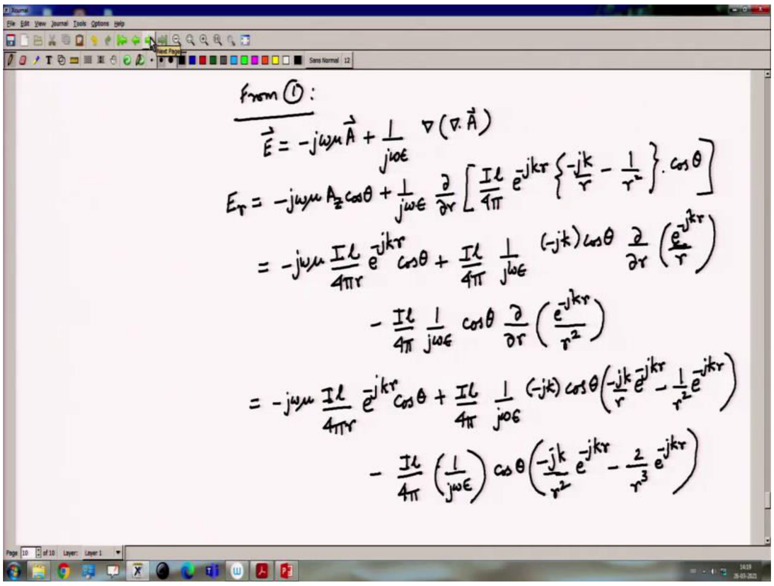

So, now we know that from equation 1, E is equal to minus j omega mu A plus 1 by j omega epsilon grad of divergence A, grad of divergence grad of divergence. So, Er therefore, the r component of the electric field will be equal to minus j omega mu Az cos theta plus 1 by j omega mu, the r component of grad of divergence A which we derived before which is del del r Il by 4 pi e to the power minus jkr minus jk by r minus 1 by r square times cos theta.

And that is equal to minus j omega mu substituting for z, Il by 4 pi r e to the power minus j kr cos theta plus Il by 4 pi 1 by j omega epsilon minus jk cos theta del del r will work on e to the power minus j kr by r minus Il by 4 pi 1 by j omega epsilon cos theta del del r is going to work on e to the power minus j kr by r square. And that will give rise to minus j omega mu Il by 4 pi r e to the power minus j kr cos theta plus Il by 4 pi 1 by j omega epsilon minus jk cos theta times minus jk by r e to the power minus j kr minus 1 by r square e to the power minus j kr, for the first term.

For the second term and then for the third term minus Il by 4 pi 1 by j omega epsilon, 1 by j omega epsilon cos theta minus jk by r square e to the power minus j kr minus 2 by r cube to the power minus j kr.

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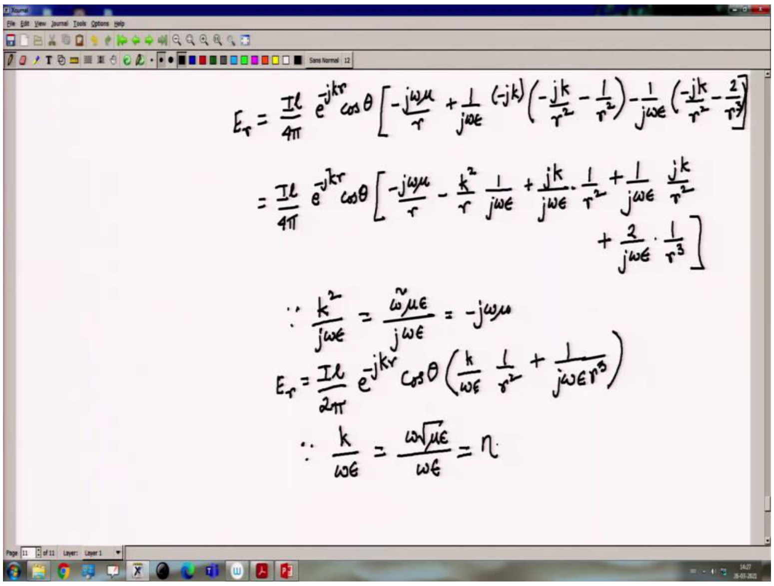

This can be further simplified as Er equal to Il by 4 pi e to the power minus j kr cos theta times minus j omega mu by r plus 1 by j omega epsilon times minus jk times minus jk by r square minus 1 by r square minus 1 by j omega epsilon minus jk by r square minus 2 by r cube.

And this can be further written as Il by 4 pi e to the power minus j kr cos theta times minus j omega mu by r minus k square by r 1 by j omega epsilon plus jk by j omega epsilon 1 by r square plus 1 by j omega epsilon jk by r square plus 2 by j omega epsilon times 1 by r cube. Since, now k square by j omega epsilon is omega squared mu epsilon by j omega epsilon that is equal to minus j omega mu. Er can be written as Il by 2 pi e to the power minus j kr times cos theta k by omega epsilon times 1 by r square plus 1 by g omega epsilon r cube.

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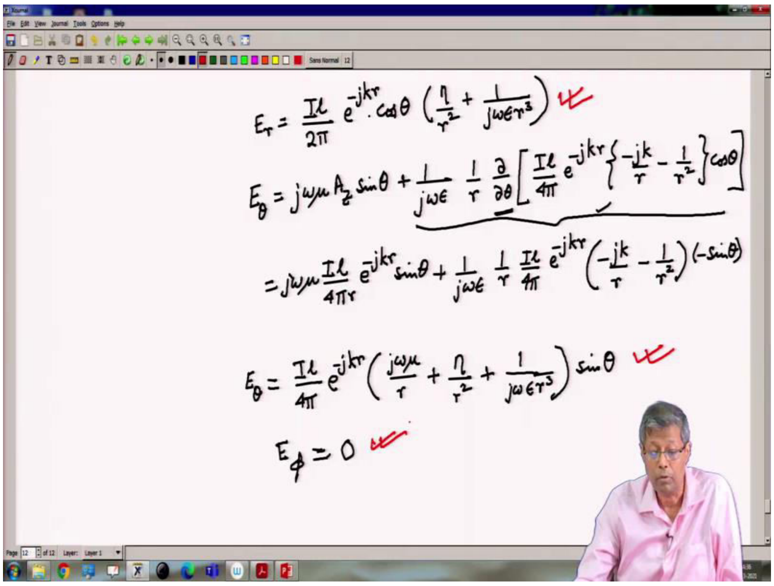

Now, since k by omega epsilon equal to omega root mu epsilon by omega epsilon equal to eta is root of mu by epsilon, the intrinsic impedance of free space we can write the value of Er as Il by 2 pi e to the power minus j kr times cos theta eta by r square plus 1 by j omega epsilon r q. So, this completes the derivation of the radial component of the electric field radiated by the electric current point source.

Now, for the theta directed electric field we have that equal to j omega mu Az sine theta plus 1 by j omega epsilon 1 by r del del theta Il by 4 pi e to the power minus j kr times minus jk by minus jk by r minus 1 by r square times cos theta. So, this is the theta component of grad of divergence of A times 1 by Eg omega epsilon. So, this is the theta component of 1 by Eg omega epsilon grad of divergence of A.

And that now on substitution of Az becomes equal to j omega mu Il by 4 pi r e to the power minus j kr sine theta and we need to do just this theta differentiation plus 1 by j omega epsilon 1 by r Il by 4 pi e to the power minus j kr minus jk by r minus 1 by r square times the differentiation with respect to theta which is minus sine theta times the differentiation of cos theta with respect to theta, which is minus sine theta.

And then combining the terms together we have and then combining the terms together we can finally write E theta as Il by 4 pi e to the power minus j kr j omega mu by r plus eater by r square plus 1 by j omega epsilon silent 1 by j omega epsilon r cube times sine theta. So, this is the expression for E theta.

Finally, E phi is 0 because E does not have any component along the phi direction. So, the first term of the electric filed which is minus j omega mu A that is 0 and the second term if you look at the expression for grad of divergence of A, if you look at the second term, at the third term, if you look at the third term of the grad of divergence of A, that is del del phi of this term. So, this term has no phi component. So, therefore, the phi component of grad of divergence of A is 0.

And therefore, E phi becomes equal to 0. So, this completes the derivation of the three electric field components radiated by the point current source. We note that E phi is 0 and the electric field is along the radial direction and the theta direction. So, we conclude this lecture. Next we are going to go to the magnetic current source. Thank you.