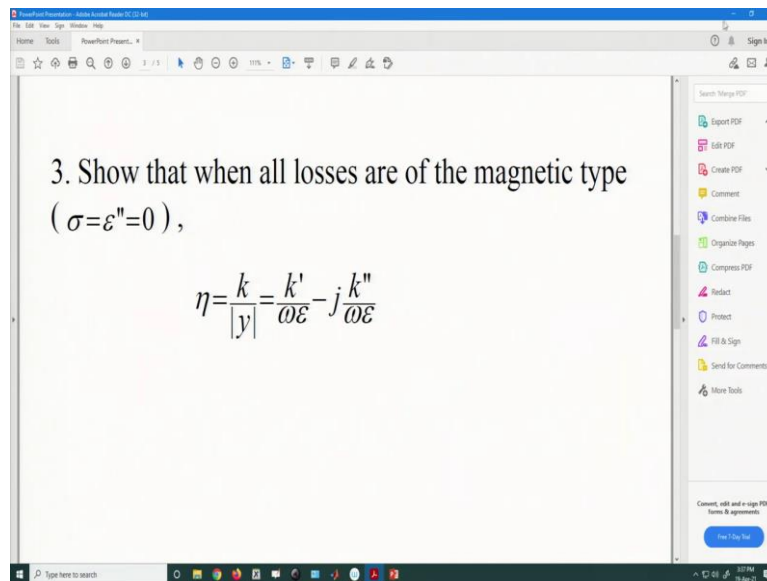


**Advanced Microwave Guided-Structures and Analysis**  
**Professor. Bratin Ghosh**  
**Department of E & ECE**  
**Indian Institute of Technology, Kharagpur**  
**Lecture No. 25**  
**Radiation from an Electric Current Source Tutorials**

Hello everyone, let us solve some few more numerical problems on relation between wave numbers.

(Refer Slide Time: 0:26)



3. Show that when all losses are of the magnetic type ( $\sigma = \epsilon'' = 0$ ),

$$\eta = \frac{k}{|y|} = \frac{k'}{\omega\epsilon} - j \frac{k''}{\omega\epsilon}$$

So, to start with the first problem states that

3. Show that when all losses are of the magnetic type ( $\sigma = \epsilon'' = 0$ ),

$$\eta = \frac{k}{|y|} = \frac{k'}{\omega\epsilon} - j \frac{k''}{\omega\epsilon}$$

So, let us start solving this.

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37 Given,  $\sigma = \epsilon'' = 0$

$$\therefore \eta = \sqrt{\frac{\hat{z}}{y}} \quad ; \quad \hat{y} = \sigma + j\omega\epsilon \quad ; \quad \hat{z} = j\omega\mu$$
$$= j\omega\epsilon$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$
$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{j\omega\mu}{jk}$$
$$\therefore \eta = \frac{j\omega\mu}{j\omega\epsilon}$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$
$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{j\omega\mu}{jk}$$
$$\therefore \eta = \frac{j\omega\mu}{j\omega\epsilon}$$
$$= \frac{\omega\mu}{\omega\epsilon} = \frac{k' - jk''}{\omega\epsilon} = \frac{k'}{\omega\epsilon} - \frac{jk''}{\omega\epsilon}$$

where,  $k' \rightarrow$  intrinsic phase constant  
 $k'' \rightarrow$  intrinsic attenuation constant.

So, we are given that

37 Given,  $\sigma = \epsilon'' = 0$

$$\therefore \eta = \sqrt{\frac{\hat{z}}{y}} \quad ; \quad \hat{y} = \sigma + j\omega\epsilon \quad ; \quad \hat{z} = j\omega\mu \\ = j\omega\epsilon$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{jk}{g}$$

$$\therefore \eta = \frac{jk}{j\omega\epsilon}$$

we know:

$$\hat{z} = jk\eta \quad \text{and} \quad \hat{y} = \frac{jk}{\eta}$$

$$\therefore \eta = \frac{\hat{z}}{jk} = \frac{jk}{g}$$

$$\therefore \eta = \frac{jk}{j\omega\epsilon}$$

$$= \frac{k}{\omega\epsilon} = \frac{k' - jk''}{\omega\epsilon} = \frac{k'}{\omega\epsilon} - \frac{jk''}{\omega\epsilon}$$

where,  $k' \rightarrow$  intrinsic phase constant

$k'' \rightarrow$  intrinsic attenuation constant.

(Refer Slide Time: 4:14)

4. Show that for nonmagnetic dielectrics

$$\left. \begin{aligned} k' &\approx w\sqrt{\mu\epsilon'}\left(1+\frac{1}{8Q^2}\right) \\ k'' &\approx \frac{w\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{1}{8Q^2}\right) \\ \mathcal{R} &\approx \sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{3}{8Q^2}\right) \\ \chi &\approx \frac{\epsilon''}{2\epsilon'}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{5}{8Q^2}\right) \end{aligned} \right\} Q \gg 1$$

where  $Q = \frac{w\epsilon'}{\sigma + w\epsilon''}$

The next problem is that

4. Show that for nonmagnetic dielectrics

$$\left. \begin{aligned} k' &\approx w\sqrt{\mu\epsilon'}\left(1+\frac{1}{8Q^2}\right) \\ k'' &\approx \frac{w\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{1}{8Q^2}\right) \\ \mathcal{R} &\approx \sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{3}{8Q^2}\right) \\ \chi &\approx \frac{\epsilon''}{2\epsilon'}\sqrt{\frac{\mu}{\epsilon'}}\left(1-\frac{5}{8Q^2}\right) \end{aligned} \right\} Q \gg 1$$

where  $Q = \frac{w\epsilon'}{\sigma + w\epsilon''}$

So again, we will start solving.

(Refer Slide Time: 5:32)

47  $Q_s = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''}$

$$\hat{z} = j\omega\mu \quad ; \quad \hat{y} = \sigma + j\omega\epsilon$$
$$= \sigma + j\omega(\epsilon' - j\epsilon'')$$
$$k = \sqrt{-\hat{z}\hat{y}}$$
$$= \sqrt{-j\omega\mu(\sigma + j\omega(\epsilon' - j\epsilon''))}$$

So, we it is given that quality factor is written as

$$Q_s = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''}$$
$$\hat{z} = j\omega\mu \quad ; \quad \hat{y} = \sigma + j\omega\epsilon$$
$$= \sigma + j\omega(\epsilon' - j\epsilon'')$$
$$k = \sqrt{-\hat{z}\hat{y}}$$
$$= \sqrt{-j\omega\mu(\sigma + j\omega(\epsilon' - j\epsilon''))}$$

(Refer Slide Time: 7:05)

$$\begin{aligned}
 k &= \sqrt{-\hat{z} \hat{y}} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega(\epsilon' - j\epsilon''))} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega\epsilon' + \omega\epsilon'')} \\
 &= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''} \\
 &= \sqrt{-j\omega\mu (\sigma + \omega\epsilon'') + \omega^2\mu\epsilon'} \\
 &= \sqrt{\omega^2\mu\epsilon' \left(1 - \frac{j\omega\mu(\sigma + \omega\epsilon'')}{\omega^2\mu\epsilon'}\right)}
 \end{aligned}$$

Now, we will simplify this just open the braces, we will get

$$\begin{aligned}
 k &= \sqrt{-\hat{z} \hat{y}} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega(\epsilon' - j\epsilon''))} \\
 &= \sqrt{-j\omega\mu (\sigma + j\omega\epsilon' + \omega\epsilon'')} \\
 &= \sqrt{-j\omega\mu\sigma + \omega^2\mu\epsilon' - j\omega^2\mu\epsilon''} \\
 &= \sqrt{-j\omega\mu (\sigma + \omega\epsilon'') + \omega^2\mu\epsilon'} \\
 &= \sqrt{\omega^2\mu\epsilon' \left(1 - \frac{j\omega\mu(\sigma + \omega\epsilon'')}{\omega^2\mu\epsilon'}\right)}
 \end{aligned}$$

(Refer Slide Time: 9:21)

$$\begin{aligned}
&= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j\omega \mu (\sigma + \omega \epsilon'')}{\omega^2 \mu \epsilon'}\right)} \\
&= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j(\sigma + \omega \epsilon'')}{\omega \epsilon'}\right)} \quad \alpha = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''} \\
&= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
&\text{NOTE: } (1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
&\therefore k = \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right.
\end{aligned}$$

Then we can write this

$$\begin{aligned}
&= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j\omega \mu (\sigma + \omega \epsilon'')}{\omega^2 \mu \epsilon'}\right)} \\
&= \sqrt{\omega^2 \mu \epsilon' \left(1 - \frac{j(\sigma + \omega \epsilon'')}{\omega \epsilon'}\right)} \quad \alpha = \frac{\omega \epsilon'}{\sigma + \omega \epsilon''} \\
&= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
&\text{NOTE: } (1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
&\therefore k = \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right.
\end{aligned}$$

$$\begin{aligned}
&= \omega \sqrt{\mu \epsilon'} \left\{1 - j \frac{1}{\alpha}\right\}^{1/2} \quad \checkmark \\
&\text{NOTE: } (1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots \\
&\therefore k = \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{1}{2} \cdot \frac{j}{\alpha} + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \frac{j^2}{\alpha^2}\right. \\
&\quad \left. - \frac{1}{3!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \frac{j^3}{\alpha^3} + \dots \right\} \\
&= \omega \sqrt{\mu \epsilon'} \left\{1 - \frac{j}{2\alpha} + \frac{1}{8\alpha^2} + \frac{j}{16\alpha^3} + \dots \right\}
\end{aligned}$$

(Refer Slide Time: 13:05)

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - j \frac{1}{Q_3} \right\}^{1/2}$$

NOTE:  $(1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$

$$\therefore K = \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{1}{2} \cdot \frac{1}{Q_3} + \frac{1}{2!} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \frac{j^2}{Q_3^2} - \frac{1}{3!} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{-3}{2} \right) \frac{j^3}{Q_3^3} + \dots \right\}$$

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{j}{2Q_3} + \frac{1}{8Q_3^2} + \frac{j}{16Q_3^3} + \dots \right\}$$

(Refer Slide Time: 15:33)

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - j \frac{1}{Q_3} \right\}^{1/2}$$

NOTE:  $(1-x)^r = 1 - rx + \frac{1}{2!} r(r-1)x^2 - \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$

$$\therefore K = \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{1}{2} \cdot \frac{1}{Q_3} + \frac{1}{2!} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \frac{j^2}{Q_3^2} - \frac{1}{3!} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{-3}{2} \right) \frac{j^3}{Q_3^3} + \dots \right\}$$

$$= \omega \sqrt{\mu \epsilon'} \left\{ 1 - \frac{j}{2Q_3} + \frac{1}{8Q_3^2} + \frac{j}{16Q_3^3} + \dots \right\}$$

$$K = \omega \sqrt{\mu \epsilon'} \left( 1 + \frac{1}{8Q_3^2} \right) - j \omega \sqrt{\mu \epsilon'} \left( \frac{1}{2Q_3} - \frac{1}{16Q_3^3} \right) + \dots$$

Comparing the above eq with

$$K = K' - jK'' \quad \text{--- (1)}$$

$$\therefore \boxed{K' = \omega \sqrt{\mu \epsilon'} \left( 1 + \frac{1}{8Q_3^2} \right)} \rightarrow \text{Proved.}$$



So, let us write it as separating real and imaginary parts, separating the real and imaginary parts,

$$\kappa = \omega \sqrt{\mu \epsilon} \left( 1 + \frac{1}{8\sigma^2} \right) + \omega \sqrt{\mu \epsilon} \left( \frac{j}{2\sigma} + \frac{j}{16\sigma^3} \right)$$

$$= \omega \sqrt{\mu \epsilon'} \left( 1 + \frac{1}{8\sigma^2} \right) - j \omega \sqrt{\mu \epsilon'} \left( \frac{1}{2\sigma} - \frac{1}{16\sigma^3} \right) + \dots$$

Comparing the above eq with ①

$$\kappa = \kappa' - j \kappa'' \quad \text{--- ①}$$

$$\therefore \boxed{\kappa' = \omega \sqrt{\mu \epsilon'} \left( 1 + \frac{1}{8\sigma^2} \right)} \rightarrow \text{Proved.}$$

So, this we got this is proved.

(Refer Slide Time: 19:06)

The screenshot shows a digital whiteboard with the following handwritten derivation for  $\kappa''$ :

$$\kappa'' = \omega \sqrt{\mu \epsilon'} \left( \frac{1}{2\sigma} - \frac{1}{16\sigma^3} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'}}{2\sigma} \left( 1 - \frac{1}{8\sigma^2} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'} (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left( 1 - \frac{1}{8\sigma^2} \right)$$

$$\boxed{\kappa'' = \frac{\mu}{\epsilon'} \frac{\omega \epsilon''}{2} \left( 1 - \frac{1}{8\sigma^2} \right)} \quad (\text{Proved}).$$

And for  $\kappa''$  we will take the imaginary part.

$$k'' = \omega \sqrt{\mu \epsilon'} \left( \frac{1}{2Q} - \frac{1}{16Q^3} \right)$$

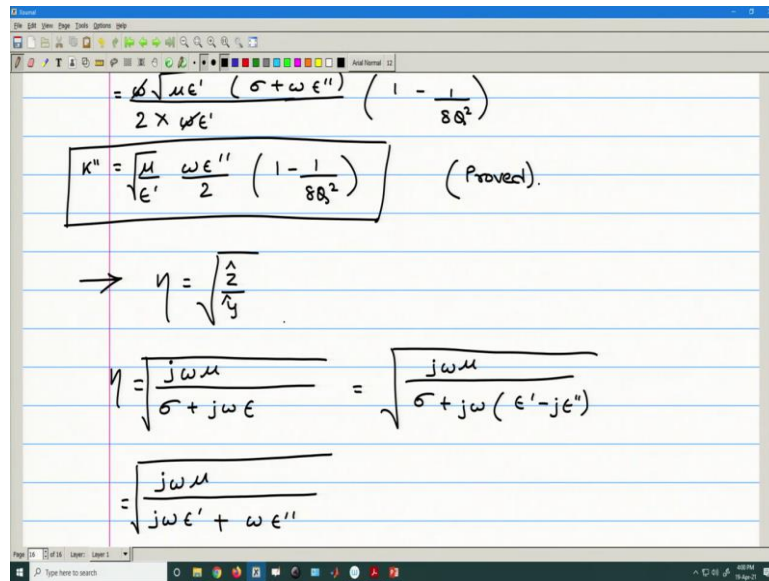
$$= \frac{\omega \sqrt{\mu \epsilon'}}{2Q} \left( 1 - \frac{1}{8Q^2} \right)$$

$$= \frac{\omega \sqrt{\mu \epsilon'} (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left( 1 - \frac{1}{8Q^2} \right)$$

$$\boxed{k'' = \sqrt{\frac{\mu}{\epsilon'}} \frac{\omega \epsilon''}{2} \left( 1 - \frac{1}{8Q^2} \right)} \quad (\text{Proved}):$$

So, this is also proved. Now further resistance and the reactants. So, for that we will start with the eta.

(Refer Slide Time: 21:56)



$$= \frac{\mu \epsilon' (\sigma + \omega \epsilon'')}{2 \times \omega \epsilon'} \left( 1 - \frac{1}{8\delta^2} \right)$$

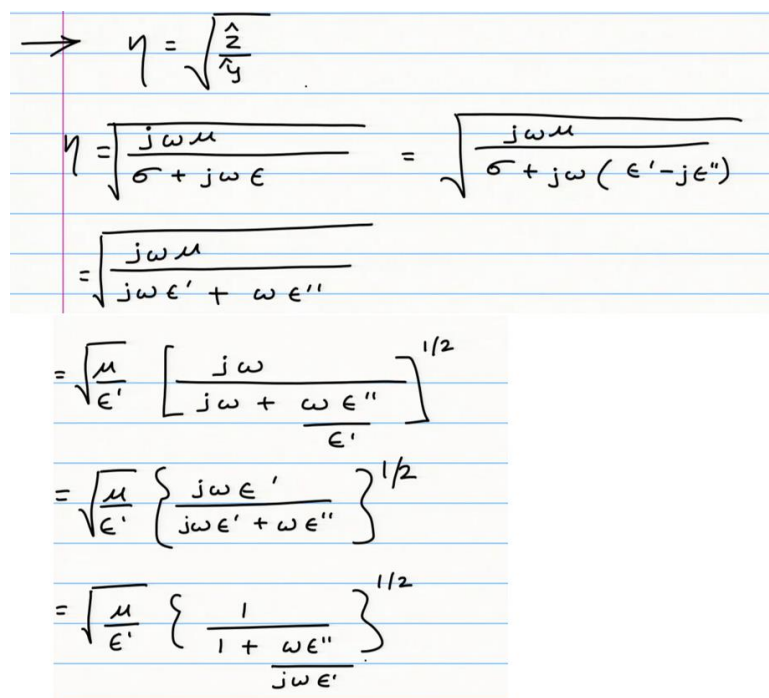
$$K'' = \frac{\mu}{\epsilon'} \frac{\omega \epsilon''}{2} \left( 1 - \frac{1}{8\delta^2} \right) \quad (\text{Prove it}).$$

$$\rightarrow \eta = \sqrt{\frac{Z}{Y}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

So, first and second done, for third and fourth we will start with we know eta is given by



$$\rightarrow \eta = \sqrt{\frac{Z}{Y}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega(\epsilon' - j\epsilon'')}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\epsilon' + \omega\epsilon''}}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[ \frac{j\omega}{j\omega + \frac{\omega\epsilon''}{\epsilon'}} \right]^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

(Refer Slide Time: 24:11)

$$\begin{aligned}
 &= \sqrt{\frac{\mu}{\epsilon'}} \left[ \frac{j\omega}{j\omega + \frac{\omega \epsilon''}{\epsilon'}} \right]^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega \epsilon'}{j\omega \epsilon' + \omega \epsilon''} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega \epsilon''}{j\omega \epsilon'}} \right\}^{1/2}
 \end{aligned}$$

That is we can write it as

$$\begin{aligned}
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega \epsilon'}{j\omega \epsilon' + \omega \epsilon''} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega \epsilon''}{j\omega \epsilon'}} \right\}^{1/2} \\
 &= \sqrt{\frac{\mu}{\epsilon'}} \left[ \frac{1}{1 + \frac{1}{j\omega \epsilon'}} \right]^{1/2} \\
 \therefore \eta &= \sqrt{\frac{\mu}{\epsilon'}} \left( 1 + \frac{1}{j\omega \epsilon'} \right)^{-1/2}
 \end{aligned}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left( 1 + \frac{1}{j\omega \epsilon'} \right)^{-1/2}$$

NOTE:

$$(1+x)^r = 1 + rx + \frac{r(r-1)}{2!} x^2 + \frac{r(r-1)(r-2)}{3!} x^3 + \dots$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^3 + \dots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\omega \epsilon'}\right)^3 + \dots \right]$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 - \frac{1}{2j\omega \epsilon'} - \frac{3}{8\omega^2 \epsilon'^2} + \frac{15}{48} \frac{1}{j\omega^3 \epsilon'^3} \dots \right]$$

Separating the real and imaginary parts:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 - \frac{3}{8\omega^2 \epsilon'^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left[ -\frac{1}{2j\omega \epsilon'} + \frac{15}{48} \frac{1}{j\omega^3 \epsilon'^3} \dots \right]$$

(Refer Slide Time: 25:58)

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{j\omega\epsilon'}{j\omega\epsilon' + \omega\epsilon''} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left\{ \frac{1}{1 + \frac{\omega\epsilon''}{j\omega\epsilon'}} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[ \frac{1}{1 + \frac{1}{j\beta}} \right]^{1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left( 1 + \frac{1}{j\beta} \right)^{-1/2}$$

(Refer Slide Time: 27:04)

$$\sqrt{\frac{\mu}{\epsilon'}} \left[ 1 + \frac{1}{j\beta} \right]^{-1/2}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon'}} \left( 1 + \frac{1}{j\beta} \right)^{-1/2}$$

NOTE:

$$(1+x)^r = 1 + rx + \frac{1}{2!} r(r-1)x^2 + \frac{1}{3!} r(r-1)(r-2)x^3 + \dots$$

$$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 + \left(-\frac{1}{2}\right)\left(\frac{1}{j\beta}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{j\beta}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(\frac{1}{j\beta}\right)^3 + \dots \right]$$

(Refer Slide Time: 29:50)

Handwritten derivation on a digital whiteboard:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 + \left(-\frac{1}{2}\right) \left(\frac{1}{j\alpha}\right) + \frac{1}{2!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{1}{j\alpha}\right)^2 + \frac{1}{3!} \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{1}{j\alpha}\right)^3 + \dots \right]$$

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 - \frac{1}{2j\alpha} - \frac{3}{8\alpha^2} + \frac{15}{48} \frac{1}{j\alpha^3} \dots \right]$$

Separating the real and imaginary parts:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 - \frac{3}{8\alpha^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left[ -\frac{1}{2j\alpha} + \frac{15}{48} \frac{1}{j\alpha^3} \dots \right]$$

So, separating the real and imaginary parts, separating the real and imaginary parts as we will get

Handwritten derivation showing the separation of real and imaginary parts:

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[ 1 - \frac{3}{8\alpha^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left( \frac{j}{2\alpha} - \frac{5j}{16\alpha^3} \right)$$

$$\boxed{R = \sqrt{\frac{\mu}{\epsilon'}} \left( 1 - \frac{3}{8\alpha^2} \right)} \rightarrow (\text{Proved})$$

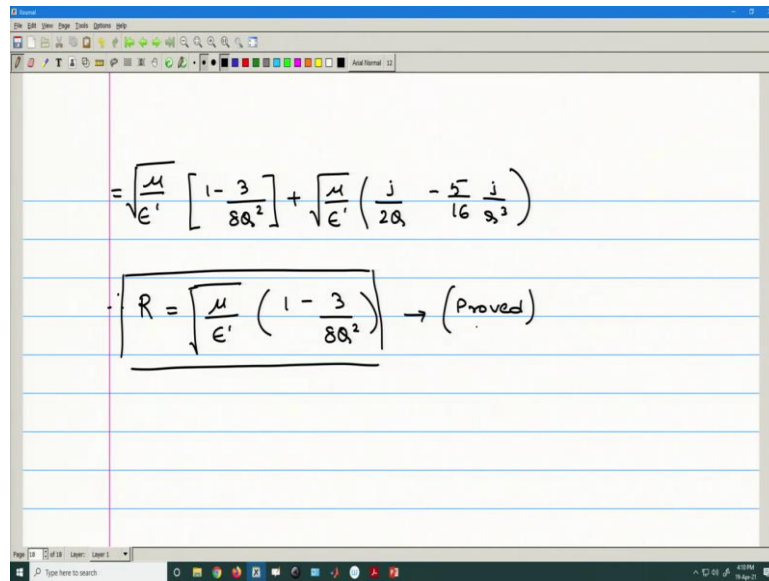
$$X = \sqrt{\frac{\mu}{\epsilon'}} \left( \frac{1}{2\alpha} - \frac{5}{16\alpha^3} \right)$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \frac{1}{2\alpha} \left[ 1 - \frac{5}{8\alpha^2} \right]$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \frac{1 \cdot \omega \epsilon''}{2 \cdot \omega \epsilon'} \left[ 1 - \frac{5}{8\alpha^2} \right]$$

$$\boxed{X = \sqrt{\frac{\mu}{\epsilon'}} \frac{\epsilon''}{2\epsilon'} \left( 1 - \frac{5}{8\alpha^2} \right)} \rightarrow (\text{Proved})$$

(Refer Slide Time: 32:33)



The image shows a digital whiteboard with a blue border and a toolbar at the top. The whiteboard contains two lines of handwritten mathematical equations. The first line is an equality between two square root terms. The second line shows a boxed equation for 'R' followed by an arrow pointing to the word 'Proved'.

$$= \sqrt{\frac{\mu}{\epsilon'}} \left[ \frac{1-3}{80^2} \right] + \sqrt{\frac{\mu}{\epsilon'}} \left( \frac{j}{20} - \frac{5-j}{16} \frac{j}{80^2} \right)$$
$$\therefore \boxed{R = \sqrt{\frac{\mu}{\epsilon'}} \left( \frac{1-3}{80^2} \right)} \rightarrow \text{(Proved)}$$

(Refer Slide Time: 33:55)

So, our next class we will also solve some few more problems on wave numbers and then we will move into electric current source. Thank you.