Advanced Microwave Guided-Structures and Analysis
Professor. Bratin Ghosh
Department of E & ECE
Indian Institute of Technology, Kharagpur
Lecture No. 24
Wave Equation and Solution Tutorials

Hello everyone, today we will start solving some numerical problems on wave equation and solution and the relation between wave numbers in a homogeneous medium.

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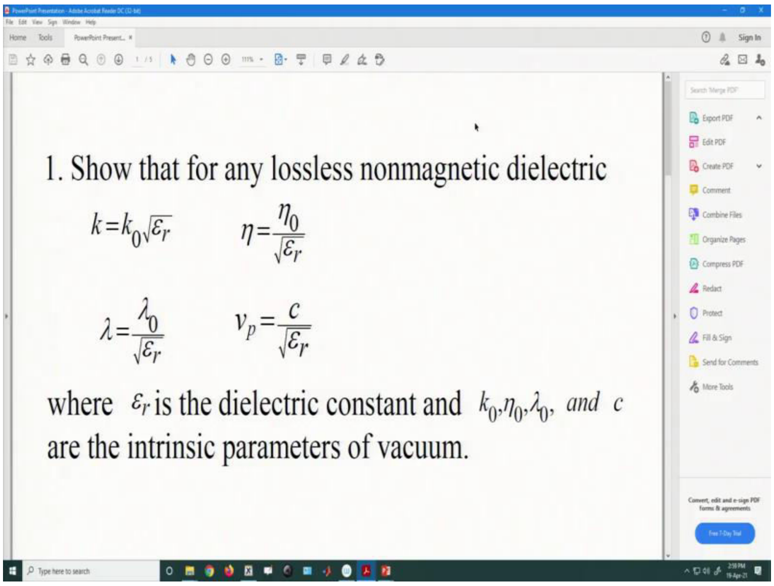

So, our first problem is shown above. So, let us start solving it.

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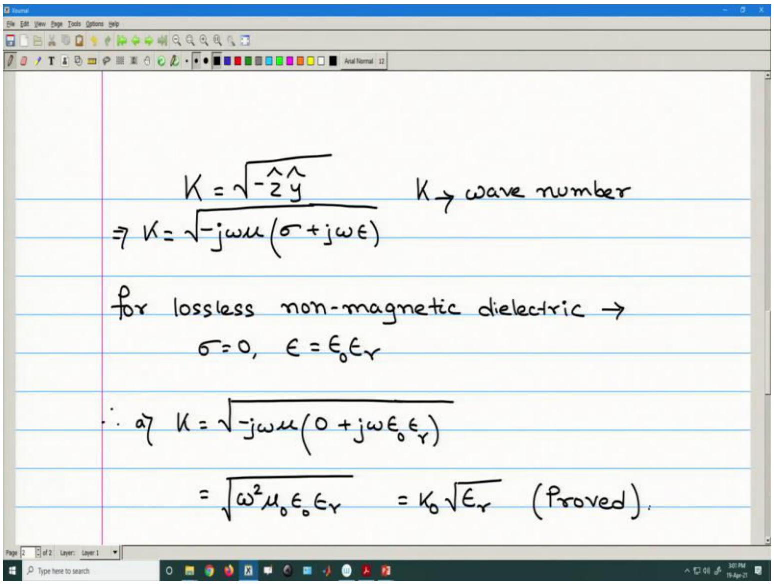

So, to start with we know that k is given by root under minus of $\sqrt{-\hat{z}\hat{y}}$, where k is wave number and z and y is impedivity and admmitivity of the medium respectively. So, substituting the values $k = \sqrt{-j\omega\mu(\sigma + j\omega\varepsilon)}$

Now, we know that for lossless nonmagnetic, for lossless nonmagnetic dielectric sigma is 0, epsilon is epsilon naught multiplied by epsilon r. Therefore, the first thing that we need to prove this k so, we can write that k equals to $k = \sqrt{-j\omega\mu(0 + j\omega\varepsilon_r\varepsilon_o)}$

Solving this, we get k equals to k naught root under epsilon are, the first thing is proved, fine.

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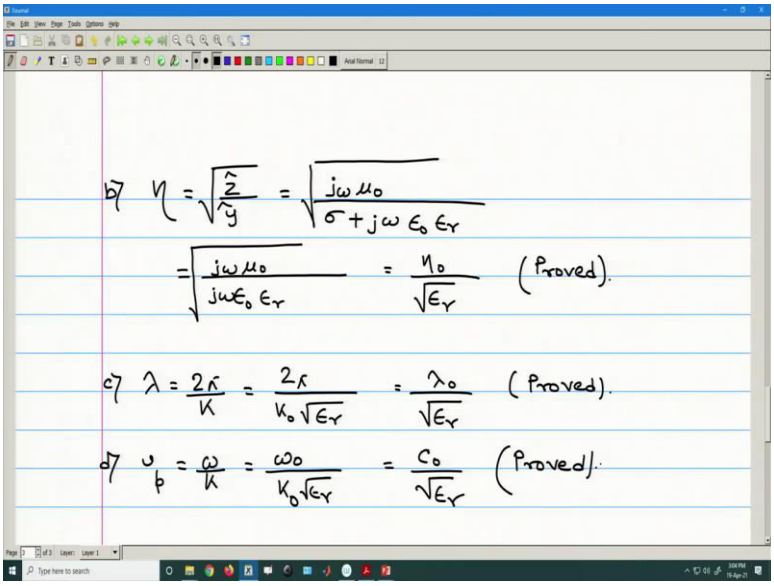

Now, the next thing eta is the given $\eta = \sqrt{\frac{\hat{z}}{\hat{y}}}$, now again on substitution $\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon_o\varepsilon_r)}}$

Now, again, sigma in this case will be 0. So, we have j omega mu naught by j omega epsilon naught epsilon r. This gives us $\frac{\eta_o}{\sqrt{\varepsilon_r}}$

For the next part, the next part we have lambda we know is 2 pi upon k. So, substituting for k from a like we got k equals to k naught epsilon r, so, we will substitute in place of k we will get 2 pi by k naught under root of epsilon r. So, k naught again can be written as 2 pi epsilon. So, this will cancel out and we were left with lambda naught upon root over of epsilon r. So again, this is proved.

For the next part that is d, d is phase velocity Vp. So, Vp phase velocity we know is omega by k that is omega naught upon k naught epsilon r that gives us c naught upon epsilon r. So, this is also proved. So, the first problem is solved.

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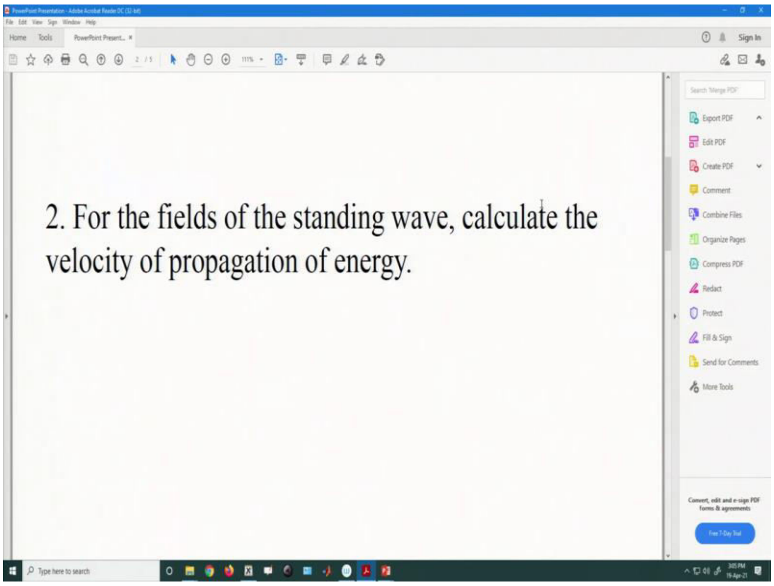

Moving on to the next problem we have, the next problem is for the fields of the standing wave we need to calculate the velocity of propagation of energy. So, we will start solving the second one, fine.

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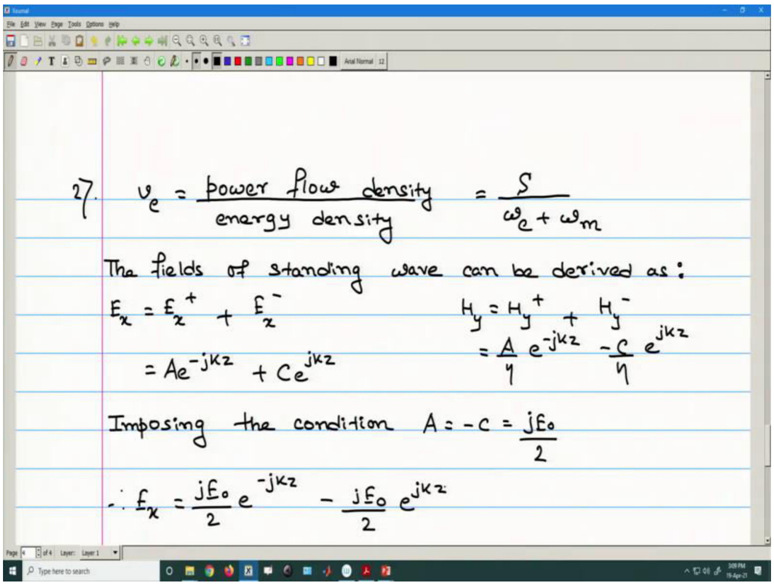

So, we need to find out the velocity of propagation of energy for a standing wave. So, the fields of a standing wave can be derived as first of all the velocity of propagation of energy is given by $\frac{S}{w_e + w_m}$, fine. where we and wm are the electric and magnetic energy densities, fine.

So, now to calculate the velocity of propagation of energy, we need to find out we and wm as well. So to start with, we will first find out the fields of a standing wave. So, the fields of standing wave going can be derived as we can write like suppose we can write Ex equals to $E_x^+ + E_x^-$. That is equal to $Ae^{-jkz} + Ce^{jkz}$ where A and C are the respective amplitudes fine now, we will impose the condition.

Here, the superscript plus denotes a plus traveling wave and the superscript minus denotes a minus traveling wave. So, similarly, Hy can be written as $H_y^+ + H_y^-$ that is $\frac{A}{\eta}e^{-jkz} - \frac{C}{\eta}e^{jkz}$ where A and C are the respective amplitudes. Fine.

Now, we will impose the condition as imposing the condition A = -C equals j E naught by 2. Since they are out of phase we will get Ex as j E naught by 2 e to the power minus j kz minus j E naught by 2 e to the power j kz.

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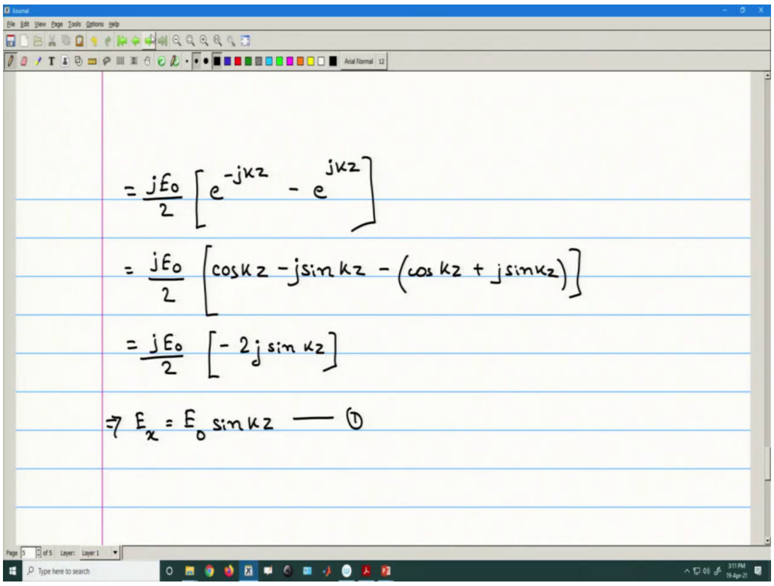

That gives us j E naught by 2 e to the power minus j kz minus e to the power j kz. Now, solving this we can write j E naught by 2; e to the power minus j kz can be written as cos kz minus j sine kz. So, e to the power minus j kz is that is cos of kz minus j sine kz minus of cos kz plus j sine kz.

Now, simplifying and opening the braces we get j E naught by 2 minus of 2 j sine kz. So, this 2, 2 gets cancel out and we get Ex as $E_o \sin(kz)$, fine. So, we denote this equation as equation number 1. Now, similarly, we will calculate Hy fine.

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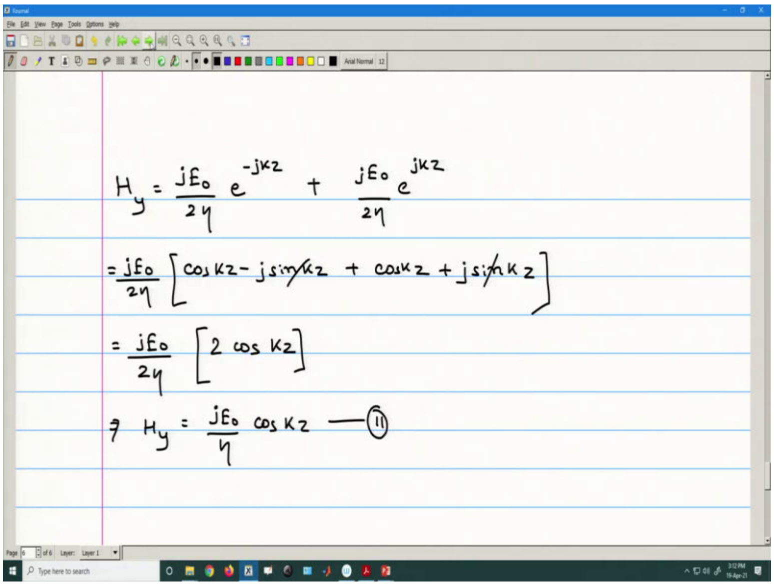

So, to start with we can write Hy is j of E naught by e2 eta e to the power minus j kz plus j E naught by 2 eta e to the power of j kz. Again simplifying it in a similar manner we get j E naught by 2 eta and this e to the power minus j kz will be cos of kz minus j sine kz plus e to the power j kz will be cos kz plus j sine kz, fine.

Thus, this jk, j sine kz, j sine kz gets cancel out and we are left with j e naught by 2 eta 2 cos of kz. This 2, 2 cancels and we are having H of y is $\frac{jE_o}{\eta}\cos(kz)$. Let us denote this equation as equation number 2. Now, we are having Ex and Hy. Now we will find the instantaneous fields.

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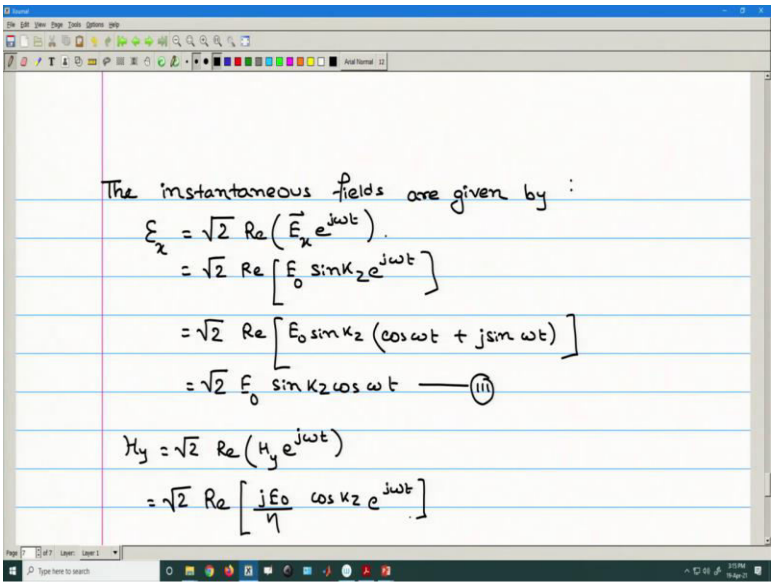

Therefore, the instantaneous fields are given by like $E_x(inst) = \sqrt{2}\,\mathrm{Re}(\vec{E}_x e^{j\omega t})$. So, we can write it as root 2 real part of Ex like we got Ex as E naught sine kz. So, in place of Ex we will write E naught sine of kz e to the power of j omega t that equals to root 2 real part of E naught sine kz e to the power j omega t, we will write it as cos of omega t plus j sine omega t.

That is equal to we will take only the real part. So, we will have $\sqrt{2}E_o \sin(kz)\cos(\omega t)$. So, this is the instantaneous field. Denote this as equation number 3. Similarly, we will find the instantaneous Hy that is H of y is given by root 2 real part of Hy e to the power j omega t.

So, again substituting for Hy, we got Hy as j E naught by eta cos of kz. So, we will substitute Hy in place of it we will write root 2 real part of in place of Hy we will write j E naught by eta cos of kz. And then e to the power j omega t.

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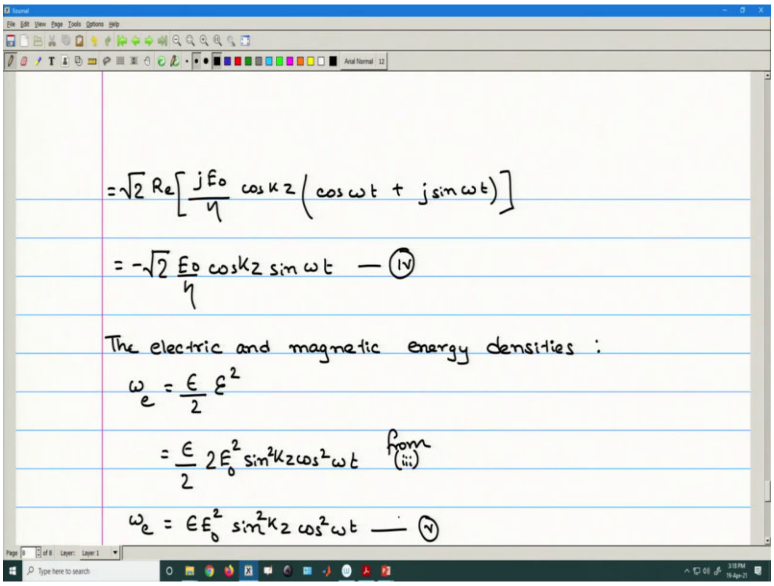

Next, we will write it as root 2 real part of j E naught by eta cos of kz and E to the power j omega t, we are writing it as cos of omega t plus j sine omega t. Now, we will take the real part. So, real part is we will have roots 2 this minus j squared it is minus time, this minus is due to this j into j j square, then real part is $\sqrt{2}\,\frac{jE_o}{\eta}\cos(kz)\cos(\omega t)$. Let us give this equation as equation number 4.

So, now we have derived the instantaneous fields Ex and Hy. Now, we will write the electric and magnetic energy densities that is We and Wm. So, we will write it as the electric and magnetic energy densities can be written as We that is electric energy density is given by epsilon by 2 into E square that is equal to epsilon by 2 E square.

This instantaneous effect we have derived it as root 2 E naught sine kz cos omega t that is from equation number 3. So, substituting from 3, we get E square as 2 E naught square sine square kz cos square omega t, fine. So, this 2, 2 cancel out and we get We as $\varepsilon E_o^2 \sin^2(kz)\cos^2(\omega t)$. So, we give this equation as equation number 5. Similar manner we will also calculate Wm.

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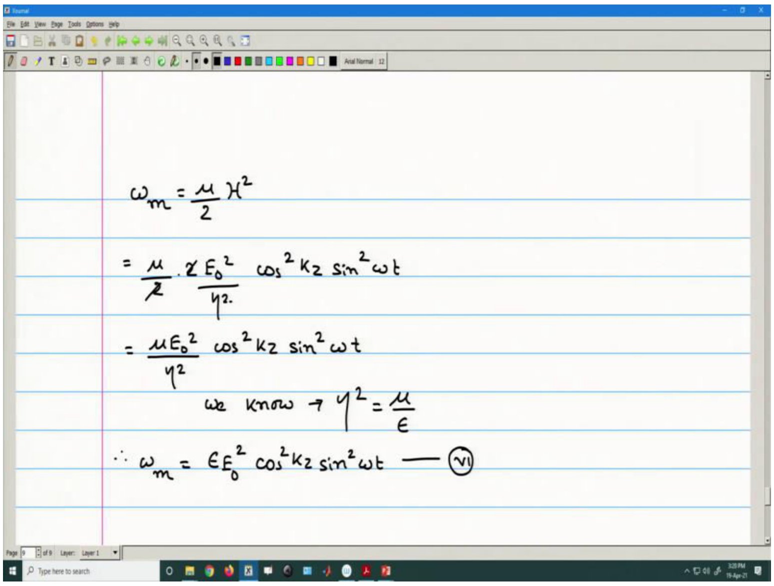

So, Wm is mu by 2 H square. Therefore, against substituting for the value of H from equation number 4 we will get mu by 2 multiplied by H square that is 2 E naught square by eta square cos square kz sine square omega t. So, 2 2 cancels out we will get mu E naught square by eta square cos square kz sine square omega t.

Now, we know that, we also know that eta square is mu by epsilon therefore, we can write Wm as epsilon in place of eta square we are substituting mu by epsilon. So, epsilon gets cancel out, mu mu mu gets cancelled out we will left with $\varepsilon E_o^2 \cos^2(kz)\sin^2(\omega t)$. Let us get this as equation number 6. So, now, we have We and Wm now we will calculate S.

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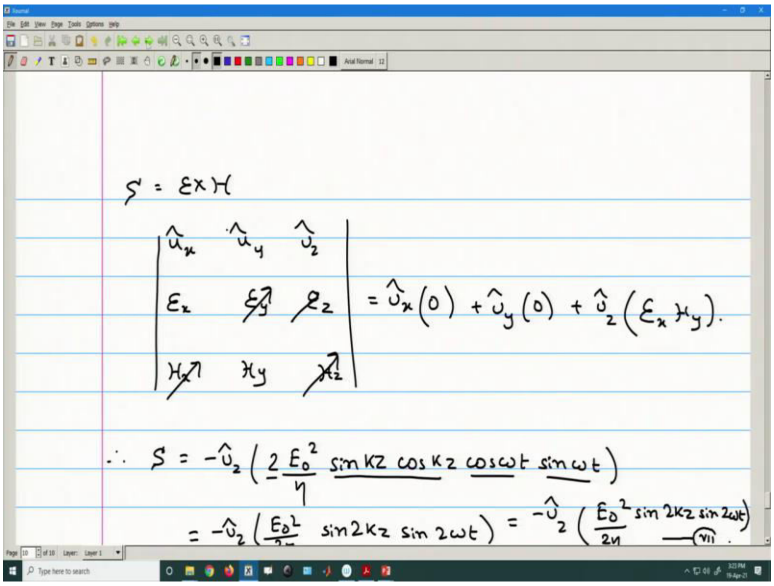

So, **S** is given as **E** cross **H**. So, we will have like your ux, u cap y, u cap z. We will having Ex y Ez, again Hx, Hy, Hz, fine. That is equal to for ux see here Ey Ez is not there Hx Hz is not there. So, we will calculate as ux we will get u of x is 0 plus uy is also 0 plus uz we have Ex multiplied by Hy, fine.

Therefore, we can write S as minus of uz because Ex and Hy we are having one minus sign. So, we will have Ex multiplied by Hy so, we will get from multiplying equation number 3 and 4. So, Ex multiplied by Hy we will get minus of 2 E naught square by eta sine of kz cos of kz cos omega t sine omega t.

So, in this too, we can right sine kz cos kz and this 2 we can write it as sine 2 kz this one and this one we can write it as sine of 2 kz and again taking one 2 we can write this as sine of 2 omega t and that comes out as E naught square by 2 eta. So, S is minus u of z E naught square by 2 eta sine of 2 kz sine of 2 omega t. We will denote this expression as equation number 7. Now, now we will calculate the velocity of propagation of energy.

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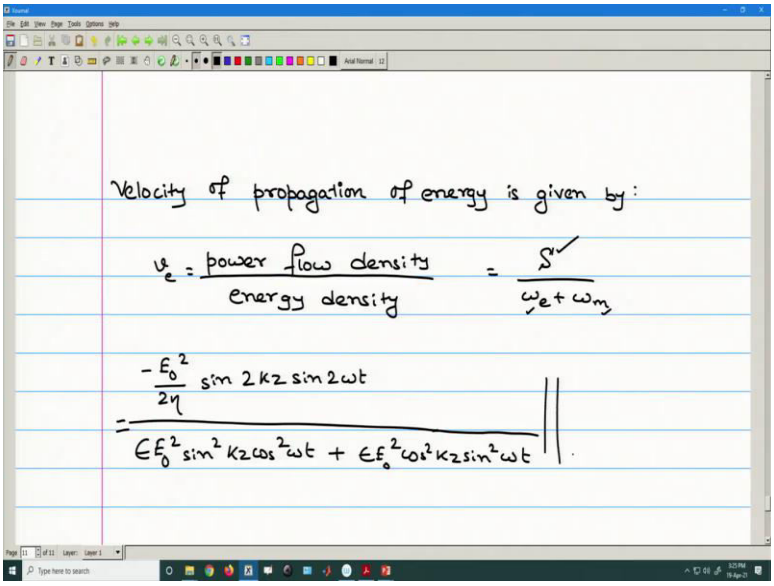

So, we can write like this velocity of propagation of energy you know it is given as, is given by Ve equals to power flow density divided by energy density that this ratio of S by We plus Wm. So, we are equipped with S, we are equipped with We, and we are equipped with Wm. So, just we will substitute the individual values that is equal to, so S we got minus of a E naught square by 2 eta sine of 2 kz sine of 2 omega t divided by epsilon E naught square that is We.

We, we got epsilon E naught square sine square kz cos square omega t plus again epsilon E naught square cos square kz sine square omega t. So, basically what we did we substituted the value of S and We and Wm. Now, we need to simplify this equation.

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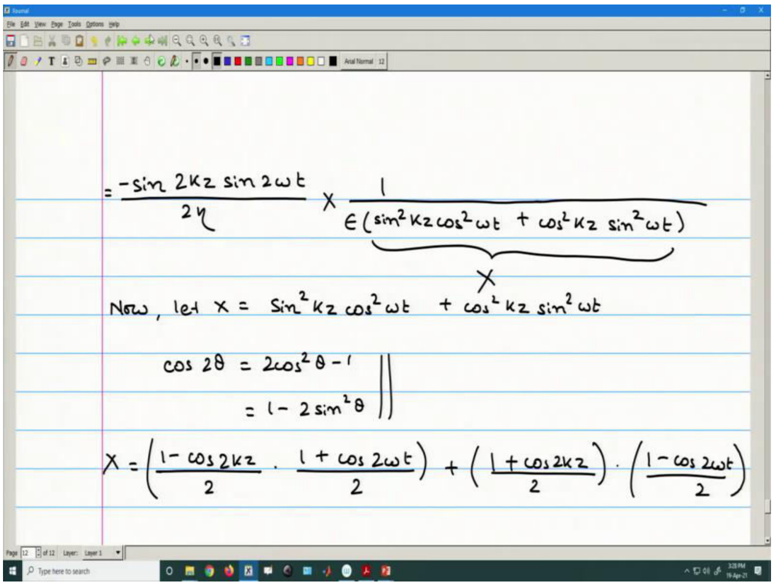

So, we can write it as, so, E naught square E naught square will get cancel out, we will be left with minus of sine 2 kz sine of 2 omega t by 2 eta into 1 by epsilon sine square kz cos square omega t plus cos square kz sine square omega t. Now, in order to simply, further simplify we are, let this be considered as A, fine So, we will write or suppose we write it as, we denote it as X suppose.

So, we now solve this X. Now, let X be sine square kz cos square omega t plus cos square kz sine square omega t, fine. Now, we know that we can write cos 2 theta as 2 cos square theta minus 1 and minus and 1 minus 2 sine square theta. So, using this trigonometric identities, we will simplify this as, we can write X as so, in place of sine square kz we will write 1 minus of course 2 kz by 2 multiplied in place of cos square omega t we will write 1 plus cos 2 omega t by 2 plus 1 plus for cos square kz we will write 1 plus cos of 2 kz by 2 multiplied by 1 minus cos 2 omega t by 2.

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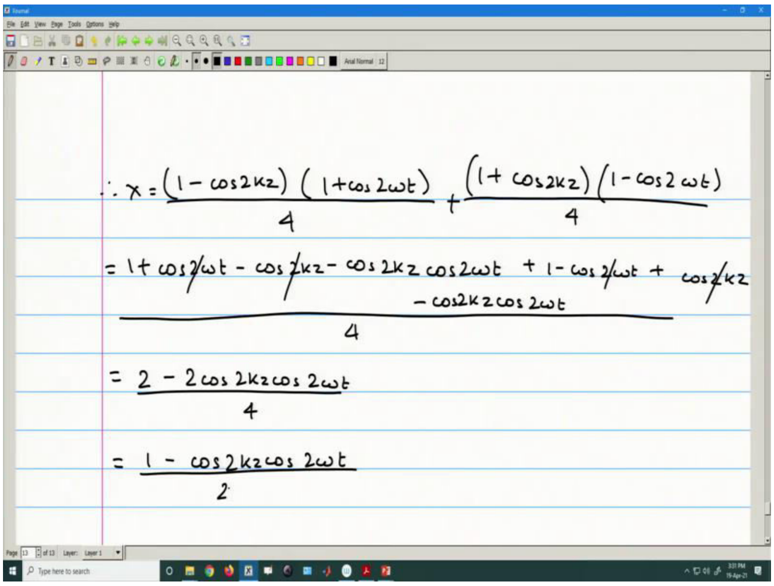

Now, therefore, we can simplify it as 1 minus cos of 2 kz 1 plus cos of 2 omega t by 4 plus 1 plus costs of 2 kz multiplied by 1 minus cos of 2 omega t by 4 So, we will just multiply and open up the braces, then we will get 1 for this one, 1 plus cos of 2 omega t minus cos of 2 kz minus cos of 2 kz cos of 2 omega t. Then plus 1 minus of cos 2 omega t plus cos of 2 kz minus costs of 2 kz cos omega t time 4.

So, this term cos 2 omega t gets cancelled out this cos 2 omega t and this cos t omega t cancel out and again this cos 2 kz cos 2 kz cancels out. Now, we will have 2 minus of 2 cos of 2 kz cos of 2 omega t divided by 4, this is what? 1 minus cos of 2 kz cos of 2 omega t by 2. So, now, after this simplification we will substitute this value in place of Ve.

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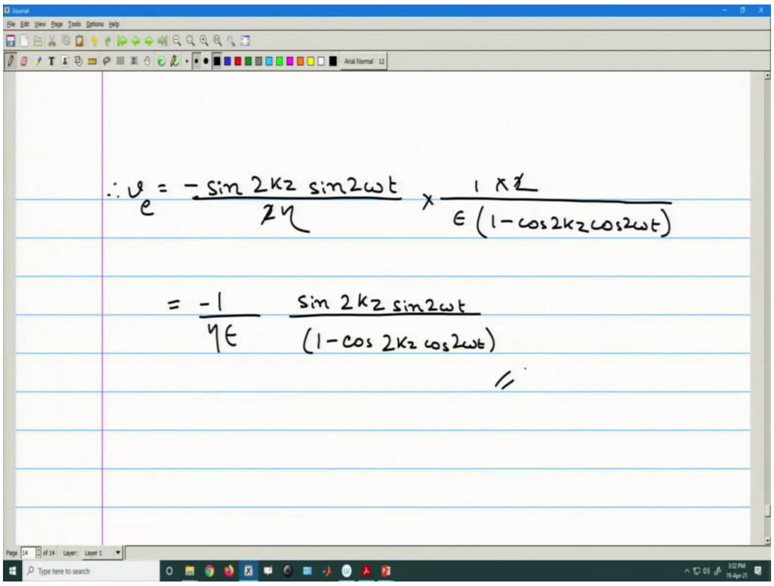

So, now, let us see what happens when substituting this value in Ve get minus of sine 2 kz sine 2 omega t by 2 eta multiplied by 1 by epsilon and this we simplified as 1 minus cos of 2 kz cos of 2 omega t this 2 will go upwards and this 2 2 we can cancel out and thus we will have it as minus 1 by eta epsilon sine of 2 kz sine of 2 omega t divided by 1 minus cos of 2 kz cos 2 omega t. So, this is the velocity of propagation of energy. So, in next class we will also solve some few more problems. So, thank you