Advanced Microwave Guided-Structures and Analysis
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Lecture 26
Radiation from a Magnetic Current Source

Welcome to this session of lecture on the radiation from a magnetic current source in a homogeneous medium. Just as we saw that the radiation from an electric current source is given through the magnetic vector potential, dual to those relationships we will discover that the radiation from a magnetic current source is given by the electric vector potential.

So, let us come to those equations, let us derive those equations and finally we will move on to the radiated field components. Let us go to the lecture.

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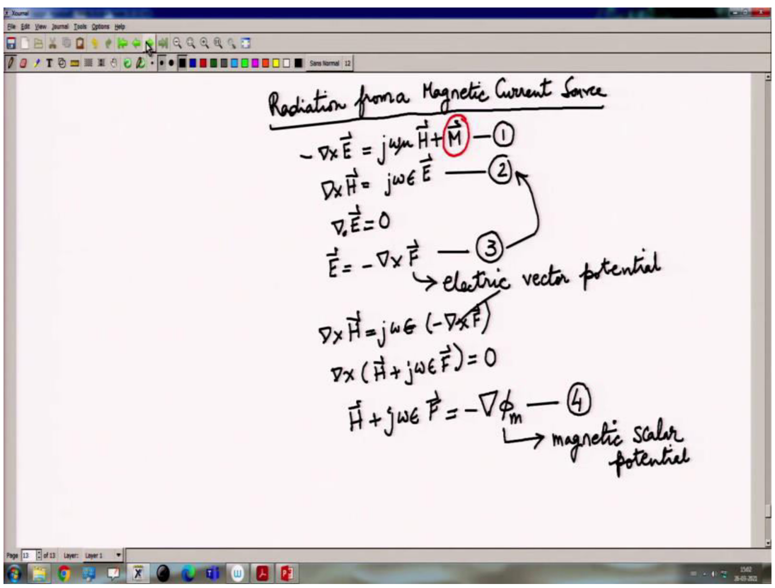

So, the magnetic current source actually is a fictitious source, nothing called the magnetic current source exists, but we are also going to show the relevance of the magnetic current source in the electromagnetic field computation problems, why it is needed, why it is introduced, the magnetic current source in electromagnetics actually is as important as the electric current source.

We will soon see how the magnetic current source accomplishes its task and how it simplifies or I should say enables the field computation. Without the magnetic current source we cannot solve an electromagnetic slot coupled field problem. So, first the radiation from the magnetic current source, so we will write first of all the Maxwell's equations, which is minus curl of **E** is j omega mu **H** plus **M**.

Let us call this equation 1 and curl of **H** as j omega epsilon **E** which is equation number 2. So, we have the magnetic current source which is **M**, and no electric current source. Now, in a homogeneous media the divergence of equation 2 is 0, so the divergence of curl of any vector is 0 and therefore the divergence of **E** is 0. So, any divergence free vector can be expressed as the curl of another vector, so **E** becomes equal to minus curl of **F**, where **F** is the electric vector potential.

So, now substituting 3 into 2 similar to the electric current source problem having the steps are conceptually similar to the electric current source problem. We have curl of **H** equal to j omega epsilon minus curl of **F** where we have curl of **H** plus j omega epsilon **F** equal to zero. Any vector which is free of curl is the gradient of some scalar, so **H** plus j omega epsilon **F** becomes equal to minus grad of phi m, where phi m is the magnetic scalar potential, so we call this equation number 4. So, we now substitute 3 and 4 into equation number 1.

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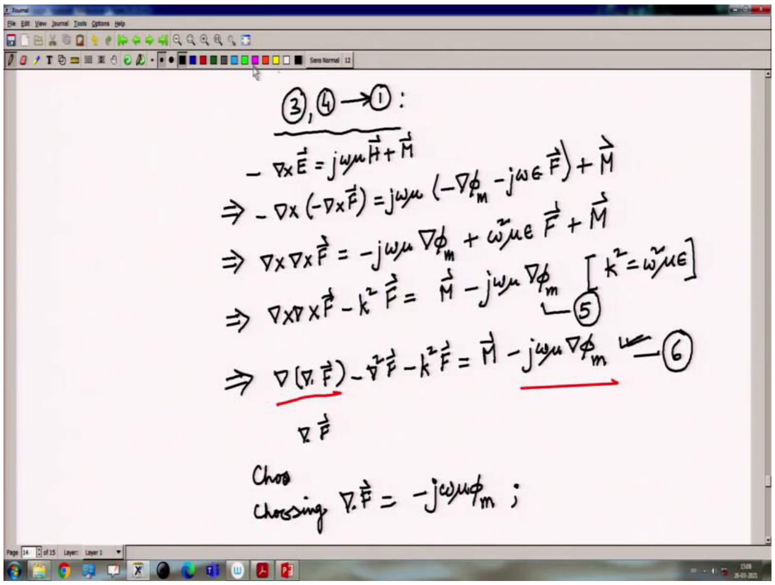

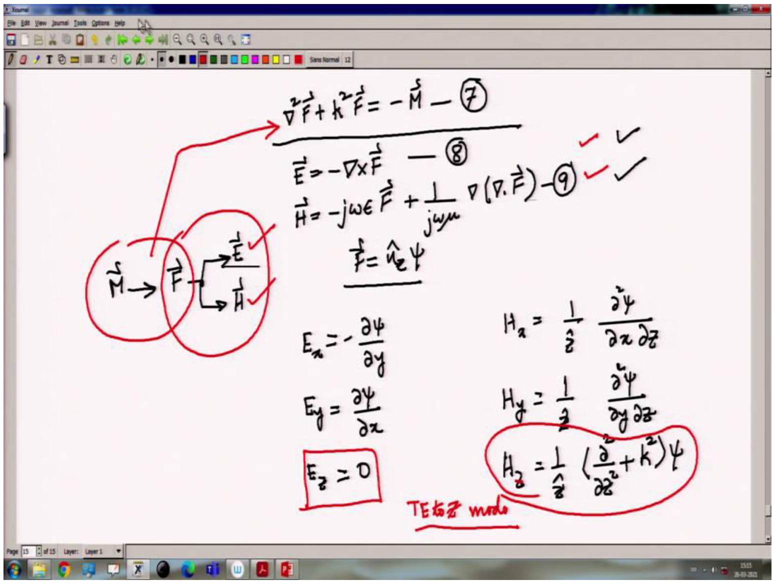

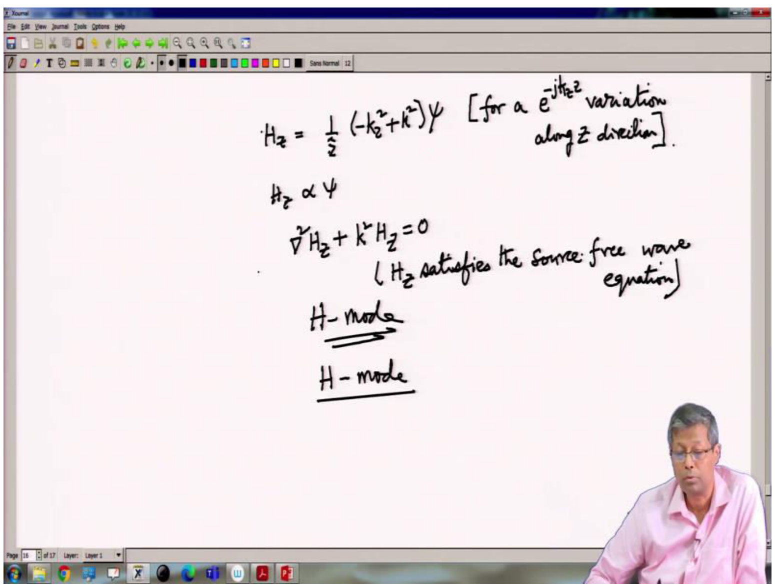

So, 3 and 4 are substituted into equation number 1, so we then obtain minus curl of **E**. we have minus curl of **E** equal to j omega mu **H** plus **M**, which we will now imply, minus curl minus curl of **F** equal to j omega mu minus grad of phi m minus j omega epsilon **F** plus **M**, and that is going to imply curl curl **F**, curl curl **F** equals to minus j omega mu grad phi m plus omega square mu epsilon **F** plus **M**.

And that is going to imply curl curl **F** minus k square **F** is **M** minus j omega mu grad phi m with k square equal to omega square mu epsilon. So, we call this equation number 5. Through the vector identity the same vector identity we used for curl curl **A** for the case of the electric current source, we can write equation 5 as grad of divergence **F** minus grad square **F** minus k square **F** equal to **M** minus j omega mu grad phi m.

Now, curl of **F** was specified by equation number 3, we are yet free to choose divergence of **F.** So, now we will choose divergence of **F** accordingly such that this term and this term neatly cancels out and therefore we choose the divergence of **F** as minus j omega mu phi m choosing this equation which we can call equation 6, simplifies to grad square **F** plus k square **F** equal to minus **M** which is equation 7.

So, this is the Helmholtz equation corresponding to the magnetic current source. So, similar to the case of the electric current source the solutions to these Helmholtz equations can now be easily given as **E** equal to minus curl of **F**, which we already know, and **H** equal to minus j omega epsilon **F** plus 1 by j omega mu grad of divergence of **F**.

So, we call this 8, and we call this 9. So, we have a magnetic current source and from there we have the electric vector potential **F** and from there we have the expressions for the electric and magnetic fields. Like the case of the electric current source, this is a one shot process via equations number 8 and 9, while this process, the computation of **F** through **M** will come by the Helmholtz equations that will be surrounded by the boundary conditions of the problem.

In case the magnetic current source is not given it will be assumed to be a point magnetic current source and thereafter we will compute the electric vector potential **F** by the application of the boundary conditions and therefore the solution to 7 will be obtained. After the solution to 7 is obtained we will find the solutions to **E** and **H** rapidly by the applications of equations number 8 and 9.

In addition, if we choose for instance, **F** is equal to uz psi if you make this choice, we can easily find out the fields from these two equations, the electric and magnetic fields by substituting **F** is equal to uz psi, I am not going to work through this you should be working through this and finding out the value of **E** and **H** which will be dual to the electric current source, which will be very-very similar to the electric current source, but it will be dual to the case of the electric current source.

So, let me write down the expressions of the fields which you should be verifying. So, if **F** equal to uz psi then we have Ex as minus del psi del y, Ey as del psi del x and Ez will be 0, similarly Hx will be 1 by z cap, del square psi del x del z, Hy as 1 by z cap del square psi del y del z and Hz as 1 by z cap del square del z square plus k square psi.

Now, you see that the z-component of the electric field is 0 and thus this will constitute a transverse electric to z mode, this constitutes the transverse electric to z mode. Also, if you look at the expression for Hz, if we assume a variation of e^{-jkz}, we can write down the value for Hz as 1 by z cap minus kz square plus k square psi.

For e^{-jkz} variation along the z direction, and therefore Hz becomes proportional to psi. And because psi satisfies the source free wave equation, Fz will satisfy the source free wave equation through 7, so, psi will satisfy the source free wave equation.

And therefore, because Hz is proportional to psi, Hz will also satisfy the source free wave equation and we will have grad square Hz plus k square Hz as 0. So, Hz satisfies the source free wave equation. As such the TE to z mode is also called the H mode. So, let us stop here, we will be continuing from here to find out the spherical wave components radiated by the magnetic current source, thanks.